

**Original scientific paper**

## **BERGSTRÖM-BOYCE VS. HYPERELASTIC RUBBER MODELS IN STRUCTURAL ANALYSIS OF TIRES**

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**Abstract.** *New viscoelastic/viscoplastic material models, such as the Bergström-Boyce (BB) model, are known to bring advantages in finite element analysis (FEA) of rubber-based components. To test if the same is true in FEA of tires, a study was performed in which the hyperelastic Yeoh model, as well as the BB and dynamic Bergström-Boyce (DBB) models were used to characterize the tread of an existing 205/65 R16 tire. Curve fitting results for all material models as well as the results of the footprint and steady state rolling tire analyses are presented in the paper. There are notable differences between the obtained results at higher strain rates, when viscoelastic material behavior is dominant.*

**Key Words:** *Tire, Viscoelastic/viscoplastic, Material Models, Structural Analysis, Curve Fitting, Finite Element Analysis*

### 1. INTRODUCTION

The accuracy of the results of the tire analysis based on FEM, as well as the results of the analysis of any product made partly or wholly of rubber, largely depends on the material model used to describe the mechanical behavior of its structural components [1, 2]. Those components can be either entirely made of rubber or rubber-based composites. If a rubber-based composite is modeled using rebar elements, its rubber matrix is modeled using the same material models that are used for modeling of rubber components [3].

In numerous research studies, the mechanical behavior of the tire was modeled as time independent, nonlinear elastic [4]. Thereby, hyperelastic material models [5] were used, which neglect the viscoelastic characteristics of the rubber that are reflected in phenomena such as hysteresis, relaxation or creep. However, these characteristics may be very pronounced in various types of rubber with fillers that are normally used in tire design. Thus, it may be expected that the use of more advanced material models, which account for time-dependent

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rubber behavior, may lead to more accurate modeling of mechanical behavior of rubber, especially when phenomena like rolling resistance or thermal dissipation are modeled.

We performed an extensive curve-fitting study, involving a number of hyperelastic and viscoelastic material models applied to different sets of experimental data on filled rubbers. Due to a large volume of data, its complete results will be published separately, and only the most relevant subset is presented here. Material models considered in curve fitting were:

- Yeoh (Y) [6, 7]
- Bergström-Boyce (BB) [8, 9]
- Dynamic Bergström-Boyce (DBB) [10]

The first two models are amongst the most popular ones used in tire design, while BB and DBB represent new viscoelastic/viscoplastic material models that are expected to have a higher accuracy of prediction than hyperelastic ones. Prediction accuracy of the Bergström-Boyce model was assessed in several studies [11-14]. Ghoreishy et al. [15] studied the effect of the use of different carbon black and their blends on the time-dependent mechanical behavior of a typical rubber compounds used in the tread of passenger car tire as well as the accuracy of prediction of the Bergström-Boyce material model, where the equilibrium response was modeled by the Yeoh model. They have performed tests according to the ASTM: D412-C and ISO7743 and compared the test results with simulation results. Only at higher strains there was a slight deviation of predicted behavior from the experimentally determined one. They explained this deviation as a consequence of debonding phenomenon between polymer-filler and filler-filler that occur at higher strains which change the structure of the material. Since the Bergström-Boyce model does not take the interaction between polymer networks and fillers into consideration, the accuracy of the model decreased. In another study by Ghoreishy et al. [16] it was shown that Bergström-Boyce was quite capable of predicting even the mechanical behavior of the tire tread compounds reinforced by silica and carbon black at both low and high strain values.

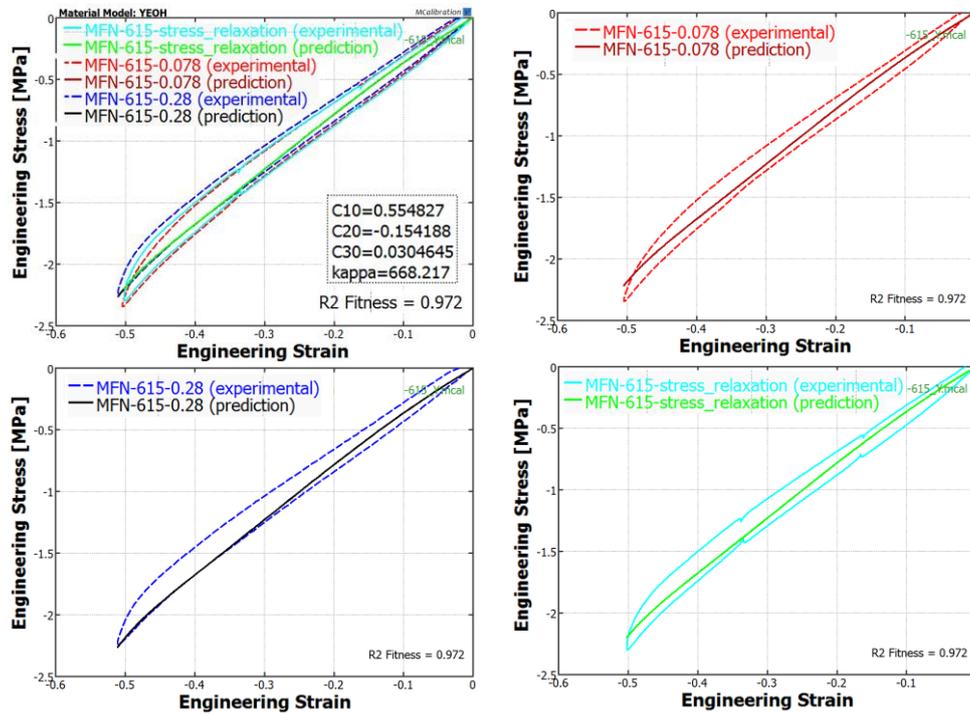
In this paper, the accent is put on a study in which the tire tread was modeled using the three mentioned rubber models while tire behavior during vertical loading, acceleration and braking was simulated by means of FEA [17, 18].

## 2. CURVE FITTING

Experimental data was obtained by uniaxial compression testing of the rubber mixture TG-615, manufactured by “TIGAR technical rubber” Serbia, which has mechanical characteristic similar to the ones of tire tread. It is characterized by Shore hardness value of 60 and its base is a pure natural rubber. The mixture is reinforced by moderate amount of carbon black particles. Testing was performed through loading-unloading cycle of uniaxial compression at strain rates -0.078, -0.28 and stress relaxation. Curve fitting was performed using MCalibration software. The obtained material parameters were transferred to FEA software ABAQUS for further analysis, via specialized software PolyUMod. Mechanical behavior was predicted for a full load-unload cycle.

The comparison of experimental data and behavior of rubber specimen predicted by Yeoh model is shown in Fig. 1. Due to the mentioned limitations of hyperelastic models, unloading behavior could not be predicted. The average  $R^2$  value for all tests was 0.972.

The Yeoh model fit tends towards the unloading curve at low strains and towards the loading curve for higher strains.



**Fig. 1** Comparisons of experimental data and behavior predicted by Yeoh model

The comparison of experimental data and behavior of rubber specimen predicted by the Bergström-Boyce model is shown in Fig. 2. Average  $R^2$  value for all tests was 0.895. The prediction was unrealistic at strains greater than 0.4. The predicted strain rate dependence and hysteresis are less pronounced than in the experimental data set, while the stress relaxation prediction is in agreement with the experimental data set. Despite the differences between the predicted and the experimental behavior, the obtained model parameters can be used for further simulations with confidence, especially in the lower strain region important in tire analysis (20-40%), as the average error is slightly above 10%.

The comparison of experimental data and behavior of rubber specimen predicted by the Dynamic Bergström-Boyce model is shown in Fig. 3. Average  $R^2$  value for all tests was 0.997, which indicates a perfect fit of predicted and experimental data sets. Moreover, strain rate dependence, stress relaxation and value of dissipated energy (hysteresis) are also predicted with a very high accuracy and the Dynamic Bergström-Boyce model achieves almost a perfect fit to test data.

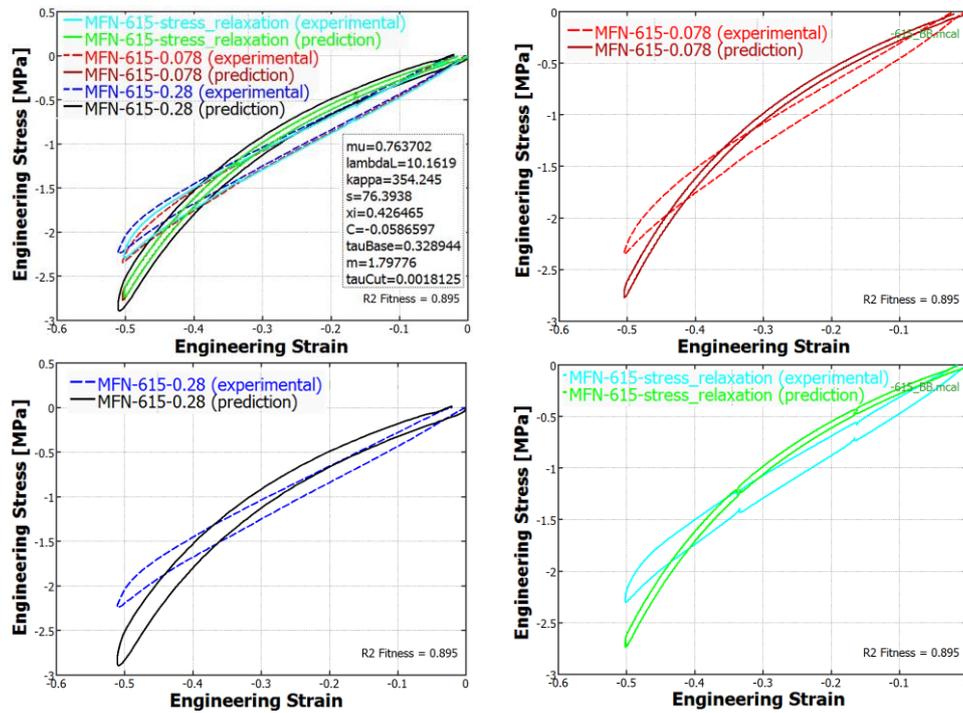


Fig. 2 Experimental data vs prediction by the Bergstöm-Boyce model

### 3. TRIAL SIMULATIONS IN ABAQUS USING VISCOELASTIC/VISCOPLASTIC RUBBER MATERIAL MODELS

The curve-fitting study described in previous chapter had shown clear advantages of viscoelastic/viscoplastic material models in FEA of rubber-based components. Thus, it was expected that these would also bring the advantage in tire FEA. A comprehensive study was performed in which several rubber models were used to perform the footprint and steady state rolling analyses of an existing 205/65 R16 tire.

All the analyses in the study were conducted using simplified tread tire FEM model [17, 19] with rather coarse mesh, as a very large number of analyses had to be run. All rubber materials except tire tread were defined as hyperelastic, Yeoh. Tread material was defined by curve fitting in MCalibration to TG-615 mixture experimental data, using four selected material models and analysis options shown in **Table 1**.

The first subset of analyses was based on viscoelastic FEA procedures in which the surface was moved 30 mm towards the tire axis and back to starting position, with analysis times varying from 0.001 to 10000 seconds. When analysis time was set to 0.001 sec, the tire was moving relative to the surface at the velocity of  $60 / 0.001 = 60000$  mm/sec, i.e. 216 km/h. Analysis time of 0.01 sec corresponded to tire velocity of 21.6km/h, 0.1 sec to 2.16 km/h, etc. Those speeds were expected to cover the range of velocities that may be expected in tire exploitation. To capture the full range of viscoelastic rubber behavior, smaller velocities were also considered. Finally, to approximate the strain rate in tire during

steady state rolling analyses, the velocities of 50 and 80 km/h were also taken into account, with rather coarse mesh, as a very large number of analyses had to be run. All rubber materials except tire tread were defined as hyperelastic, Yeoh. Tread material was defined by curve fitting in MCalibration to TG-615 mixture experimental data, using four selected material models and analysis options shown in Table 1.

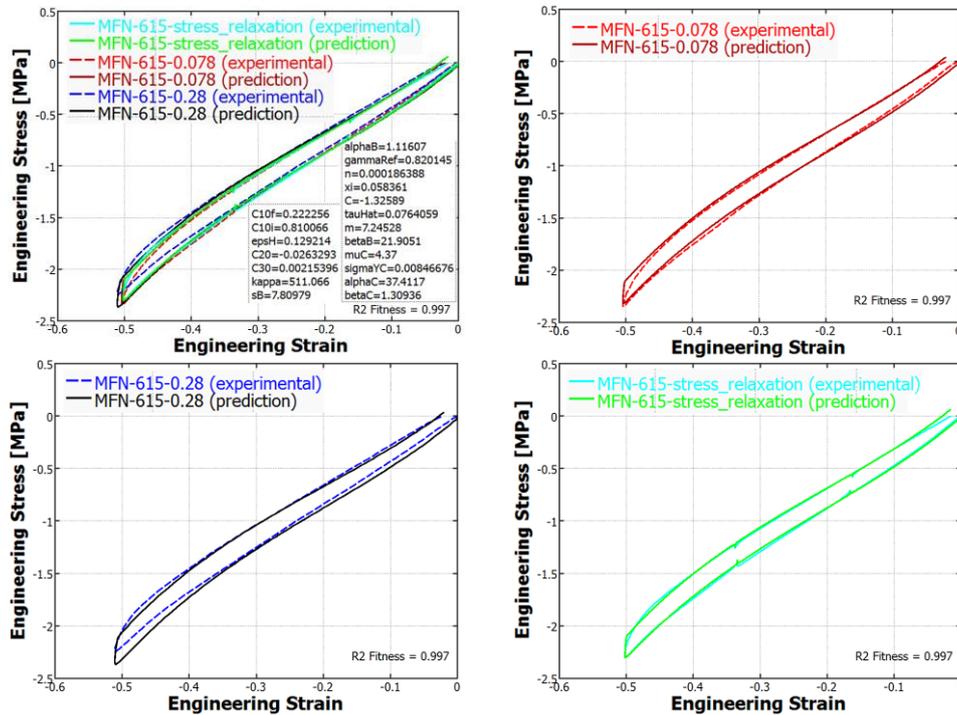


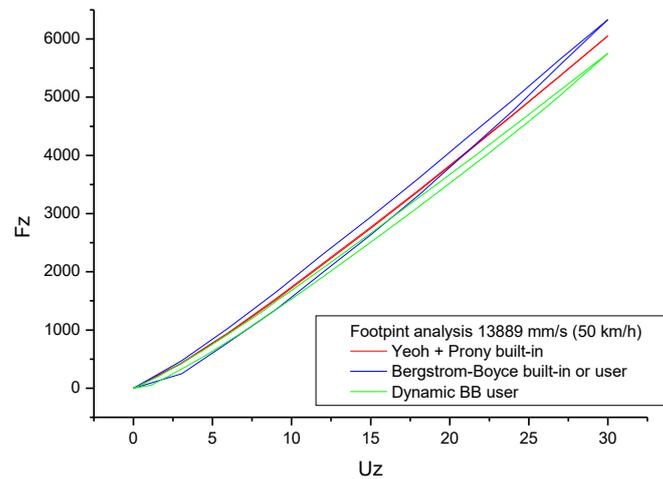
Fig. 3 Comparisons of experimental data and behavior predicted by Dynamic BB model

Table 1 Material models used in viscoelastic/viscoplastic tire material modeling study

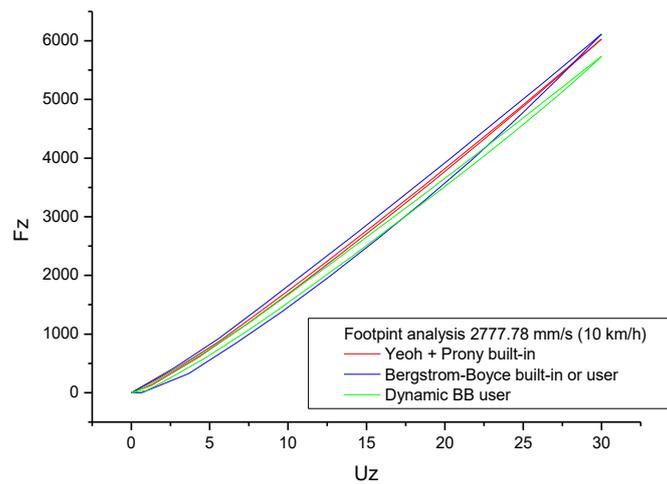
No.	Material model	Viscoelastic material behavior definition	Material model parameters	Abaqus analysis procedure
1.	Yeoh	*VISCOELASTIC, Prony series (3 terms)	MCalibration	Built-In
2.	Bergstrom - Boyce	*HYSTERESIS	MCalibration	Built-In
3.	Bergstrom - Boyce	User	MCalibration	User subroutine, PolyUMod library
4.	Dynamic Bergstrom - Boyce	User	MCalibration	User subroutine, PolyUMod library

The next set of figures (Fig. 4 - 7) brings the comparison of footprint results obtained using the Yeoh material model with Prony series (Y+P), the Bergstrom-Boyce (BB) model (built-in or user) and the Dynamic Bergstrom-Boyce (DBB) model for several values of tire axle velocity (relative and normal to the ground). On one hand, it may be

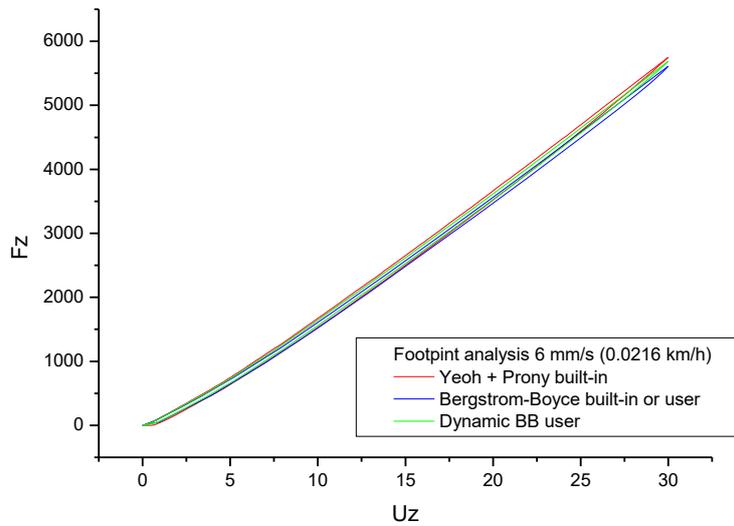
seen that the difference between predictions by various models gets smaller with decrease of speed. On the other, the BB model predicts the largest difference in sensitivity of load-deflection curves to strain rate, as well as the largest hysteresis. The Y+P model curve is constantly above the DBB curve except for very small velocities, while the BB curve is the highest of all for large velocities and the lowest for lowest velocities. The Yeoh model constantly predicts very small hysteresis, while the BB model predicts very large hysteresis for velocities between 13889 and 600 mm/s. All models converge at very small velocities, with hysteresis becoming minor or negligible.



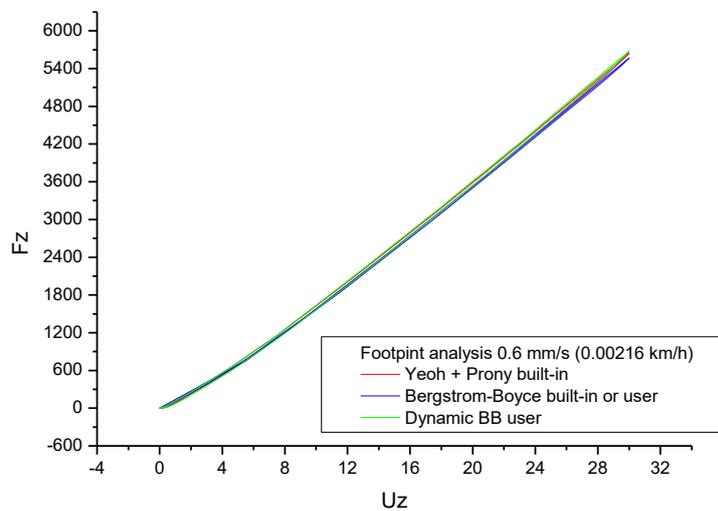
**Fig. 4** Loading-unloading curve obtained by footprint analysis, at tire axle velocity of 13889 mm/s (50 km/h) relative to the surface, loading-unloading time 0.00432 s



**Fig. 5** Loading-unloading curve obtained by footprint analysis, at tire axle velocity of 2777.78 mm/s (10 km/h) relative to the surface, loading-unloading time 0.0216 s

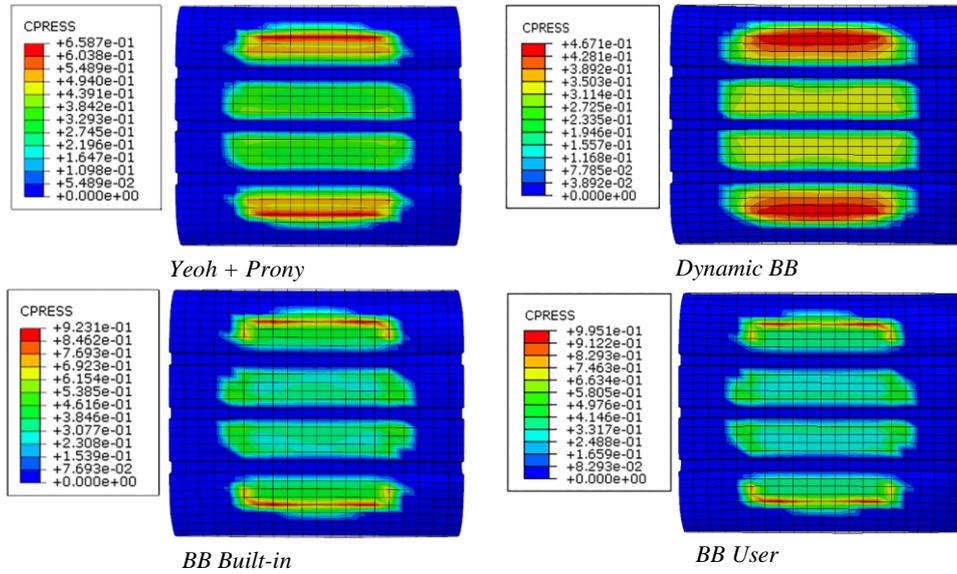


**Fig. 6** Loading-unloading curve obtained by footprint analysis, at tire axle velocity of 6 mm/s (0.0216 km/h), loading-unloading time 10 s

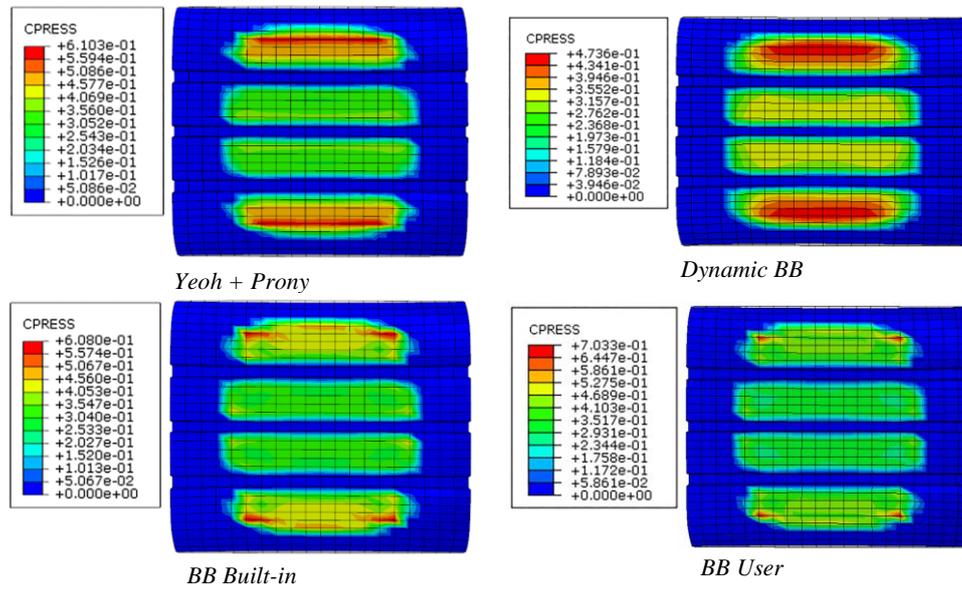


**Fig. 7** Loading-unloading curve obtained by footprint analysis, at tire axle velocity of 0.6 mm/s (0.00216 km/h), loading-unloading time 100 s

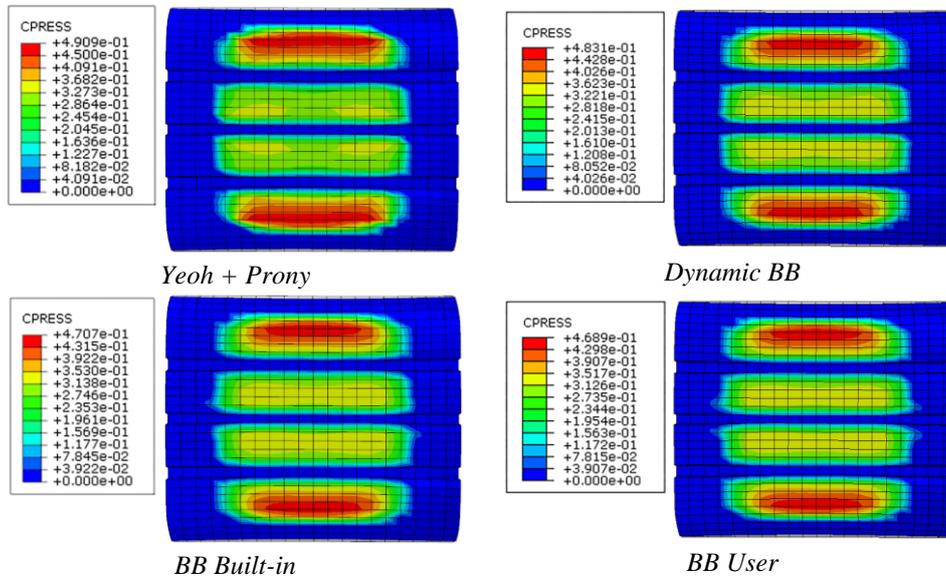
A series of figures showing contact stress at the footprint at deflection of 30 mm, obtained using different material models and different strain rates, is presented next (Fig. 8 - Fig. 10). The differences in predicted stress at high strain rates are very obvious. The BB model predicts the largest differences and the contact pressure it predicts at high strain rates is very uneven. On the other hand, the Dynamic BB was quite insensitive to strain rate change. Behavior of the Y+P model was something in between those two.



**Fig. 8** Contact stress at the footprint, at deflection of 30mm, and tire axle velocity of 6000 mm/s (21.6 km/h), corresponding to loading-unloading time of 0.01 s

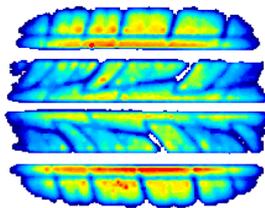


**Fig. 9** Contact stress at the footprint, at deflection of 30mm, and tire axle velocity of 600 mm/s (2.16 km/h), corresponding to loading-unloading time of 0.1 s



**Fig. 10** Contact stress at the footprint, at deflection of 30mm, and tire axle velocity of 6 mm/s (0.0216 km/h), corresponding to loading-unloading time of 10 s

At this moment, there are no experimental results related to contact pressures of modeled tire at different speeds that can be compared with numerically obtained data. Experimentally obtained contact pressure distribution at the footprint, for a similar, statically loaded, 175/70 R13 tire is shown in Fig. 11. Although there exists a significant difference in local contact pressures between experimental and numerical results, which is a consequence of the use of the simplified model, global predictions are generally similar, as largest values of contact pressure are located close to tire shoulders. In order to make a relevant comparison of the results, experimental data should be obtained for the 205/65 R16 tire moving towards the axle or rolling at different rolling speeds and compared to numerical results obtained using detailed tread tire models.

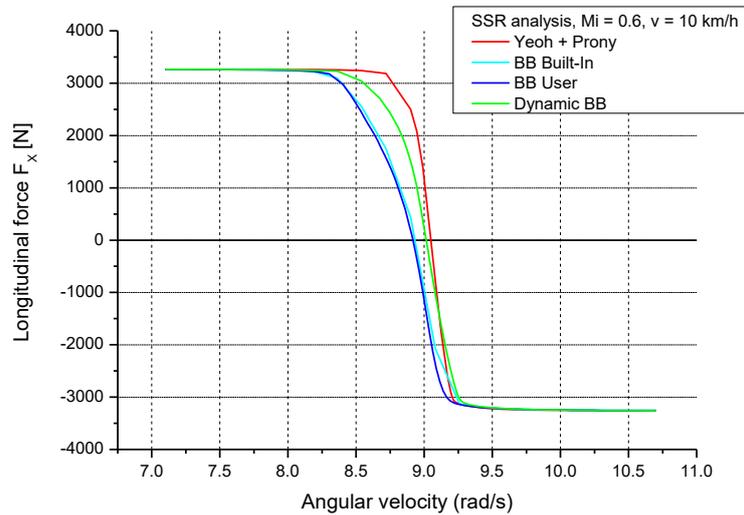


**Fig. 11** Contact pressure at the footprint of a 175/70 R13 tire obtained using XSENSOR tire sensors

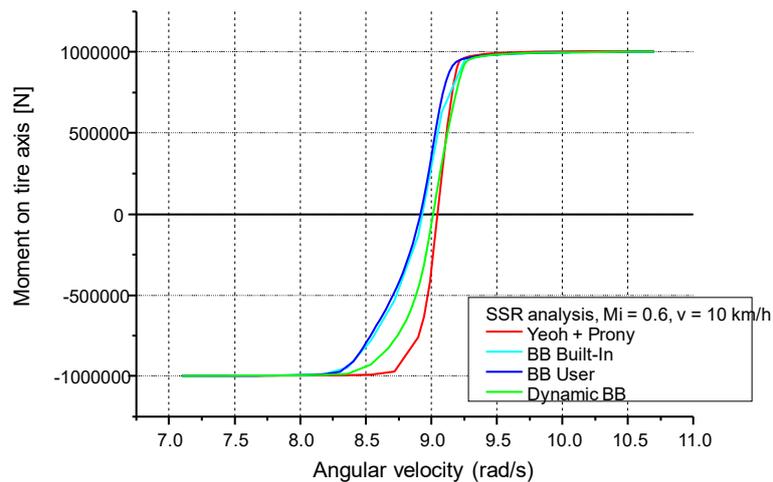
Although it is stated in the Abaqus documentation that hysteresis part of the Built-in Bergstrom-Boyce material model is not active in steady state analysis, results obtained by

this procedure and various material models are shown in next section. Inactivity of hysteresis in SSR means that only hyperelastic part of material model is active, which is a notable limitation concerning tire analysis. It should be examined if this is the case with user subroutine-based BB model and DBB model.

Fig. 12 and Fig. 13 show the dependence of longitudinal force and moment on the axle obtained by steady state rolling analysis. The responses obtained using different models are notably diverse, even in cases when only the hyperelastic part is active.

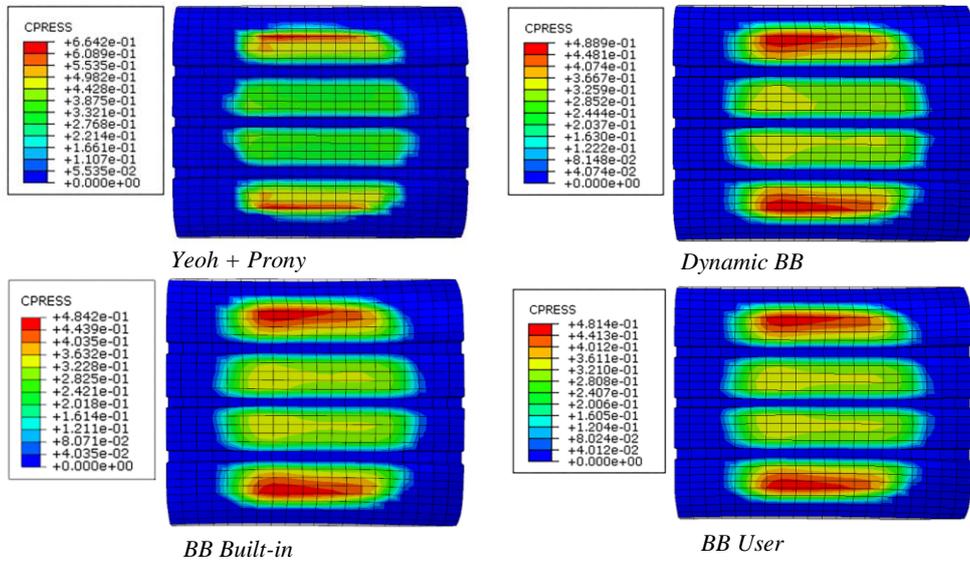


**Fig. 12** Longitudinal force obtained by steady state rolling analysis at 10km/h using different material models

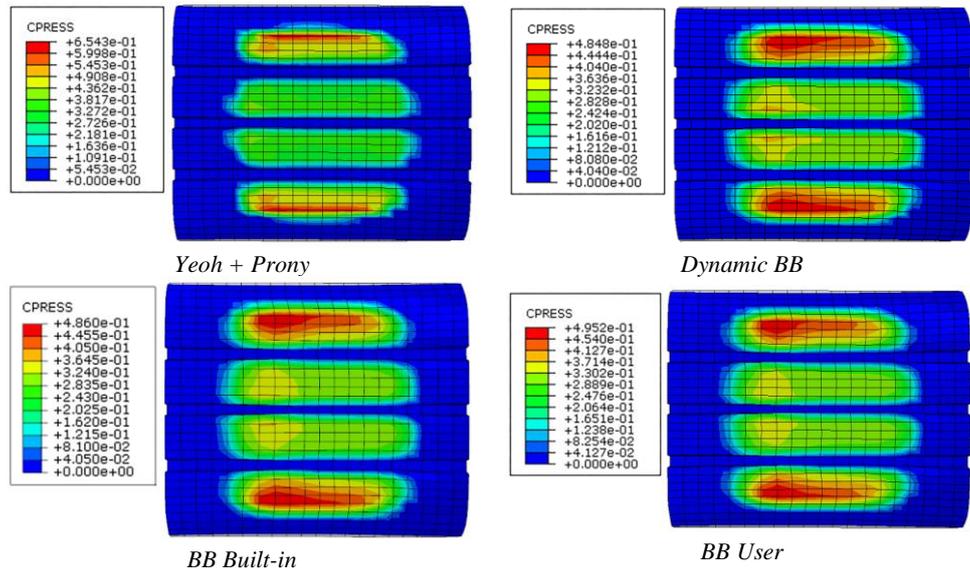


**Fig. 13** Moment on tire axis obtained by steady state rolling analysis at 10km/h using different material models

Finally, Fig. 14 and 15 bring comparison of footprint stress at various moments of braking-to-acceleration analysis, obtained using different material models.



**Fig. 14** Contact stress at the footprint at full braking, 50 km/h, obtained using different models



**Fig. 15** Contact stress at the footprint at 40% of braking-to-acceleration step, 50 km/h, obtained using different models

#### 4. CONCLUSION

Structural analyses of a vertically loaded and rolling tire were performed, in which the selected viscoelastic/viscoplastic and hyperelastic rubber material models were used to model the tread. The results of the analyses were presented and compared. Significant differences in the results obtained by various models were demonstrated, especially at higher strain rates when the contribution of viscoelastic part of material models was the greatest. At lower strain rates, all material models except the Yeoh + Prony yielded similar results. At higher strain rates, the results obtained using the Yeoh + Prony and the DBB models were similar, while the results obtained using the BB model were notably different.

The results of the current study are considered as giving a general insight into behavior of viscoelastic/viscoplastic models in FEA of tires. Further studies, in which all rubber materials included in tire structure would be accurately described using viscoelastic/viscoplastic material models, should give a clearer picture of the possible advantages/disadvantages of viscoelastic/viscoplastic models. Numerical results ought to be compared with experimental ones, obtained at various tire velocities, to establish the reliable conclusions concerning the use of new material models.

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