

## ASSESSMENT OF VELOCITY ACCURACY OF AIRCRAFT IN THE DYNAMIC TESTS USING GNSS SENSORS

**Kamil Krasuski<sup>1</sup>, Adam Ciećko<sup>2</sup>, Grzegorz Grunwald<sup>2</sup>,  
Damian Wierzbicki<sup>3</sup>**

<sup>1</sup>Military University of Aviation, Institute of Navigation, Dęblin, Poland

<sup>2</sup>University of Warmia and Mazury in Olsztyn, Faculty of Geoengineering,  
Institute of Geodesy and Civil Engineering, Olsztyn, Poland

<sup>3</sup>Military University of Technology, Faculty of Civil Engineering and Geodesy,  
Institute of Geospatial Engineering and Geodesy Warsaw, Poland

**Abstract.** *The paper presents a new model for determining the accurate and reliable flight speed of an aircraft based on navigation data from the three independent Global Navigation Satellite System (GNSS) receivers. The GNSS devices were mounted on-board of a Cessna 172 aircraft during a training flight in south-eastern Poland. The speed parameter was determined as the resultant value based on individual components from 3 independent solutions of the motion model. In addition, the standard deviation of the determined flight speed values for the Cessna 172 aircraft was determined in the paper. The resultant on-ground and flight speed of the Cessna 172 aircraft ranged from 0.23 m/s to 74.81 m/s, while the standard deviation of the determined speed values varied from 0.01 m/s to 1.07 m/s. In addition, the accuracy of research method equals to -0.46 m/s to +0.61 m/s, in respect to the RTK-OTF solution. The RMS parameter as an accuracy term amounts to 0.07 m/s for the presented research method.*

**Key Words:** *Aircraft, Velocity, GNSS Receiver, Flight Test, Accuracy*

### 1. INTRODUCTION

The basic parameters of flight dynamics include navigation data, among which the most important are coordinates, altitude, flight speed and orientation angles. The use of GNSS receivers in aviation allows for the determination of the above-mentioned navigation parameters of the aircraft [1, 2, 3]. The aircraft position established on the

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**Corresponding author:** Kamil Krasuski

Military University of Aviation, 08-521 Dęblin, Dywizjonu 303 nr 35 Street, Poland

E-mail: k.krasuski@law.mil.pl

basis of the GNSS technique is determined in accordance with ICAO recommendations using BLh(B-Latitude, L-Longitude, h-ellipsoidal height) ellipsoid coordinates [4]. In turn, the flight speed can be estimated based on the difference of: XYZ geocentric coordinates or ENU (E-Easting, N-Northing, U-Up) local coordinates whereas the HPR (H-Heading, P-Pitch, R-Roll) orientation angles are typically specified in the ENU local coordinates [5]. The determination of aircraft navigation parameters is very important for maintaining and ensuring the continuity of flight mechanics [6, 7, 8]. Therefore, the real-time monitoring of the aircraft flight navigation parameters is of key importance in terms of the safety of flight operations.

The motivation of this work is to determine the flight speed of an aircraft using a GNSS sensor. The undertaken scientific problem has already been presented in many research papers. Cannon et al. [9] presented a mathematical model of numerical simulation for determining the flight speed of an aircraft based on navigation data from two GNSS receivers. Szarmes et al. [10] presented a very interesting solution in which the Doppler effect based on GNSS data was used to determine the flight speed of an aircraft. Krasuski [11] described an extended solution using the Doppler effect. Namely, the GPS code measurements and the Doppler measurement at L1 frequency were used to determine the flight speed of the aircraft. Ćwiklak et al. [12] presented a mathematical model for determining the flight speed of an aircraft based on data from one on-board GPS receiver. In this case, the flight speed of the aircraft was determined in the ENU local coordinates. However, Kozuba and Krasuski [13] proposed the solution of the aircraft flight speed model for the on-board GLONASS receiver in the XYZ geocentric coordinates. A very interesting solution for determining speed was published by He [14], who determined the aircraft flight speed by using two on-board GPS/GLONASS receivers. Salazar [15] presented two models for determining the flight speed of an aircraft with the use of GPS sensors - the Kennedy model and the EVA model. The speed results obtained from both models are convergent for a single on-board GPS receiver. Van Graas and Soloviev [16] presented a mathematical model for estimating the flight speed of an aircraft employing GPS autonomous code positioning of the SPP (Single Point Positioning) mode and code differential DGPS (Differential Global Positioning System) mode. Various researchers presented interesting research results regarding the determination of flight speed based on a multi-sensor solution. Wang et al. [17] and Wu et al. [18] showed the results of aircraft flight speed tests based on a solution from a GPS sensor and an INS sensor, while the flight speed readings of the aircraft from the GPS sensor and Pitot tubes were published and compared by the Foster et al. [19].

As part of the presented work, a new solution for determining the flight speed of an aircraft was presented based on readings from 3 GNSS receivers installed on board the plane. The resultant aircraft flight speed value is determined on the basis of three independent readings. The research experiment used real navigation data from on-board GNSS receivers mounted on-board Cessna 172 aircraft. The proposed solution is innovative in the aspect of improving the navigation indications of aircraft flight mechanics. Presented research considerations were carried out on a large sample of navigation data acquired from three on-board GNSS receivers.

## 2. MATERIALS AND METHODS

In the study real GNSS data collected from on-board receivers, located in the cockpit of Cessna 172 aircraft, were used. The flight took part in south-eastern Poland near Dęblin airport (EPDE). There were 3 GNSS receivers of different brands and configurations on-board the Cessna 172 aircraft, as follows [20]:

- Thales Mobile Mapper Pro receiver using GPS L1 code positioning,
- Thales Mobile Mapper Pro receiver using GPS L1 code augmented with EGNOS,
- Topcon HiPer Pro - a dual-system GNSS receiver using GPS/GLONASS code observations.

All 3 receivers recorded navigation data with an interval of 1 second. Typical accuracy of position determination for examined GNSS solutions was in the range of 1 to 5 m. All GNSS receivers were placed in the cockpit of a Cessna aircraft, very close to each other, as shown in Fig. 1.



**Fig. 1** The on-board GNSS receivers in Cessna 172 aircraft

The location of GNSS sensors in the cockpit allowed the determination of basic flight navigation parameters, including position, time and speed. It should be noted that GNSS receivers did not have any direct impact on the work of other flight instruments. Nor did they disturb the pilot in any way. In flight mechanics, the reliable and accurate recording of flight parameters is crucial. Equipping the aircraft with 3 GNSS sensors enables the verification of aircraft flight parameters in real time. The effective verification can also be made in post-processing mode. For every second of the flight, the position of the aircraft is determined by each GNSS sensor. The three-dimensional position can be given in the form of XYZ geocentric coordinates or BLh ellipsoidal coordinates. On the basis of collected coordinates of the flight position, the components of the flight speed of the aircraft in the XYZ geocentric or ENU topocentric coordinates are determined.

In the first stage of research, individual components of the aircraft flight speed are determined. In the presented work, the components of the Cessna 172 flight speed were calculated based on the XYZ geocentric coordinates for all 3 GNSS receivers independently, as given below:

$$\begin{cases}
V_X^{GPS} = \frac{\Delta X^{GPS}}{\Delta t}; V_Y^{GPS} = \frac{\Delta Y^{GPS}}{\Delta t}; V_Z^{GPS} = \frac{\Delta Z^{GPS}}{\Delta t} \\
V_X^{EGNOS} = \frac{\Delta X^{EGNOS}}{\Delta t}; V_Y^{EGNOS} = \frac{\Delta Y^{EGNOS}}{\Delta t}; V_Z^{EGNOS} = \frac{\Delta Z^{EGNOS}}{\Delta t} \\
V_X^{GPS/GLO} = \frac{\Delta X^{GPS/GLO}}{\Delta t}; V_Y^{GPS/GLO} = \frac{\Delta Y^{GPS/GLO}}{\Delta t}; V_Z^{GPS/GLO} = \frac{\Delta Z^{GPS/GLO}}{\Delta t}
\end{cases} \quad (1)$$

where  $(V_X^{GPS}, V_Y^{GPS}, V_Z^{GPS})$  are flight speed components along the XYZ axes based on readings from the Thales Mobile Mapper Pro receiver (GPS L1 solution),  $(V_X^{EGNOS}, V_Y^{EGNOS}, V_Z^{EGNOS})$  are flight speed components along the XYZ axes based on readings from the Thales Mobile Mapper Pro receiver (GPS L1 + EGNOS solution),  $(V_X^{GPS/GLO}, V_Y^{GPS/GLO}, V_Z^{GPS/GLO})$  are flight speed components along the XYZ axes based on readings from the Topcon Hiper Pro receiver (GPS/GLONASS solution),  $\Delta t$  is an observation interval,  $\Delta t = 1s$ ,  $\Delta X^{GPS} = X_i^{GPS} - X_{i-1}^{GPS}$  is a coordinate increment along the X axis based on readings from the Thales Mobile Mapper Pro receiver (GPS L1 solution) for the  $\Delta t$  time interval,  $\Delta X^{EGNOS} = X_i^{EGNOS} - X_{i-1}^{EGNOS}$  is a coordinate increment along the X axis based on readings from the Thales Mobile Mapper Pro receiver (GPS L1 + EGNOS solution) for the  $\Delta t$  time interval,  $\Delta X^{GPS/GLO} = X_i^{GPS/GLO} - X_{i-1}^{GPS/GLO}$  is a coordinate increment along the X axis based on readings from the Topcon Hiper Pro receiver (GPS/GLONASS solution) for the  $\Delta t$  time interval,  $\Delta Y^{GPS} = Y_i^{GPS} - Y_{i-1}^{GPS}$  is a coordinate increment along the Y axis based on readings from the Thales Mobile Mapper Pro receiver (GPS L1 solution) for the  $\Delta t$  time interval,  $\Delta Y^{EGNOS} = Y_i^{EGNOS} - Y_{i-1}^{EGNOS}$  is a coordinate increment along the Y axis based on readings from the Thales Mobile Mapper Pro receiver (GPS L1 + EGNOS solution) for the  $\Delta t$  time interval,  $\Delta Y^{GPS/GLO} = Y_i^{GPS/GLO} - Y_{i-1}^{GPS/GLO}$  is a coordinate increment along the Y axis based on readings from the Topcon Hiper Pro receiver (GPS/GLONASS solution) for the  $\Delta t$  time interval,  $\Delta Z^{GPS} = Z_i^{GPS} - Z_{i-1}^{GPS}$  is a coordinate increment along the Z axis based on readings from the Thales Mobile Mapper Pro receiver (GPS L1 solution) for the  $\Delta t$  time interval,  $\Delta Z^{EGNOS} = Z_i^{EGNOS} - Z_{i-1}^{EGNOS}$  is a coordinate increment along the Z axis based on readings from the Thales Mobile Mapper Pro receiver (GPS L1 + EGNOS solution) for the  $\Delta t$  time interval,  $\Delta Z^{GPS/GLO} = Z_i^{GPS/GLO} - Z_{i-1}^{GPS/GLO}$  is a coordinate increment along the Y axis based on readings from the Topcon Hiper Pro receiver (GPS/GLONASS solution) for the  $\Delta t$  time interval,  $i$  stands for the current epoch of observation  $\Delta t$  and  $i - 1$  is the previous epoch of observation  $\Delta t$ .

Based on Eq. (1), the resultant flight velocity of the Cessna 172 aircraft was calculated based on the XYZ coordinates for all 3 GNSS receivers independently as presented below:

$$\begin{cases}
V_{total}^{GPS} = \sqrt{(V_X^{GPS})^2 + (V_Y^{GPS})^2 + (V_Z^{GPS})^2} \\
V_{total}^{EGNOS} = \sqrt{(V_X^{EGNOS})^2 + (V_Y^{EGNOS})^2 + (V_Z^{EGNOS})^2} \\
V_{total}^{GPS/GLO} = \sqrt{(V_X^{GPS/GLO})^2 + (V_Y^{GPS/GLO})^2 + (V_Z^{GPS/GLO})^2}
\end{cases} \quad (2)$$

where  $V_{total}^{GPS}$  is the resultant flight speed based on readings from the Thales Mobile Mapper Pro receiver (GPS L1 solution),  $V_{total}^{EGNOS}$  is the resultant flight speed based on readings from the Thales Mobile Mapper Pro receiver (GPS L1 + EGNOS solution) and  $V_{total}^{GPS/GLO}$  stands for the resultant flight speed based on readings from the Topcon Hiper Pro receiver (GPS/GLONASS solution).

In the final step, velocity components of Eq. (1) and the resultant values for each GNSS receiver, Eq. (2), allow for the Cessna 172 plane velocity determination based on the entire GNSS sensor array, as follows:

$$V_{total}^{aircraft} = \frac{V_{total}^{GPS} + V_{total}^{EGNOS} + V_{total}^{GPS/GLO}}{n} \quad (3)$$

where  $V_{total}^{aircraft}$  is the total resulting airspeed of the Cessna 172 and  $n = 3$ .

The parameter  $V_{total}^{GPS}$  means the resultant speed of vessel movement in flight mechanics using GNSS sensors. The standard deviation for  $V_{total}^{GPS}$  parameter is also defined according to:

$$StdV_{total}^{aircraft} = \sqrt{\frac{[d_V d_V]}{n-1}} \quad (4)$$

where  $StdV_{total}^{aircraft}$  is a standard deviation of the total resultant flight speed of the Cessna 172 and  $d_V$  is a correction, difference between parameters  $V_{total}^{aircraft}$  and  $V_{total}^{GPS}$ ,  $V_{total}^{EGNOS}$ ,  $V_{total}^{GPS/GLO}$ , according to:

$$d_V = \begin{bmatrix} V_{total}^{aircraft} - V_{total}^{GPS} \\ V_{total}^{aircraft} - V_{total}^{EGNOS} \\ V_{total}^{aircraft} - V_{total}^{GPS/GLO} \end{bmatrix}$$

### 3. RESULTS

The results of the tests are presented in Section 3. First, the flight velocity components for XYZ axes were determined for 3 GNSS sensors independently, according to Eq. (1). Table 1 presents the results of individual speed components along the XYZ axes based on 3 solutions: GPS L1, GPS L1+EGNOS and GPS/GLONASS. The results show that the minimum flight speed along the X axis based on data from 3 GNSS receivers ranges from -48.79 m/s to -48.72 m/s. Whereas, the maximum flight speed along the X axis based on data from 3 GNSS receivers is from +56.44 m/s to +56.84 m/s. The minimum flight speed along the Y axis based on data from 3 GNSS receivers stretches from -61.83 m/s to -61.44 m/s. While, the maximum flight speed along the Y axis is between +61.25 m/s and +61.34 m/s. The minimum flight speed along the Z axis based on data from 3 GNSS receivers is from -49.03 m/s to -48.91 m/s. whereas, the maximum flight speed along Z-axis is from +41.11 m/s to +41.54 m/s.

**Table 1** Results of aircraft velocity along XYZ axes for each GNSS receiver

GNSS receiver	Minimum range of velocity component along X axis [m/s]	Maximum range of velocity component along X axis [m/s]	Minimum range of velocity component along Y axis [m/s]	Maximum range of velocity component along Y axis [m/s]	Minimum range of velocity component along Z axis [m/s]	Maximum range of velocity component along Z axis [m/s]
Thales Mobile Mapper Pro (GPS L1 solution)	-48.72	+56.44	-61.44	+61.25	-49.03	+41.11
Thales Mobile Mapper Pro (GPS L1 + EGNOS solution)	-48.73	+56.48	-61.45	+61.26	-49.01	+41.14
Topcon Hiper Pro (GPS/GLONASS solution)	-48.79	+56.84	-61.83	+61.34	-48.91	+41.54

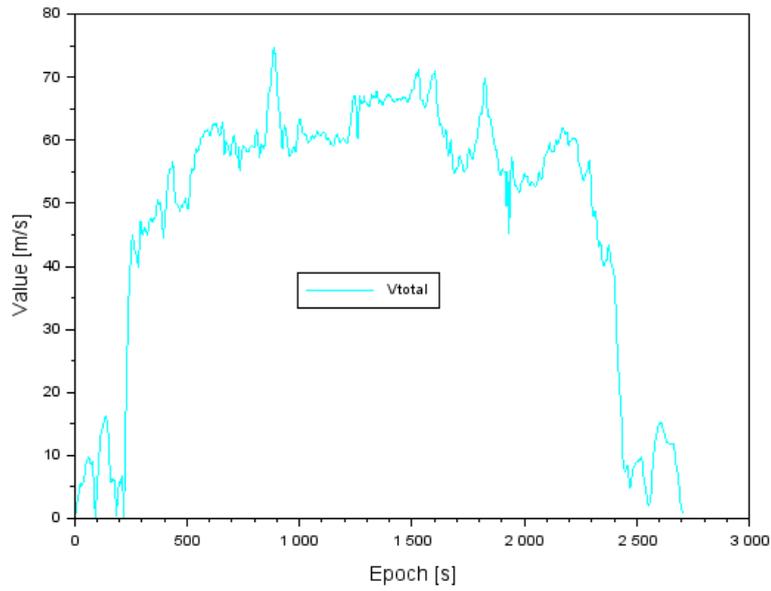
Table 2 displays the resultant flight velocity of the Cessna 172 aircraft based on 3 solutions: GPS L1, GPS L1+EGNOS and GPS/GLONASS, according to Eq. (2). The results show that the minimum resultant flight speed based on the data from 3 GNSS receivers ranges from +0.11 m/s to +0.42 m/s. While, the maximum resultant flight speed based on data from 3 GNSS receivers stretches from +74.50 m/s and +75.56 m/s.

**Table 2** Results of total aircraft velocity for each GNSS receiver

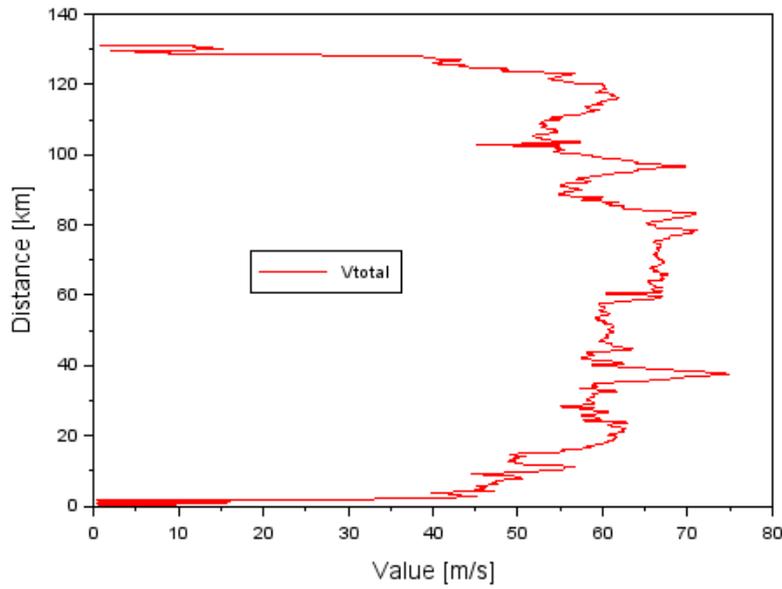
GNSS receiver	Minimum range of velocity [m/s]	Maximum range of velocity [m/s]
Thales Mobile Mapper Pro (GPS L1 solution)	+0.12	+74.51
Thales Mobile Mapper Pro (GPS L1 + EGNOS solution)	+0.11	+74.50
Topcon Hiper Pro (GPS/GLONASS solution)	+0.42	+75.56

Fig. 2 shows the relevant results of the test, i.e. the total resultant velocity for the frame of all GNSS sensors installed in the Cessna 172, according to Eq. (3). Based on the obtained test results, the total flight speed of the frame of 3 GNSS sensors placed on board the Cessna 172 aircraft is between +0.23 m/s and +74.81 m/s. The average flight speed is +48.53 m/s. It can be observed that for the first 200 measurement epochs, the flight speed was less than 20 m/s. Starting from the epoch 250, the flight speed increased to over 40 m/s. The maximum speed can be observed in about the 750 epoch and from the 2400 epoch, the flight speed starts to drop down to around 10 m/s.

Fig. 3 shows the results of the total resultant velocity for a frame of all GNSS sensors installed in a Cessna 172, as a function of the distance travelled by the aircraft. It can be seen that after passing a point of 3 kilometers, the flight speed increases to over 40 m/s. At a distance of around 40 km, the speed rises to a maximum value of about 75 m/s. Up to 130 km of the route, the speed of flight is over 40 m/s, then it starts to fall systematically during approach and landing at the airport.

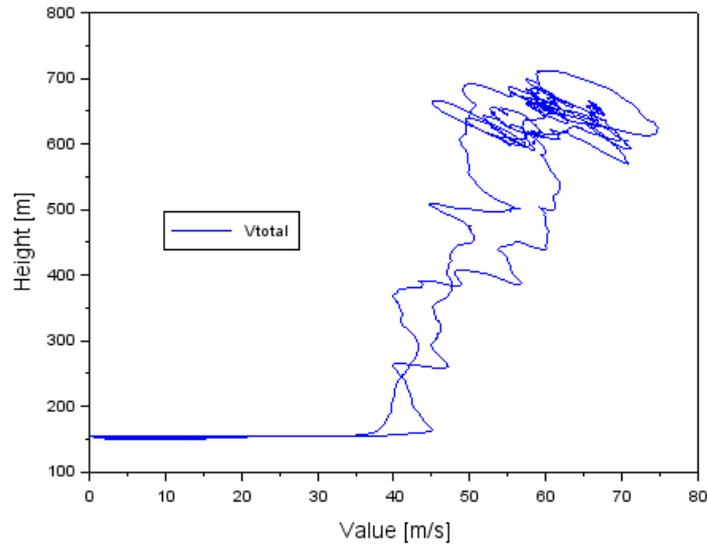


**Fig. 2** The total velocity of GNSS sensor array installed in Cessna 172 aircraft as a function of time



**Fig. 3** The total velocity of GNSS receivers array in Cessna 172 aircraft as a function of distance

Fig. 4 displays the results of the total resultant velocity for a frame of all GNSS sensors installed in the Cessna 172, as a function of aircraft flight altitude. It is worth noting that from an altitude of about 200 m, the flight speed increases to over 40 m/s. The highest flight speed values are visible, with a maximum flight altitude of 600-700 m. At altitudes from 200 m to 500 m, the flight speed ranges from 40 m/s to 60 m/s.



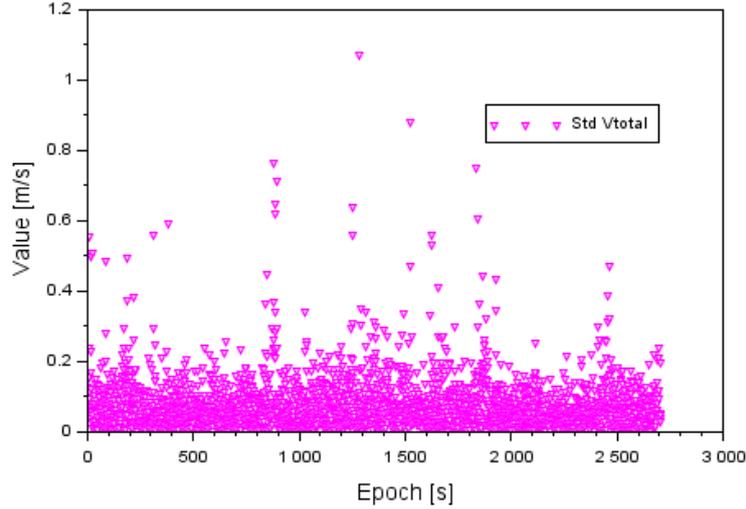
**Fig. 4** The total velocity of GNSS receivers array in Cessna 172 aircraft as a function of flight altitude

**Table 3** Results of  $d_V$  parameter

Parameter	Minimum value [m/s]	Maximum value [m/s]	Mean value [m/s]	Median value [m/s]
$d_V = V_{total}^{aircraft} - V_{total}^{GPS}$ parameter	-0.53	+0.61	+0.01	+0.01
$d_V = V_{total}^{aircraft} - V_{total}^{EGNOS}$ parameter	-0.46	+0.62	+0.01	+0.01
$d_V = V_{total}^{aircraft} - V_{total}^{GPS/GLO}$ parameter	-1.24	+0.88	-0.01	-0.01

Table 3 shows correction values  $d_V$  as the difference between the total speed  $V_{total}^{aircraft}$  and resultant speeds for 3 different receivers:  $V_{total}^{GPS}$ ,  $V_{total}^{EGNOS}$ ,  $V_{total}^{GPS/GLO}$  (see Eq. (4)). From the results obtained it can be concluded that the dispersion of the parameter results ranges from -1.24 m/s to +0.88 m/s. It should be noted that the nature of the parameter resembles a white noise model whose mean values are close to 0. In the example under consideration, the mean value of the parameter is  $\pm 0.01$  m/s.

Fig. 5 presents the standard deviation results for the total resultant velocity for the frame of all GNSS sensors installed in the Cessna 172, according to Eq. (4). The value of the standard deviation for all measurement epochs ranges from 0.01 m/s to 1.07 m/s. In addition, the mean and median of the  $StdV_{total}^{aircraft}$  parameter equals 0.08 m/s and 0.06 m/s respectively. In about 74% cases the  $StdV_{total}^{aircraft}$  parameter is less than 0.1 m/sec whereas in over 94% cases the  $StdV_{total}^{aircraft}$  parameter is less than 0.2 m/s.



**Fig. 5** The standard deviation of total velocity of GNSS receivers array in Cessna 172 aircraft

#### 4. DISCUSSION

As part of the discussion, the results obtained from the proposed research method were verified. For this purpose, the results of  $V_{total}^{aircraft}$  parameter were compared with the flight reference speed determined by the RTK-OTF differential technique. The reference flight  $V_{RTK}^{ref}$  speed was determined on the basis of precise GPS phase observations using the RTK-OTF differential technique [21]. First, the reference position of the aircraft from the RTK-OTF solution was determined, and then the reference speed of the Cessna 172 flight, according to the formula:

$$\{V_{RTK}^{ref} = \sqrt{(V_X^{RTK})^2 + (V_Y^{RTK})^2 + (V_Z^{RTK})^2} \quad (5)$$

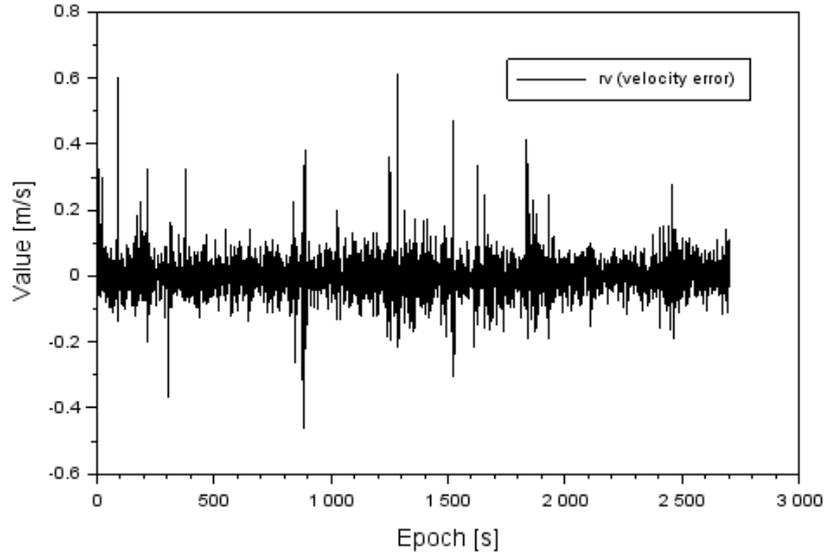
where  $V_{RTK}^{ref}$  is the reference value of  $v$  of aircraft based on RTK-OTF solution while  $(V_X^{RTK}, V_Y^{RTK}, V_Z^{RTK})$  are the flight speed components along the XYZ axes based on RTK-OTF solution.

Reference value speed  $V_{RTK}^{ref}$  was from 0.23 m/s to 74.81 m/s for the entire measuring cycle. Comparison of parameter  $V_{total}^{aircraft}$  results and  $V_{RTK}^{ref}$  enables the determination of speed errors and, additionally, determines the accuracy of the presented test method. Resultant speed errors  $V_{total}^{aircraft}$  are defined as follows:

$$r_V = V_{total}^{aircraft} - V_{RTK}^{ref} \quad (6)$$

where  $r_V$  is the velocity error.

Fig. 6 shows the results of speed errors  $r_V$  as a function of observation time. Speed error values are between -0.46 m/s and +0.61 m/s. The average value of speed error is -0.01 m/s; therefore, the nature of the changes in the  $r_V$  parameter resembles white noise. It is worth noting that over 91% of the obtained  $r_V$  parameter values are within  $\pm 0.1$  m/s.



**Fig. 6** The values of velocity error  $r_V$  for presented research method

A statistical measure of accuracy in the form of an RMS mean square error [22] was also determined for the  $r_V$  parameter, as follows:

$$RMS = \sqrt{\frac{[r_V^2]}{N}} \quad (7)$$

where  $RMS$  is accuracy and  $N$  is a number of measurement epochs.

The RMS error value for the analyzed aviation test is 0.07 m/s. Therefore, it can be concluded that, for the presented research method, high accuracy was achieved. As a part of the discussion, the resultant aircraft flight speed was also determined based on the readings from 3 GNSS receivers as a weighted average mathematical model as below:

$$V_{total}^{aircraft} = \frac{p_{GPS} \cdot V_{total}^{GPS} + p_{EGNOS} \cdot V_{total}^{EGNOS} + p_{GPS/GLO} \cdot V_{total}^{GPS/GLO}}{p_{GPS} + p_{EGNOS} + p_{GPS/GLO}} \quad (8)$$

where  $p_{GPS}$  stands for a speed weight using the GPS solution,  $p_{EGNOS}$  is a speed weight using the EGNOS solution and  $p_{GPS/GLO}$  is a speed weight using the GPS/GLONASS solution.

The implemented calculation scheme assumes that the speed weights are respectively:

$$\begin{cases} p_{GPS} = \frac{1}{n_{S_{GPS}}} \\ p_{EGNOS} = \frac{1}{n_{S_{EGNOS}}} \\ p_{GPS/GLO} = \frac{1}{n_{S_{GPS/GLO}}} \end{cases} \quad (9)$$

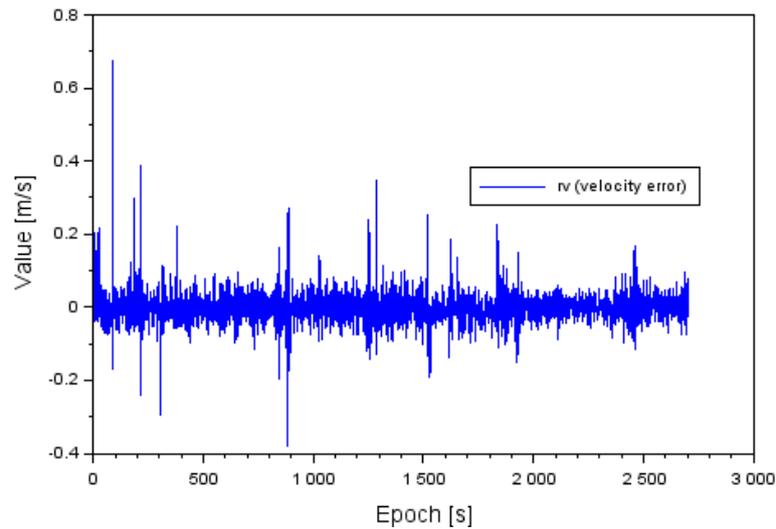
where  $n_{S_{GPS}}$  is a number of GPS satellites,  $n_{S_{EGNOS}}$  is a number of GPS+EGNOS satellites and  $n_{S_{GPS/GLO}}$  stands for a number of GPS/GLONASS satellites.

Therefore, it can be concluded that weighting according to Eq. (8) takes place as a function of the number of tracked GNSS satellites, which were used in the calculation process of determining the aircraft XYZ coordinates. Table 4 shows the results of determining the resultant aircraft flight speed based on the mathematical model (3) and (8). It can be stated that the results of the flight speed on the basis of both test methods are close to within  $\pm 0.03$  m/s to  $\pm 0.08$  m/s. Therefore, the resultant speed performance based on Eq. (8) including the weighing process is similar to the results of the speed calculated as the arithmetic average for 3 GNSS receivers.

**Table 4** Comparison results of the total velocity of aircraft based on Eqs. (3) and (8)

Total velocity of aircraft	Minimum range of velocity [m/s]	Maximum range of velocity [m/s]
Velocity model based on Eq. (3)	+0.23	+74.81
Velocity model based on Eq. (8)	+0.20	+74.73

In the last stage of the discussion, the accuracy of the research method was determined from Eq. (8) based on Eqs. (6) and (7). Fig. 7 shows aircraft flight speed errors calculated as the difference between the weighted average speed and  $V_{RTK}^{ref}$  parameter. Speed error values



**Fig. 7** The values of velocity error as a difference between weighted average of velocity and the RTK-OTF solution

take results from  $-0.38$  m/s to  $+0.67$  m/s. It is worth noting that over 97% of the obtained  $r_V$  parameter results are within  $\pm 0.1$  m/s. In addition, the RMS error is less than 0.05 m/s. Therefore, the accuracy of this method is relatively high.

## 5. CONCLUSIONS

The paper presents the results of the flight speed test of the Cessna 172 aircraft during a training flight at Dęblin airport (EPDE) in south-eastern Poland. Until now, in the investigated research paper, the readings from a single GNSS receiver or another measuring sensor have been used to determine the aircraft speed. In the analyzed example, the authors of the paper decided to use real data from 3 GNSS receivers placed in the cockpit to determine the flight speed of the Cessna 172 aircraft. In the mathematical model of speed, individual components were determined and finally the resultant value was calculated for the whole measurement frame of 3 GNSS sensors. The presented research method has its advantages because it is based on a multi-receiver GNSS solution, making the result independent of on-board avionics and most importantly - it gives pilots additional information and navigation data concerning aircraft flight mechanics in real time. Therefore, the presented research method can be applicable in the area of flight technology of manned and unmanned aircraft.

The obtained research results show that the flight speed of the Cessna 172 aircraft for the entire GNSS sensor frame was from  $+0.23$  m/s to  $+74.81$  m/s. In the work the flight speed parameter was determined as a function of time, distance travelled by the plane and flight altitude. In turn, the standard deviation parameter was also determined for the resultant flight speed value. The standard deviation value for all measurement epochs ranges from 0.01 m/s to 1.07 m/s.

The paper analyzes the accuracy of the research method presented. The aircraft flight speed errors were determined as between  $-0.46$  m/s and  $+0.61$  m/s. In addition, an RMS error was determined, whose value is 0.07 m/s. The article also presents a model for determining the weighted average flight speed parameter. The accuracy of determining the speed from the weighted average model in relation to the RTK-OTF solution is from  $-0.38$  m/s to  $+0.67$  m/s, while the RMS error is less than 0.05 m/s. In future, the authors plan to develop their scientific research on the use of GNSS sensors to determine the flight speed of aircraft. It should be noted that the authors intend to use other methods or systems to determine the flight speed of aircraft. It is planned to use the Doppler effect and use the INS system to determine the flight speed of the aircraft. The combination of several measurement methods or systems can be very useful in determining the resultant aircraft flight speed. Research tests are in the experimental phase.

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