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# PARAMETRIC STUDY OF A CNC TURNING PROCESS USING DISCRIMINANT ANALYSIS

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**Abstract**. *In the present day manufacturing scenario, computer numerical control (CNC) technology has evolved out as a cost effective process to perform repetitive, difficult and unsafe machining tasks while fulfilling the dynamic requirements of high dimensional accuracy and low surface finish. Adoption of CNC technology would help an organization in achieving enhanced productivity, better product quality and higher flexibility. In this paper, an endeavor is put forward to apply discriminant analysis as a multivariate statistical tool to investigate the effects of speed, feed, depth of cut, nose radius and type of the machining environment of a CNC turning center on surface roughness, tool life, cutting force and power consumption. Simultaneous discrimination analysis develops the corresponding discriminant function for each of the responses taking into account all the input parameters together. On the contrary, step-wise discriminant analysis develops the same functions while considering only those significant input parameters influencing the responses. Higher values of hit ratio and cross-validation percentage prove the application of both the discriminant functions as effective prediction tools for achieving enhanced performance of the considered CNC turning operation.*

**Key Words**: *CNC Turning, Discriminant Analysis, Process Parameter, Response, Hit Ratio, Cross-validation*

## 1. INTRODUCTION

In manufacturing and metalworking industries, turning is the most basic material removal process where a single-point wedge-shaped cutting tool is employed to remove material from the surface of a rotating cylindrical workpiece. The cutting tool is advanced linearly in a direction parallel to the axis of rotation of the workpiece [1]. Turning is an extremely precise process that can attain a surface finish of 0.5-1 μm [2]. The turning center or lathe provides the power for turning the workpiece at a given rotational speed, and feeding the cutting tool at a specified rate and depth of cut, facilitating material

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removal in the form of chips [3, 4]. In order to cope up with the present-day requirements of high productivity and low production cost with enhanced product quality, conventional multi-spindle lathes are now being gradually substituted by the high performance computer numerical control (CNC) machine tools due to their ease of setting, operation, repeatability and accuracy. In CNC machining technology, there is an automated control of machine tools through dedicated instructions stored in memory to machine complex workpieces to fulfill the requirements of higher dimensional accuracy and better surface finish under the occasional supervision of an operator. Its various advantageous features, like program storage and editing facility, ability to store multi-part programs, tool offset and compensation, ability to send and receive data from a variety of sources etc. have made the CNC technology an almost indispensible tool in the present-day highly competitive manufacturing environment [5]. A schematic diagram illustrating the CNC turning process is shown in Fig. 1.

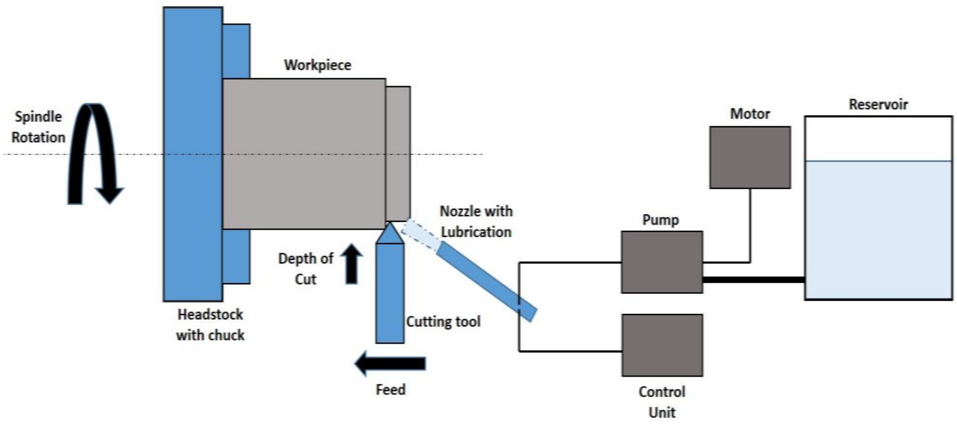


**Fig. 1** Schematic representation of CNC turning process

It has been observed that the machining performance of a CNC turning center with respect to material removal rate (MRR), surface roughness (SR), tool life (TL), cutting force (CF), power consumption (PC), tool wear, etc. is greatly affected by the settings of its different input parameters, like feed rate, spindle speed, depth of cut, type of the cutting fluid, machining environment, etc. [6]. Researchers have already applied several approaches to identify the best settings of multiple input parameters of the CNC turning processes for attaining higher productivity with the desired quality level. Occasionally, the manufacturer's operating manuals are consulted or the expert operator's knowledge is sought to determine the optimal parametric combination of a CNC turning process. Unfortunately, these intuitive and conservative approaches do not always lead to the best machining performance of a CNC process under a given machining environment. Thus, to determine the optimal operating levels of various input parameters during CNC turning operation on a given work material, it has become essential to examine the effects of those input parameters on the process outputs (responses). Keeping this objective in mind, this paper aims at the application of discriminant analysis for a CNC turning process in order to develop the corresponding discriminant functions showing the influences of the considered process parameters on the responses, as well as to single out the most significant parameter for each of the responses. In simultaneous estimation of discriminant analysis, the developed functions consist of all the input parameters of the CNC turning process, while in step-wise analysis, only the significant parameters are taken into account in the developed functions. The performance of both the estimation procedures is validated based on the values of hit ratio and cross-validation percentage.

## 2. SURVEY OF THE LITERATURE

Considering feed rate, cutting speed and depth of cut as the input parameters during CNC turning operation of SAE 8822 alloy steel, Kanakaraja et al. [7] determined their best settings based on Taguchi methodology. Singh and Sodhi [8] adopted response surface methodology (RSM) to determine the optimal settings of feed rate, depth of cut and cutting speed for attaining improved values of MRR and SR in CNC turning on aluminium-7020 alloy material. During hard turning operation of AISI 4340 steel on a CNC turret lathe, Rashid et al. [9] investigated the influences of feed rate, spindle speed and depth of cut on SR values of the machined components using Taguchi methodology. While taking into consideration depth of cut, spindle speed and feed rate as the parameters of a CNC turning process, Rudrapati et al. [10] analyzed their effects on SR of the machined components. The said process was later optimized using teaching-learning-based optimization algorithm. Park et al. [11] applied RSM technique for establishing the relationships between various machining parameters, i.e. cutting speed, feed rate, nose radius, edge radius, rake angle and relief angle, and cutting energy and energy efficiency. Non-dominated sorting genetic algorithm-II (NSGA-II) was adopted for multi-objective optimization and development of the Pareto optimal solutions. The optimal parametric setting was finally determined using technique of order preference by similarity to the ideal solution (TOPSIS). Arunkumar et al. [12] applied Taguchi methodology to establish the optimal intermixture of depth of cut, speed, feed rate and coolant type during CNC machining of LM6 aluminum alloy for having better SR values. Applying RSM technique, Nataraj and Balasubramanian [13] established the optimal settings of cutting speed, depth of cut and feed rate for achieving better values of SR, intensity of vibration and work-tool interface temperature while machining hybrid metal matrix composites. Gadekula et al. [14] employed Taguchi methodology for optimization of a CNC turning process while treating feed rate, spindle speed and depth of cut as the input parameters, and MRR and SR as the responses. Rathore et al. [15] studied the influences of feed rate, depth of cut, spindle speed and coolant type on SR properties of AA 6463 materials. The weights of the responses were determined using principal component analysis and the optimal parametric mix was identified based on grey relational analysis (GRA) technique. Sahoo et al. [16] applied weighted aggregate sum product assessment (WASPAS) method for parametric optimization of a CNC turning process for achieving minimum tool vibration and SR of 6063-T6 aluminum components. Vijay Kumar et al. [17] studied the effects of feed rate, depth of cut and spindle speed on SR and MRR during CNC turning on EN 19 stainless steel material. Based on Taguchi's $L_{18}$ mixed orthogonal array experimental design plan, Syed Irfan et al. [18] optimized the settings of cutting speed, feed rate and depth of cut while performing CNC turning operation on EN45 spring steel material. The MRR and SR were treated as the responses. While machining aluminium-2014 alloy, Aswal et al. [19] considered cutting speed, depth of cut and feed rate as the input parameters of a CNC turning operation, and investigated their effects on SR.

It has been revealed from the review of the existent literature that various multi-criteria decision-making tools, i.e. TOPSIS, GRA, WASPAS, etc. have already been employed by the past researchers for parametric optimization of CNC turning processes. Taguchi methodology has become a popular technique among the research community for single objective optimization of CNC turning processes. The relationships between the CNC turning parameters and responses have also been investigated through the deployment of RSM technique. Both RSM technique and discriminant analysis are explicit methods having clear, transparent and unambiguous underlying mathematical principles with similar computation time. However, there are some drawbacks of RSM technique. It attempts to fit data to a polynomial even though many systems cannot be well explained by second order polynomials. It becomes necessary to decrease the range of the independent variables, if the system cannot be explained by the regression equation computed through RSM technique. On the other hand, discriminant analysis develops a causal model which maximizes the group difference by computing weights associated with the independent variables. Hence, it becomes an effective tool in evaluating the effect of each independent variable on the dependent variable based on its ability to separate the group differences. Besides this, the range of the independent variable does not affect the solution accuracy. Thus, it can be considered capable of effective parametric analysis of varied machining processes.

## 3. Discriminant Analysis

Discriminant analysis is a multivariate statistical technique used for categorizing a set of observations into predefined groups [20]. It can be considered as a profile analysis, where it evaluates differences between groups based on a set of independent variables. It establishes the link between the categorical (nominal or non-metric) dependent variables and metric independent variables. The discriminant function, computed from this analysis, has a linear relationship between two or more independent variables and can be expressed as below [21]:

$$Z_{qr} = \alpha + \beta_1 X_{1r} + \beta_2 X_{2r} + \ldots\ldots\ldots + \beta_n X_{nr} \tag{1}$$

where $Z_{qr}$ is the score of discriminant function $q$ for object $r$, $\alpha$ is the intercept, $X_{nr}$ is the independent variable $n$ for object $r$ and $\beta_n$ is the discriminant coefficient for independent variable $n$.

The discriminant analysis tests the hypothesis of equality of group means for each of the dependent variables. The group mean, also called group centroid, is the arithmetic mean of the discriminant scores for all the objects belonging to a single group. The group centroid denotes the most characteristic location of an object in a group, and the distance between the groups can be explained by comparing their centroids. It also enables prediction of the group where a certain element can be classified based on the closeness of its discriminant score to the group centroid. The discriminant function is said to be statistically significant if there is a substantial difference between the group centroids [21]. The statistical significance of the function is calculated by comparing the spread of the discriminant score for each group and therefore, by testing the intersection between the groups. A small intersection represents significant separation between the groups due to the discriminant function, while a large intersection denotes poor differentiating power of the function. Multiple discriminant functions can be developed provided that the dependent variables comprise more than two groups. The number of functions computed equals to

($g$ – 1), where $g$ is the number of groups, with different discriminant scores calculated by each function. In this paper, however, the analysis is conducted with each dependent variable consisting of two groups, where their relations with a combination of independent variables are established with the help of a single discriminant function. In this analysis, the responses of the considered CNC turning process are considered as the dependent variables, while the turning parameters are treated as the independent variables.

The steps of discriminant analysis are illustrated through a flowchart in Fig. 2. At first, the problem statement and purpose of the analysis are identified. The purpose of this

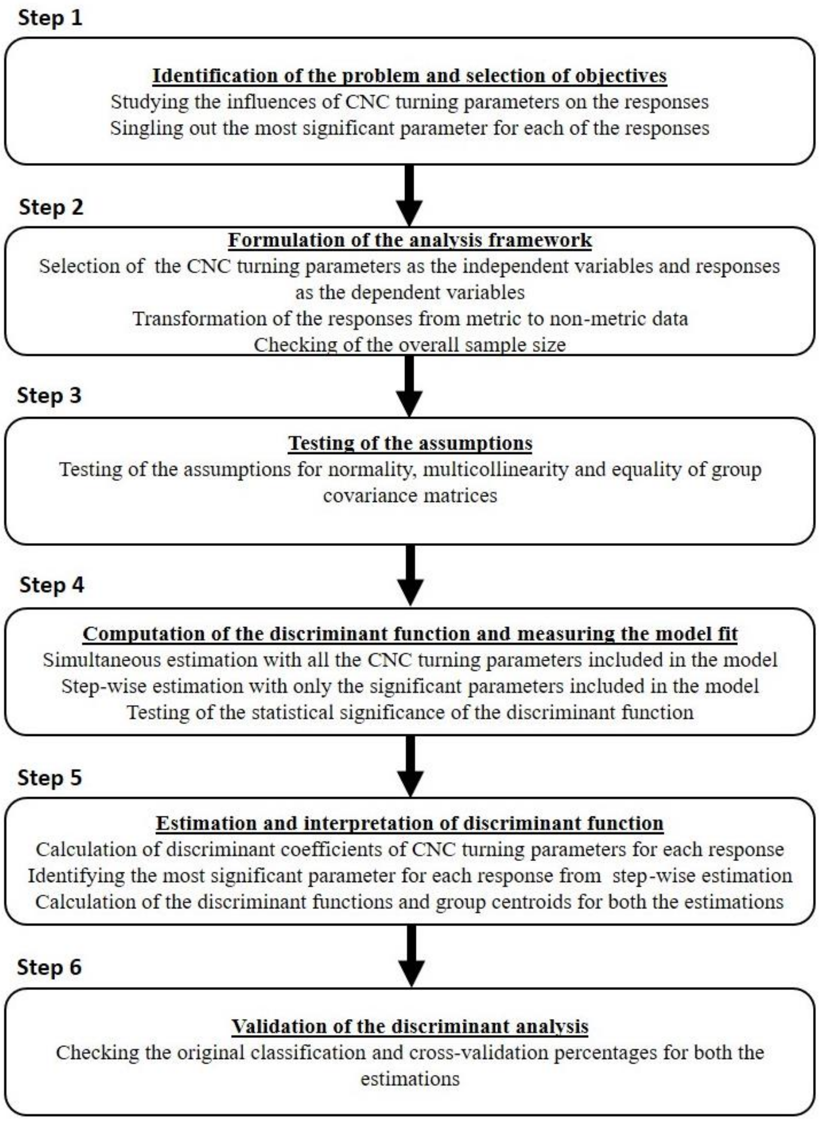


**Fig. 2** Flowchart showing steps of the discriminant analysis

paper is to demonstrate the application of discriminant analysis to evaluating the effects of the considered CNC turning parameters on the responses while identifying the most significant parameter influencing each of the responses. The analysis framework is then formulated. Determination of the independent and dependent variables takes place, followed by classification of the dependent variables into corresponding binary categories. If a dependent variable is metric, it needs to be transformed into non-metric data. Checking of the sample size is also required in this step. Pituch and Stevens [22] advised that the ratio between the sample size and number of independent variables should be 20:1, with a minimum of 20 elements in the group containing the least number of objects.

After this step, the corresponding assumptions of discriminant analysis, i.e. multivariate normality, multicollinearity and homogeneity of covariance matrices need to be validated. The independent variables can be tested for univariate normality while calculating their skewness and kurtosis values, which can be considered as adequate for validation of multivariate normality [21, 22]. Multicollinearity indicates high inter-correlations between two or more independent variables. It poses problems in determination of the significance of an independent variable because the influences of the independent variables are confounded due to high correlations between them, making its absence as a mandatory requirement [22]. Multicollinearity can be tested using variance inflation factor (VIF) and tolerance values. The VIF measures how much larger the variance would be for multicollinear data than the orthogonal data, where its most preferred value is 1 [23]. Tolerance is the reciprocal of VIF. Homogeneity of covariance matrices or homoscedasticity specifies whether the covariance matrix for each group is equal to each other and is verified using the Box's *M* test, which considers equality of the within-class covariance matrices as the null hypothesis. Thus, non-rejection of the null hypothesis is desired, which can be denoted by an insignificant result.

The developed discriminant function can be interpreted by assessing the unstandardized and standardized coefficients of the independent variables and structure matrix. The contribution of an independent variable to the ability of the discriminant function to separate and classify objects into the related groups is determined by the absolute value of its standardized discriminant coefficient. As the independent variables are quantified in different scales, it is recommended to compare their relative contributions based on standardized coefficients. Larger is the absolute value of the standardized coefficient, higher is the discriminating power of the independent variable. The influence of an independent variable on the discriminant function can also be explained using the corresponding structure matrix. The structure coefficients, also known as structure correlations, are the correlations between the independent variables and discriminant function. Thus, structure coefficient can be treated as the factor loading of an independent variable on the discriminant function, allowing measurement of the relative closeness of the variable to the discriminant function. In step-wise discriminant analysis, structure correlations can be computed even for those variables not included in the model. The unstandardized coefficient, computed for each of the independent variables in the model, is utilized for formulating the discriminant function. The discriminant function yields the discriminant score for different values of the independent variables. These scores are instrumental in cross-validation and classification of the objects into the corresponding groups. The objects are classified into groups based on their discriminant scores and closeness to the group centroids. The cut-off scores, considered to determine the groups into which the related objects are classified, are computed using group centroids. The cut-off score ($Z_C$) between two groups is calculated using the following equations:

a) For unequal groups:

$$Z_C = (N_A Z_B + N_B Z_A) / (N_A + N_B) \quad (2)$$

where $N_A$ and $N_B$ are the group sizes, and $Z_A$ and $Z_B$ are the group centroids, respectively.

b) For equal groups:

$$Z_C = (Z_A + Z_B) / 2 \quad (3)$$

In the validation stage, accuracy of the discriminant function in separating and classifying objects into the relevant groups is measured, based on two approaches, i.e. hit ratio and cross-validation. Hit ratio is a measure of actual percentage of correct classification of objects by the developed discriminant function. Along with the hit ratio calculation, cross-validation must be carried out to validate the results in order to apply the function for classification of the subsequent objects into appropriate groups. Discriminant analysis aims at maximization of the separation between two groups based on sample-specific error [24]. Since the errors may differ for different samples of objects, it becomes necessary to cross-validate the results, which provides the predictive accuracy of the function along with its suitability of application for a wider range of samples. In this paper, the leave-one-out procedure of cross-validation is applied [22], where one element from the sample is systematically excluded and the discriminant function is estimated based on the remaining elements in the sample. The excluded element is then classified into one of the two groups according to its discriminant score. This process repeats till every element in the sample is excluded and classified. Higher values of hit ratio and cross-validation percentage are desired to validate the function's suitability and potentiality as a multivariate prediction tool.

Discriminant analysis has certain similarities and dissimilarities with regression analysis and analysis of variance (ANOVA). All these techniques have one dependent variable and one or more independent variables. However, ANOVA and regression analysis are concerned with continuous dependent variables, while discriminant analysis has categorical dependent variables [25]. On the other hand, regression and discriminant analysis deal with continuous independent variables while ANOVA has categorical independent variables. Both regression and discriminant analysis can predict values (although of different data types) and study the influence of independent variables on dependent variables, while ANOVA is used to ascertain the effects of independent variables on dependent variables.

Mathematically, discriminant analysis is similar to one-way multivariate ANOVA (MANOVA), with the difference being in the variable data types. In MANOVA, like ANOVA, the classification is on the basis of the categorical independent variables, while in discriminant analysis, the classification is on the basis of the values that the dependent variables obtain.

## 4. Discriminant Analysis for a CNC Turning Process

Gupta et al. [26] applied Taguchi methodology along with fuzzy logic reasoning approach for multi-response optimization of a high speed CNC turning operation on AISI P20 tool steel material using TiN coated tungsten carbide inserts. Speed (S), feed (F), depth of cut (D), nose radius (NR) and environment (E) were selected as the input parameters (independent variables), and SR (in μm), TL (in min), CF (in N) and PC (in W) were the responses (dependent variables). Taking three different operating levels for each of the turning parameters, Gupta et al. [26] conducted 27 experimental runs and measured the corresponding responses values. The settings of the CNC turning parameters are provided in Table 1 and the detailed experimental plan is shown in Table 2. Based on this dataset, both the simultaneous and step-wise estimation discriminant analyses are carried out to explore the influences of the considered CNC turning parameters on each of the responses. For this purpose, IBM SPSS Statistics 25.0 software is employed.

**Table 1** CNC turning parameters along with their levels [26]

| Turning parameter | Symbol | Unit | Level 1 | Level 2 | Level 3 |
|---|---|---|---|---|---|
| Speed | S | m/min | 120 | 160 | 200 |
| Feed | F | mm/rev | 0.1 | 0.12 | 0.14 |
| Depth of cut | D | Mm | 0.2 | 0.35 | 0.5 |
| Nose radius | NR | Mm | 0.4 | 0.8 | 1.2 |
| Environment | E | | Dry | Wet | Cryogenic |

As all the response values for the said CNC turning operation are metric in nature, it is necessary to categorize them into two non-metric groups on the basis of their median values, as provided in Table 2. The values of the responses which are higher than their corresponding medians are considered as high and are categorized into Group 2. On the contrary, in Group 1, values of the responses lower than the medians are classified as low.

**Table 2** CNC turning parameters along with their levels [26]

| S (m/min) | F (mm/rev) | D (mm) | NR (mm) | E | SR (μm) | SR Group | TL (min) | TL Group | CF (N) | CF Group | PC (W) | PC Group |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 120 | 0.1 | 0.2 | 0.4 | 1 | 1.41 | 2 | 29 | 2 | 171.3 | 1 | 1066 | 1 |
| 120 | 0.1 | 0.35 | 0.8 | 5 | 0.71 | 2 | 34 | 2 | 147.5 | 1 | 1560 | 2 |
| 120 | 0.1 | 0.5 | 1.2 | 9 | 0.6 | 2 | 54.67 | 2 | 111.74 | 1 | 866 | 1 |
| 120 | 0.12 | 0.2 | 0.8 | 5 | 0.47 | 1 | 34.67 | 2 | 120.3 | 1 | 1493 | 2 |
| 120 | 0.12 | 0.35 | 1.2 | 9 | 0.19 | 1 | 51.66 | 2 | 180.6 | 2 | 987 | 1 |
| 120 | 0.12 | 0.5 | 0.4 | 1 | 1.18 | 2 | 27 | 1 | 236.2 | 2 | 1187 | 1 |
| 120 | 0.14 | 0.2 | 1.2 | 9 | 0.67 | 2 | 50 | 2 | 157.7 | 1 | 960 | 1 |
| 120 | 0.14 | 0.35 | 0.4 | 1 | 1.16 | 2 | 24.66 | 1 | 214.4 | 2 | 1134 | 1 |
| 120 | 0.14 | 0.5 | 0.8 | 5 | 0.92 | 2 | 28.33 | 2 | 286.9 | 2 | 1813 | 2 |
| 160 | 0.1 | 0.2 | 1.2 | 5 | 0.18 | 1 | 27.66 | 1 | 116.37 | 1 | 1586 | 2 |
| 160 | 0.1 | 0.35 | 0.4 | 9 | 0.45 | 1 | 47.66 | 2 | 133.33 | 1 | 1013 | 1 |
| 160 | 0.1 | 0.5 | 0.8 | 1 | 0.43 | 1 | 21.66 | 1 | 191.23 | 2 | 1240 | 1 |
| 160 | 0.12 | 0.2 | 0.4 | 9 | 0.58 | 1 | 45.66 | 2 | 125.4 | 1 | 893 | 1 |
| 160 | 0.12 | 0.35 | 0.8 | 1 | 0.72 | 2 | 20.33 | 1 | 149.43 | 1 | 1253 | 1 |
| 160 | 0.12 | 0.5 | 1.2 | 5 | 0.31 | 1 | 25.66 | 1 | 212.46 | 2 | 1773 | 2 |
| 160 | 0.14 | 0.2 | 0.8 | 1 | 0.66 | 2 | 20 | 1 | 162.93 | 1 | 1107 | 1 |
| 160 | 0.14 | 0.35 | 1.2 | 5 | 0.64 | 2 | 22.33 | 1 | 190.23 | 2 | 1533 | 2 |
| 160 | 0.14 | 0.5 | 0.4 | 9 | 0.75 | 2 | 41.33 | 2 | 177.76 | 2 | 1373 | 1 |
| 200 | 0.1 | 0.2 | 0.8 | 9 | 0.16 | 1 | 40 | 2 | 106.23 | 1 | 1053 | 1 |
| 200 | 0.1 | 0.35 | 1.2 | 1 | 0.23 | 1 | 15.67 | 1 | 208.5 | 2 | 1373 | 1 |
| 200 | 0.1 | 0.5 | 0.4 | 5 | 0.67 | 2 | 21.67 | 1 | 209.8 | 2 | 2094 | 2 |
| 200 | 0.12 | 0.2 | 1.2 | 1 | 0.4 | 1 | 14.67 | 1 | 200.2 | 2 | 1286 | 1 |
| 200 | 0.12 | 0.35 | 0.4 | 5 | 0.5 | 1 | 20.33 | 1 | 178.8 | 2 | 1866 | 2 |
| 200 | 0.12 | 0.5 | 0.8 | 9 | 0.18 | 1 | 37.66 | 2 | 168.7 | 1 | 1613 | 2 |
| 200 | 0.14 | 0.2 | 0.4 | 5 | 0.64 | 2 | 18 | 1 | 162 | 1 | 1573 | 2 |
| 200 | 0.14 | 0.35 | 0.8 | 9 | 0.31 | 1 | 34.33 | 2 | 162.5 | 1 | 1453 | 2 |
| 200 | 0.14 | 0.5 | 1.2 | 1 | 0.48 | 1 | 16.66 | 1 | 276.16 | 2 | 1667 | 2 |
| Median | | | | | 0.58 | | 27.66 | | 171.3 | | 1373 | |

It is worthwhile to mention here that among the responses, SR, CF and PC are smaller-the-better type of quality characteristics, and TL is the sole larger-the-better quality feature. Since discriminant analysis cannot be performed with categorical independent variables, type of the cutting environment is converted into three distinct classes using a 1-9 point scale (where 9 = cryogenic environment, 5 = wet environment and 1 = dry environment). For carrying out a robust discriminant analysis, number of experimental runs plays an important role. Pituch and Stevens [22] suggested a ratio of 20:1 between the number of observations and the number of independent variables, with a minimum of 20 members in the smallest group. Thus, 123 additional experimental runs are simulated to have a sample pool of 150 observations, which is in agreement with the guideline stated. All those independent and dependent variables are simulated in such a way that they must lie between their corresponding minimum and maximum values. Table 3 exhibits the number of members in each group for discriminant analysis for the four responses.

**Table 3** Members in each group for discriminant analysis

| Group | Number of members | | | |
|---|---|---|---|---|
| | SR | TL | CF | PC |
| 1 | 77 | 76 | 79 | 85 |
| 2 | 73 | 74 | 71 | 65 |

Now, the assumptions for normality, non-multicollinearity and homogeneity of covariance matrices need to be validated. For both the simultaneous and step-wise estimations of discriminant analysis, normality and multicollinearity tests would be the same, while the test for homogeneity of covariance matrices would be different. For normality test, the related skewness and kurtosis values are computed, and for multicollinearity test, tolerance and VIF values are estimated. Table 4 exhibits results of the normality and multicollinearity tests for the considered input (independent) variables.

**Table 4** Tests for normality and multicollinearity

| Input variable | Normality test | | Multicollinearity test | |
|---|---|---|---|---|
| | Skewness | Kurtosis | Tolerance | VIF |
| S | 0.111 | -1.478 | 1 | 1 |
| F | 0.037 | -1.493 | 1 | 1 |
| D | 0.000 | -1.510 | 1 | 1 |
| NR | 0.000 | -1.510 | 1 | 1 |
| E | 0.000 | -1.510 | 1 | 1 |

According to Pituch and Stevens [22], when both skewness and kurtosis values for a distribution are between -2 and +2, it should be considered as normal. In this case, values of skewness lie in the range of 0 to 0.111, while the kurtosis values are between -1.510 and -1.478. As all the skewness and kurtosis values are within the prescribed range, it can be concluded that the considered input variables follow normal distribution. Tolerance is a measure of variability in one independent variable that the other independent variables cannot explain. Its value lies between 0 and 1, with lower values indicating presence of multicollinearity. The VIF is the reciprocal of tolerance. The tolerance and VIF values are 1 for all variables, indicating that the variables are orthogonal, without multicollinearity.

### 4.1. Simultaneous Estimation

In this procedure, every single independent variable is involved in the analysis based on which the corresponding discriminant function is developed. However, before the analysis, the assumption of equality of covariance matrices needs to be tested using the Box's *M* value. The null hypothesis for this test is that the within-group covariance matrices are equal for the dependent variables. The Box's *M* values for SR, TL, CF and PC are computed as 76.789, 59.22, 60.568 and 74.954, respectively. The corresponding *p*-values are all less than 0.001, inferring that they are significant, thus rejecting the null hypothesis for the four dependent variables. However, the discriminant analysis may still be robust despite the violation of the above assumption of equality of covariance matrices as it has less importance during the analysis [27]. Table 5 shows the assessment of model fit with the help of the Wilks' lambda value. The Wilks' lambda indicates the effectiveness of the discriminant function in differentiating objects into the related groups. The lower the value of Wilks' lambda is, the higher the discriminating power of the function is. Smaller *p*-values ($p < 0.05$) also infer the same conclusion. In this case, all the four discriminant analyses exhibit low *p*-values, representing the functions' ability to effectively distinguish objects between the groups. Tables 6-8 collectively exhibit the influences of the independent variables (S, F, D, NR and E) on the responses (SR, TL, CF and PC) for the said CNC turning process.

**Table 5** Assessment of model fit for simultaneous estimation

| Output variable | Eigenvalue | Canonical correlation | Wilks' lambda | Chi-square | Df | *p*-value |
|---|---|---|---|---|---|---|
| SR | 0.644 | 0.626 | 0.608 | 72.297 | 5 | 0.000 |
| TL | 2.17 | 0.827 | 0.315 | 167.862 | 5 | 0.000 |
| CF | 0.954 | 0.699 | 0.512 | 97.453 | 5 | 0.000 |
| PC | 0.17 | 0.382 | 0.854 | 22.904 | 5 | 0.000 |

**Table 6** Group centroids for simultaneous estimation

| Group | Group centroid | | | |
|---|---|---|---|---|
| | SR | TL | CF | PC |
| 1 (Low) | 0.776 | 1.444 | -0.92 | -0.359 |
| 2 (High) | -0.818 | -1.483 | 1.023 | 0.469 |

**Table 7** Standardized discriminant function and structure coefficients for simultaneous estimation

| Input variable | SR | | TL | | CF | | PC | |
|---|---|---|---|---|---|---|---|---|
| | Std. disc. func. coeff. | Str. coeff. | Std. disc. func. coeff. | Str. coeff. | Std. disc. func. coeff. | Str. coeff. | Std. disc. func. coeff. | Str. coeff. |
| S | 0.829 | 0.636 | 0.742 | 0.271 | 0.174 | 0.089 | 0.723 | 0.663 |
| F | -0.532 | -0.35 | 0.196 | 0.062 | 0.4 | 0.214 | 0.478 | 0.419 |
| D | -0.198 | -0.123 | 0.211 | 0.078 | 0.911 | 0.685 | 0.462 | 0.405 |
| NR | 0.504 | 0.338 | 0.235 | 0.056 | 0.18 | 0.084 | 0.311 | 0.281 |
| E | 0.369 | 0.249 | -1.079 | -0.701 | -0.658 | -0.395 | 0.23 | 0.2 |

**Table 8** Unstandardized discriminant function coefficients for simultaneous estimation

| Input Variable | Unstandardized discriminant function coefficient | | | |
|---|---|---|---|---|
| | SR | TL | CF | PC |
| S | 0.029 | 0.025 | 0.005 | 0.023 |
| F | -33.764 | 12.016 | 25.000 | 29.661 |
| D | -1.61 | 1.724 | 8.892 | 3.8 |
| NR | 1.589 | 0.717 | 0.55 | 0.952 |
| E | 0.115 | -0.472 | -0.214 | 0.07 |
| Constant | -1.738 | -4.116 | -6.311 | -9.609 |

*4.1.1. Discriminant analysis for SR*

Table 6 shows that for SR response, Group 2 with higher values of SR (> 0.58 µm) has a negative centroid, while Group 1 having lower values of SR (≤ 0.58 µm) has a positive centroid. It indicates that the independent variables with negative standardized discriminant coefficients would influence the discriminant score of an observation towards the group with higher values of SR (Group 2). Similarly, the variables with positive coefficients would influence the discriminant score of an observation towards the group with lower SR values (Group 1). Table 7 shows that F and D have negative standardized discriminant function coefficients which would tend to decrease the discriminant score, moving it towards the centroid of Group 2. As a result, when the values of F and D increase, SR also increases with deterioration of surface quality of the turned components. Conversely, as S, NR and E have positive coefficients for SR in Table 7, increase in their values would significantly reduce SR. The strength of influence of each independent variable on the discriminating power of the developed function is indicated by the absolute value of its coefficient, which in turn, can be employed to compare the level of its significance on the considered dependent variable. In this case, SR depends mostly on S, followed by F, although their nature of contribution is completely opposite. The structure coefficients, which show the correlations between the independent variables and discriminant function, are 0.636, -0.35, -0.123, 0.338 and 0.249 for S, F, D, NR and E, respectively. Table 8 shows the unstandardized discriminant function coefficients, based on which the following discriminant function for SR is developed:

$$Z_{SR} = -1.738 + 0.029\times S - 33.764\times F - 1.61\times D + 1.589\times NR + 0.115\times E \quad (4)$$

The corresponding cut-off score is calculated as -0.042, which signifies that the observations with discriminant scores, estimated using Eq. (4), less than -0.042 should be classified into Group 2 (SR values more than 0.58 µm). Similarly, the observations with discriminant scores of more than -0.042 should be categorized into Group 1 (SR values less than 0.58 µm).

*4.1.2. Discriminant analysis for TL*

It can be observed from Table 6 that Group 2 with higher TL values has a negative centroid. On the other hand, the centroid of Group 1 consisting of lower values of TL is positive. Table 7 depicts that S, F, D and NR have positive coefficients, while E has negative coefficient. Thus, it can be inferred that TL would decrease with increasing values of S, F, D and NR, while it would increase with increase in the scored value of E. The TL would mostly depend on E, followed by S. The correlations between the independent variables and discriminant function are 0.271, 0.062, 0.078, 0.056 and -0.701 for S, F, D, NR and E, respectively. Now, based on the unstandardized discriminant function coefficients of Table 8, the following discriminant function for TL is derived.

$$Z_{TL} = -4.116 + 0.025\times S + 12.016\times F + 1.724\times D + 0.717\times NR - 0.472\times E \quad (5)$$

The cut-off score is equal to -0.039, which denotes that the observations whose discriminant scores, estimated using Eq. (5), are less than -0.039, should be classified into Group 2 with higher TL value (more than 27.66 min). On the other hand, observations with discriminant scores of more than the cut-off score would be in Group 1 with lower TL values (less than 27.66 min).

#### *4.1.3. Discriminant analysis for CF*

Table 6 shows that Group 2 with higher CF values has a positive centroid and Group 1 with lower CF values has a negative centroid. From Table 7, it can be propounded that S, F, D and NR have positive coefficients, while the coefficient for E is negative. As a result, CF would increase with increasing values of S, F, D and NR, and increased score for E would result in decreased value of CF. The most important turning parameter influencing CF is D, followed by E, as noticed from the absolute values of their corresponding standardized discriminant coefficients. The structure coefficients, representing the correlations between the independent variables and discriminant function, are 0.089, 0.214, 0.685 and 0.084 and -0.395 for S, F, D, NR and E, respectively. Now, based on Table 8, the following discriminant function for CF is developed.

$$Z_{CF} = -6.311 + 0.005\times S + 25.0\times F + 8.892\times D + 0.550\times NR - 0.214\times E \quad (6)$$

For CF response, the corresponding cut-off score is estimated to be 0.103. It symbolizes that the observations with discriminant scores higher than 0.103 would be assigned to Group 2 with higher CF values (more than 171.3 N). In the similar direction, the observations having discriminant scores of less than the cut-off score would be allocated to Group 1 (CF values less than 171.3 N).

#### *4.1.4. Discriminant analysis for PC*

From Table 6, it can be noticed that the centroid for Group 2 is positive, while its value is negative for Group 1. Thus, the independent variables whose standardized discriminant function coefficients are positive, would like to increase the discriminant scores of the observations moving them towards the centroid of Group 2. In Table 7, all the five independent variables have positive coefficients. Hence, increasing values of S, F, D, NR and E are all responsible for higher PC during the CNC turning operation. It can also be revealed that S and F are the two most important turning parameters influencing PC. The correlations between the independent variables and discriminant function are estimated as 0.663, 0.419, 0.405, 0.281, 0.2 for S, F, D, NR and E, respectively. Now, based on Table 8, the following discriminant function for PC is established.

$$Z_{PC} = -9.609 + 0.023\times S + 29.661\times F + 3.8\times D + 0.952\times NR + 0.070\times E \quad (7)$$

For this response, the value of the cut-off score is calculated as 0.110. It indicates that the observations with discriminant scores of more than 0.110 would be classified into Group 2 having higher PC (greater than 1373 W). Similarly, observations with discriminant scores of less than 0.110 would be included in Group 1 with lower PC values (less than 1373 W).

*4.1.5. Validation of the discriminant analysis results*

Now, it is required to validate the results derived from the simultaneous estimation-based discriminant analysis in order to justify the corresponding prediction performance. It can be observed from Table 3 that for SR response, among 150 original and simulated experimental runs, 77 have low SR values (less than 0.58 µm) and the remaining 73 observations have high SR values (more than 0.58 µm). In Table 9, the discriminant function developed for SR can correctly identify 71 Group 1 observations (out of 77) and 52 Group 2 observations (out of 73). So, the percentages of correct classifications are 92.2% and 71.2%, respectively. Hence, the hit ratio for the discriminant function for SR is 82% (123 out of 150), with a misclassification error of 18%. The prediction performance of this discriminant function is cross-validated based on leave-one-out approach, using IBM SPSS Statistics 25.0 software. For SR, the percentages of correct classification for Group

**Table 9** Classification results for simultaneous estimation method

| Output variable | Type of validation | Count (%) | Group | Predicted group membership | | Total |
|---|---|---|---|---|---|---|
| | | | | 1 | 2 | |
| SR | Original | Count | 1 | 71 | 6 | 77 |
| | | | 2 | 21 | 52 | 73 |
| | | % | 1 | 92.2 | 7.8 | 100 |
| | | | 2 | 28.8 | 71.2 | 100 |
| | Cross-validated | Count | 1 | 65 | 12 | 77 |
| | | | 2 | 21 | 52 | 73 |
| | | % | 1 | 84.4 | 15.6 | 100 |
| | | | 2 | 28.8 | 71.2 | 100 |
| TL | Original | Count | 1 | 70 | 6 | 76 |
| | | | 2 | 6 | 68 | 74 |
| | | % | 1 | 92.1 | 7.9 | 100 |
| | | | 2 | 8.1 | 91.9 | 100 |
| | Cross-validated | Count | 1 | 70 | 6 | 76 |
| | | | 2 | 6 | 68 | 74 |
| | | % | 1 | 92.1 | 7.9 | 100 |
| | | | 2 | 8.1 | 91.9 | 100 |
| CF | Original | Count | 1 | 68 | 11 | 79 |
| | | | 2 | 16 | 55 | 71 |
| | | % | 1 | 86.1 | 13.9 | 100 |
| | | | 2 | 22.5 | 77.5 | 100 |
| | Cross-validated | Count | 1 | 57 | 22 | 79 |
| | | | 2 | 16 | 55 | 71 |
| | | % | 1 | 72.2 | 27.8 | 100 |
| | | | 2 | 22.5 | 77.5 | 100 |
| PC | Original | Count | 1 | 70 | 15 | 85 |
| | | | 2 | 24 | 41 | 65 |
| | | % | 1 | 82.4 | 17.6 | 100 |
| | | | 2 | 36.9 | 63.1 | 100 |
| | Cross-validated | Count | 1 | 70 | 15 | 85 |
| | | | 2 | 24 | 41 | 65 |
| | | % | 1 | 82.4 | 17.6 | 100 |
| | | | 2 | 36.9 | 63.1 | 100 |

1 and Group 2 objects based on cross-validation are 84.4% and 71.2%, respectively. Hence, the hit ratio for cross-validation is 78% (117 out of 150). Similarly, in case of TL, both the hit-ratio and cross-validation percentages are 92%. For CF response, the hit-ratio is 82%, while the cross-validation percentage is 74.7%. Finally, for PC, 74% of the original and cross-validated grouped cases can be correctly classified. These higher values of hit-ratio prove that the discriminant functions developed on the basis of the simultaneous estimation method have the ability to correctly classify the responses into appropriate lower and higher groups.

### 4.2. Step-wise Estimation

The step-wise estimation of discriminant analysis is appropriate when only the significant independent variables need to be included in the developed discriminant function. These independent variables are selected based on the Wilks' lambda value. The variables having smaller Wilks' lambda values and maximum ability to decrease the overall Wilks' lambda, are first chosen for inclusion in the model. Before developing the model, it is assumed that the model does not have any independent variable. In each step, the variable whose '$F$ to enter' value is the largest and simultaneously higher than the entry criterion, is included in the model, while the '$F$ to remove' value is necessary to exclude any insignificant variable from further consideration. The '$F$ to enter' and '$F$ to remove' values, which would decide the entry and exit of the independent variables in the model, are estimated as 3.84 and 2.71, respectively, and are set as defaults in the software. These values relate to $p$-values of 0.05 and 0.10, respectively. This process is continued till all the significant variables, satisfying the entry criterion, are included in the model, while the insignificant variables are removed from the model.

As mentioned earlier, before the start of this analysis, testing of the assumptions is mandatory. Assumptions of normality and multicollinearity, as tested in Table 4, also hold true for step-wise discriminant analysis. The Box's $M$ test is conducted again to check whether the covariance matrices are homogenous or not. The values of the Box's $M$ for SR, TL, CF and PC are determined as 55.583, 47.129, 6.697 and 21.78, respectively. The corresponding $p$-values for SR and TL are less than 0.001, while those for CF and PC are greater than 0.001 (0.365 and 0.002, respectively). Hence, for SR and TL, the null hypothesis of equality of covariance matrices is rejected, while it cannot be rejected for CF and PC. Even though for SR and TL, the assumption of equality of covariance matrices is violated, their discriminant analyses may still be considered robust. The model fit now needs to be validated applying the overall Wilks' lambda for the discriminant functions of all the four responses in order to check their ability to separate objects into separate groups. Table 10 exhibits the eigenvalues and Wilks' lambda values for the dependent variables (SR, TL, CF and PC), testing the significance of the discriminant function for each of those variables. It can be noticed that all the $p$-values are less than 0.05, indicating satisfactory discriminating power of the developed functions. In Tables 11-14, variables entered into the models and removed from the models during step-wise discriminant analysis for the four considered responses are provided.

**Table 10** Assessment of model fit for step-wise estimation

| Output variable | Eigenvalue | Canonical correlation | Wilks' lambda | Chi-square | Df | *p*-value |
|---|---|---|---|---|---|---|
| SR | 0.619 | 0.618 | 0.618 | 70.311 | 4 | 0.000 |
| TL | 2.088 | 0.822 | 0.324 | 164.63 | 4 | 0.000 |
| CF | 0.896 | 0.687 | 0.528 | 93.687 | 3 | 0.000 |
| PC | 0.144 | 0.355 | 0.874 | 19.753 | 3 | 0.000 |

**Table 11** Variables included/not included in the model for SR

| Variable included | | | | Variable not included | | | | |
|---|---|---|---|---|---|---|---|---|
| Input variable | Tolerance | *F*-value | Wilks' lambda | Input variable | Tolerance | Min. tolerance | *F*-value | Wilks' lambda |
| E | 0.945 | 48.422 | 0.824 | D | 0.994 | 0.941 | 2.221 | 0.608 |
| S | 0.965 | 17.046 | 0.69 | | | | | |
| NR | 0.97 | 15.11 | 0.682 | | | | | |
| D | 0.983 | 8.096 | 0.652 | | | | | |

**Table 12** Variables included/not included in the model for TL

| Variable included | | | | Variable not included | | | | |
|---|---|---|---|---|---|---|---|---|
| Input variable | Tolerance | *F*-value | Wilks' lambda | Input variable | Tolerance | Min. tolerance | *F*-value | Wilks' lambda |
| E | 0.793 | 232.304 | 0.843 | F | 0.982 | 0.78 | 3.806 | 0.315 |
| S | 0.814 | 61.737 | 0.462 | | | | | |
| NR | 0.969 | 5.361 | 0.336 | | | | | |
| D | 0.983 | 4.369 | 0.334 | | | | | |

**Table 13** Variables included/not included in the model for CF

| Variable included | | | | Variable not included | | | | |
|---|---|---|---|---|---|---|---|---|
| Input variable | Tolerance | *F*-value | Wilks' lambda | Input variable | Tolerance | Min. tolerance | *F*-value | Wilks' lambda |
| D | 0.921 | 83.389 | 0.829 | S | 0.993 | 0.916 | 2.114 | 0.52 |
| E | 0.933 | 33.654 | 0.649 | | | | | |
| F | 0.967 | 11.539 | 0.569 | NR | 0.989 | 0.917 | 2.277 | 0.519 |

**Table 14** Variables included/not included in the model for PC

| Variable included | | | | Variable not included | | | | |
|---|---|---|---|---|---|---|---|---|
| Input variable | Tolerance | *F*-value | Wilks' lambda | Input variable | Tolerance | Min. tolerance | *F*-value | Wilks' lambda |
| S | 0.995 | 11.674 | 0.944 | NR | 0.998 | 0.994 | 2.177 | 0.861 |
| F | 0.997 | 4.891 | 0.903 | | | | | |
| D | 0.997 | 4.536 | 0.901 | E | 0.998 | 0.995 | 1.229 | 0.867 |

It can be revealed from Table 11 that the independent variables included in step-wise estimation of the dependent variable SR are S, F, NR and E. The independent variables that significantly influence TL are E, S, NR and D. On the other hand, D, E and F are the significant variables for CF, while S, F and D maximally influence PC. From the computed *F*-values, S is the most significant independent variable for SR, followed by F. For TL, the most significant independent variable is E, followed by S. Similarly, D has the maximum discriminating power on CF, followed by E. For response PC, S is identified as the most significant independent variable. Conversely, D is singled out as the least significant contributor for SR, while F has no discriminating power on TL. In the similar direction, S and NR do not contribute significantly to CF, and for PC, the insignificant independent variables are NR and E. In discriminant analysis, an independent variable can significantly differentiate objects into the corresponding groups only when the difference between the means of the independent variables across the groups is significant. For insignificant independent variables, the difference between their means across the groups is not enough to create sufficient separation between those two groups. Hence, for any variation in the values of insignificant variables, changes in the discriminant scores remain negligible with respect to their respective dependent variables. Likewise the simultaneous estimation method of discriminant analysis, Tables 15-17 also exhibit the effects of five independent variables of the CNC turning process on the dependent variables for step-wise estimation method.

**Table 15** Group centroids for step-wise estimation

| Group | Group centroid | | | |
|---|---|---|---|---|
| | SR | TL | CF | PC |
| 1 (Low) | 0.761 | 1.416 | -0.891 | -0.33 |
| 2 (High) | -0.802 | -1.455 | 0.992 | 0.432 |

**Table 16** Standardized discriminant function and structure coefficients for step-wise estimation

| Input variable | SR | | TL | | CF | | PC | |
|---|---|---|---|---|---|---|---|---|
| | Std. disc. func. coeff. | Str. coeff. | Std. disc. func. coeff. | Str. coeff. | Std. disc. func. coeff. | Str. coeff. | Std. disc. func. coeff. | Str. coeff. |
| S | 0.833 | 0.649 | 0.737 | 0.277 | - | -0.083 | 0.768 | 0.721 |
| F | -0.534 | -0.357 | - | -0.133 | 0.4 | 0.22 | 0.508 | 0.456 |
| D | - | 0.075 | 0.21 | 0.08 | 0.914 | 0.707 | 0.49 | 0.44 |
| NR | 0.504 | 0.344 | 0.233 | 0.057 | - | -0.095 | - | -0.041 |
| E | 0.375 | 0.254 | -1.072 | -0.715 | -0.652 | -0.407 | - | -0.039 |

**Table 17** Unstandardized discriminant function coefficients for step-wise estimation

| Input variable | Unstandardized discriminant function coefficient | | | |
|---|---|---|---|---|
| | SR | TL | CF | PC |
| S | 0.029 | 0.024 | - | 0.024 |
| F | -33.915 | - | 25.012 | 31.516 |
| D | - | 1.713 | 8.919 | 4.025 |
| NR | 1.589 | 0.712 | - | - |
| E | 0.116 | -0.469 | -0.212 | - |
| Constant | -2.312 | -2.659 | -5.051 | -9.024 |

Table 15 shows that the centroid of Group 2 with higher SR values is negative, while Group 1 having lower SR values has a positive centroid. Thus, it can be unveiled that S, NR and E with positive standardized discriminant coefficients have negative impacts on SR, while SR would increase with higher values of F. The structure coefficients which denote the correlations between the independent variables and discriminant function are estimated as 0.649, -0.357, 0.075, 0.344 and 0.254 for S, F, D, NR and E, respectively. Based on the results of step-wise estimation, the values of the standardized coefficient and structure correlation show that S has the most discriminating power on SR, maximally influencing it. Table 17 provides the unstandardized discriminant coefficients which lead to the subsequent development of the following discriminant function for SR:

$$Z_{SRS} = -2.312 + 0.029 \times S - 33.915 \times F + 1.589 \times NR + 0.116 \times E \quad (8)$$

The related cut-off score is estimated as -0.041. It denotes that the observations whose discriminant scores are higher than -0.041 would be classified into Group 1 with lower SR values. Similarly, the observations having discriminant scores of less than -0.041 would be assigned to Group 2 with higher SR values.

As in Table 15, the centroid of Group 2 is negative and that of Group 1 is positive, the independent variables having positive standardized discriminant coefficients would cause the observations to move closer to Group 1, thereby reducing TL. Therefore, S, D and NR have negative influences on TL, while an increase in the score for E would increase TL. Both the standardized coefficients and structure correlations establish that E has the maximum discriminating power on TL. The related discriminant function for TL is represented as below:

$$Z_{TLS} = -2.659 + 0.024 \times S + 1.713 \times D + 0.712 \times NR - 0.469 \times E \quad (9)$$

The cut-off discriminant score for TL is -0.039. Thus, the observations with discriminant scores of less than -0.039 would be assigned to Group 2 with higher TL values. On the contrary, observations with discriminant scores higher than the corresponding cut-off score would be classified into Group 1with lower TL values.

From Table 15, it can also be observed that as Group 2 has a positive centroid value, the independent variables with positive standardized discriminant coefficients are expected to have positive impacts on CF. Thus, with increasing values of F and D, CF would increase, while it would decrease with higher scores for E. Both the standardized coefficients and structure correlations identify D as the most significant input variable for CF. The following equation shows the developed discriminant function for CF:

$$Z_{CFS} = -5.051 + 25.012 \times F + 8.919 \times D - 0.212 \times E \quad (10)$$

For this response, the cut-off score is calculated as 0.100. It denotes that the observations with discriminant scores higher than 0.100 would be added to Group 2 with higher CF values. Similarly, the observations with discriminant scores lower than the cut-off score would be included in Group 1 with lower CF values.

Similarly for response PC, as the independent variables with positive standardized discriminant coefficients significantly influence it, increasing values of F and D would be responsible to increase PC. Based on the standardized discriminant coefficients and structure correlations, it can be propounded that S has the maximum influence on PC. The related discriminant function is developed as given below:

$$Z_{PCS} = -9.024 + 0.024 \times S + 31.516 \times F + 4.025 \times D \quad (11)$$

The cut-off score for these responses is calculated as 0.102, which signifies that the observations having discriminant scores of more than 0.102 would be assigned to Group 2 with higher PC values. On the contrary, observations with discriminant scores of less than the cutoff score would be included in Group 1 having lower PC values.

The numbers of correctly classified items, indicated by hit ratio, along with the cross-validation results for all the dependent variables are provided in Table 18. For SR response, the hit ratio is 81.3% and the cross-validation percentage is 78%. The hit ratio and cross-validation percentage for TL are both 96%. For CF, both the hit ratio and cross-validation percentage are 82%, while the hit ratio and cross-validation percentage for PC are both 74%. From these observations, it can be concluded that the developed step-wise discriminant functions for the responses have the ability to categorize the observations into the corresponding groups with minimum misclassification error.

## 5. Results and Discussion

As mentioned earlier, the aim of this paper is to study the influences of different input parameters of a CNC turning process on its responses as well as to identify the most important parameter for each of the responses. It can be unveiled from both the simultaneous and step-wise estimation methods of discriminant analysis that speed is the most significant parameter for SR and PC. On the other hand, machining environment maximally influences TL and depth of cut is the most influential parameter for CF. The coefficients of these input parameters in the discriminant function for each of the responses, along with the structure correlations, indicate their comparative strengths of influence on the responses.

In this analysis, it can be noticed that an increase in speed causes SR to decrease. The decrease in SR can be explained due to decrease in built-up-edge formation at higher temperature at the chip-tool interface at higher spindle speed [28]. An increase in feed rate leads to an increase in SR. As feed rate increases, wide and deep cracks are formed which are responsible for poor surface quality of the machined components [29]. An increase in feed rate also causes CF to increase due to the required plastic deformation and generation of excess heat in the machining area, thereby increasing tool wear and eventual deterioration of surface finish. Although SR increases with increasing values of depth of cut, it is supposed to have negligible effect on SR. The slight variation in SR is due to tool chatter, occurring at higher values of depth of cut. Better surface quality of the machined components can be achieved at higher nose radius. It can be attributed to lower strength of insert nose. At smaller nose radius of the tool, the contact length between insert tip of the tool and workpiece becomes narrower, thus reducing heat dissipation from the shear zone, causing higher stress and heat concentration at the zone, thereby increasing tool wear and SR [29]. It can also be observed that cryogenic machining environment improves SR because the machining zone temperature is effectively controlled at cryogenic environment, which simultaneously reduces adhesion between tool flank faces and chip, thus reducing tool wear and SR [30].

**Table 18** Classification results for step-wise discriminant analysis

| Output variable | Type of validation | Count (%) | Group | Predicted group membership 1 | 2 | Total |
|---|---|---|---|---|---|---|
| SR | Original | Count | 1 | 65 | 12 | 77 |
| | | | 2 | 16 | 57 | 73 |
| | | % | 1 | 84.4 | 15.6 | 100 |
| | | | 2 | 21.9 | 78.1 | 100 |
| | Cross-validated | Count | 1 | 65 | 12 | 77 |
| | | | 2 | 21 | 52 | 73 |
| | | % | 1 | 84.4 | 15.6 | 100 |
| | | | 2 | 28.8 | 71.2 | 100 |
| TL | Original | Count | 1 | 76 | 0 | 76 |
| | | | 2 | 6 | 68 | 74 |
| | | % | 1 | 100 | 0 | 100 |
| | | | 2 | 8.1 | 91.9 | 100 |
| | Cross-validated | Count | 1 | 76 | 0 | 76 |
| | | | 2 | 6 | 68 | 74 |
| | | % | 1 | 100 | 0 | 100 |
| | | | 2 | 8.1 | 91.9 | 100 |
| CF | Original | Count | 1 | 68 | 11 | 79 |
| | | | 2 | 16 | 55 | 71 |
| | | % | 1 | 86.1 | 13.9 | 100 |
| | | | 2 | 22.5 | 77.5 | 100 |
| | Cross-validated | Count | 1 | 68 | 11 | 79 |
| | | | 2 | 16 | 55 | 71 |
| | | % | 1 | 86.1 | 13.9 | 100 |
| | | | 2 | 22.5 | 77.5 | 100 |
| PC | Original | Count | 1 | 70 | 15 | 85 |
| | | | 2 | 24 | 41 | 65 |
| | | % | 1 | 82.4 | 17.6 | 100 |
| | | | 2 | 36.9 | 63.1 | 100 |
| | Cross-validated | Count | 1 | 70 | 15 | 85 |
| | | | 2 | 24 | 41 | 65 |
| | | % | 1 | 82.4 | 17.6 | 100 |
| | | | 2 | 36.9 | 63.1 | 100 |

Tool life can be defined as the time elapsed for the measured wear level of a tool to exceed an established critical value of wear. A standard measure of TL is the time to develop its maximum value of flank wear width [31]. Increased values of speed and feed rate cause higher tool flank wear, thereby decreasing TL. An increase of tool flank wear can be attributed to the increase in the concentration of compressive stress at the tool rake face in the vicinity of the cutting edge. Higher tool flank wear is also due to increase in temperature of the tool creating high cutting edge load or lowered tool hardness due to the phenomenon of thermal softening at the proximity of the cutting edge [29]. On the other hand, feed rate does not have any significant influence on TL. Tool flank wear also increases with increasing values of nose radius due to increase in CF, which can be attributed to the increase in the thrust force component [32]. Higher depth of cut implies that the contact length between the cutting edge and workpiece increases, causing deeper wear along the cutting edge, thereby decreasing TL [33]. Machining environment has significant impact on TL. It can be noticed that tool wear is minimum at cryogenic machining environment. Application of cryogenic environment improves wear resistance of the tool and decreases the temperature at the cutting zone, thereby reducing abrasion and adhesion.

With increase in spindle speed of the CNC turning process, CF is found to increase, although insignificantly. It can be attributed to the material strengthening effect induced by the strain gradient [34]. Similarly, nose radius positively influences CF. The increase in CF is due to increase of the thrust force component, along with a marginal increase in feed force and tangential force [35]. Feed rate and depth of cut also have positive influences on CF. With increase in feed and depth of cut, CF increases because the sheared chip cross-section grows larger along with the deformed metal volume, which makes the workpiece material increasingly resistant to shearing, requiring more force to remove the chips [36]. Application of cryogenic environment during CNC turning reduces CF, due to reduction in the coefficient of friction between the chip and the tool, and decrease in the chip contact length due to formation of smaller chips [37].

According to this discriminant analysis, all the five CNC turning parameters have positive influences on PC. Higher power is required for higher CF, simultaneously caused by the increases in speed, feed, depth of cut and nose radius [38]. However, an increase in PC at cryogenic machining environment can be attributed to the increase in strength and hardness of the workpiece, when cooled by the cryogenic fluid [39]. This increase in strength and hardness of the workpiece may lead to an increase in energy consumption while removing material from the workpiece surface. However, it can be noted that both the nose radius and machining environment are insignificant parameters, while speed is the most significant parameter for PC.

## 6. CONCLUSION

This paper deals with the application of discriminant analysis in a CNC turning process to explore the influences of its five input parameters on four responses, and identify the most significant parameter for each of the considered responses. After validating the corresponding assumptions, like absence of multicollinearity and missing data, normality of the independent variables, etc., two sets of discriminant functions are developed. In simultaneous estimation method, all the independent variables are considered, while in step-wise estimation method, the insignificant independent variables are excluded while developing the respective models. Based on the developed discriminant functions, it can be revealed that higher feeds are responsible for poor surface finish of the turned components, where better surface quality is achieved at higher values of speed and nose radius, and cryogenic machining environment. It is least affected by depth of cut. Similarly, higher values of speed, depth of cut and nose radius are responsible for reduced tool life. It would increase at cryogenic machining environment and remain unaffected due to feed. Cutting force would increase at higher values of feed and depth of cut. Cryogenic machining environment would cause cutting force to decrease, and speed and nose radius have no significant roles on cutting force. Finally, higher values of speed, feed and depth of cut are all responsible for more power consumption during CNC turning operation, while it remains unaffected due to changes in nose radius and machining environment. It can also be propounded that the reduced discriminant functions developed by step-wise estimation method has similar effectiveness as those formulated with the inclusion of all the independent variables. Higher values of hit ratio and cross-validation percentage conclude that both the functions are well capable of classifying objects into the corresponding binary groups.

Discriminant analysis has few limitations. It requires certain assumptions to be satisfied in order to provide satisfactory results. In discriminant analysis, with an increase in the number of independent variables, sample size must be increased as well. However, its advantages outweigh its limitations. Discriminant analysis has several advantages as an effective prediction tool. The causal relationship between the *independent* and dependent variables can be envisaged based on the developed discriminant function and computed discriminant score, which provides it an edge over the other prediction tools, like support vector machine, artificial neural network, etc. It is capable of dimensionality reduction as the dimensionality of each observation is reduced from multiple independent variables to a single attribute (discriminant score) for binary discriminant analysis. It is similar to multiple regression analysis, predicting values of dependent variables based on the developed relationship between independent and dependent variables. These benefits encourage checking the applicability of multiple discriminant analysis for modeling and parametric analysis of similar machining processes as future research interest.

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