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NON-FRICTIONAL DAMPING IN THE CONTACT OF TWO FIBERS SUBJECT TO SMALL OSCILLATIONS

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Mikhail Popov[1,2]

[1]Berlin University of Technology, Germany
[2]National Research Tomsk State University, Russia

Abstract *Structural damping is discussed for the contact of two fibers in a woven material. In the presence of both normal and tangential oscillations, structural (relaxation) damping takes place even with perfect sticking in the contact, where slip-related frictional damping disappears. For the case of an infinite coefficient of friction and small amplitudes a closed-form solution for energy lost during one oscillation cycle is obtained.*

Key Words: *Woven Composites, Structural Damping, Oscillating Contacts*

1. INTRODUCTION

The present paper is concerned with internal damping in woven materials. When fabrics are deformed, energy is dissipated in the contacts between fibers, and it is well known that at least part of this dissipation is due to friction in partial slip zones of the contact. Exact solutions for frictional damping in the contact of spheres due to tangential oscillations go back to Mindlin et. al. [1] and are also applicable to the contact of two crossed cylinders (such as the fibers in a woven material). Damping in the presence of both normal and tangential oscillations, however, has never been described exhaustively and it remains a current research topic [2],[3]. Recently it has been suggested [4] that the superposition of normal and tangential oscillations leads, in addition to slip-related frictional dissipation, to a new type of non-frictional damping that is caused by elastic relaxation due to variations in normal load and therefore contact area. It is found that in the absence of slip (an infinite coefficient of friction) and for small oscillation amplitudes, the energy dissipated during an oscillation cycle is described by

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Corresponding author: Popov Mikhail
Technische Universität Berlin, 10623 Berlin, Germany
E-mail: mikhail.popov@tu-berlin.de

$$Q = \frac{8}{3}\frac{G^*}{E^*}\frac{\partial^2 F_n}{\partial u_z^{\ 2}} u_x^{(0)2} \left| u_z^{(0)} \right| \sin^2 \phi_0 \tag{1}$$

where $u_x^{(0)}$ and $u_z^{(0)}$ are the tangential and normal oscillation amplitudes, ϕ_0 is the phase shift between the oscillations (the oscillation frequencies are identical), F_n is the normal contact force, u_z the indentation depth (relative approach of bodies). E^* and $G^*$are the reduced elastic and shear moduli and can be expressed through elastic modulus E and Poisson's ratio v as follows, when both contact partners are made from the same linear, homogeneous, isotropic material.

$$E^* = \frac{E}{2(1-\nu^2)} \tag{2}$$

$$G^* = \frac{E}{(1+\nu)(2-\nu)} \tag{3}$$

In the present work we specialize the above result for a system of two fibers crossed at right angles, representing a single mesh cell of a woven material. Small oscillations that are applied to the end of one fiber produce oscillations in the contact, leading to structural damping (and frictional damping, if slip is permitted). All assumptions from [4] (linearly elastic materials without viscous effects, infinite coefficient of friction, small amplitudes) are used here as well, so as to isolate the contribution of structural damping to overall losses.

2. Analysis

We consider a very simple model representing a mesh cell of a woven material: two fibers with circular sections (with radius R) that are crossed at right angles, Fig.1. The fibers lie in x, y - plane, while the upward-facing axis is labeled z. Three of the four fiber ends are rigidly embedded in the plane at $z = 0$, while one end is connected to a parallel guide that permits motion in x, y - plane (horizontal and vertical). These boundary conditions are neither the only possible nor necessarily the most representative of real fabrics. The above model is chosen for its simplicity, while other possible configurations are left for future work. The movable end is pre-stressed by deflecting it downwards by $W_{z,0}$, which is of the order of $2R$ in woven materials, due to symmetrical boundary conditions. Through this initial displacement, contact between the fibers is established, and base loading $F^{(0)}$ is produced in the contact. In addition, the movable end of the fiber is forced to oscillate with amplitudes ΔW_x, ΔW_z, a common frequency and phase shift ϕ_0.

Our general approach is as follows: Firstly, the oscillation amplitudes of the movable fiber end are related to force oscillations, with certain amplitudes, in the contact. Linear beam theory is used for this, while the influence of indentation depth u_z is neglected (our general assumption is that $W_{z,0} >> u_z >> \Delta W$). The force oscillations in the contact are then related to geometrical oscillations through the contact stiffness, which is itself determined by the contact configuration, and therefore $W_{z,0}$.

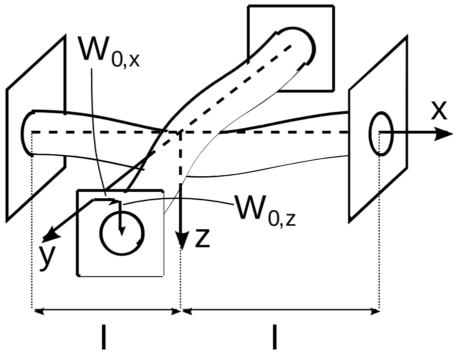

Fig. 1 Two crossed fibers are considered as elastic beams with circular cross-section

Consider a beam of length $2l$ that is stressed with a contact force F_z in the middle and deflected by $W_{0,z}$ at one end. The deflection of the central point (at $x = l$) of the beam with these boundary conditions is known to be [5]

$$W_{I,z}(l) = -\frac{F_z l^3}{24EI} + \frac{W_{0,z}}{2}, \tag{4}$$

where $I = \pi R^4/4$ is the area moment of inertia. For the beam with two fixed ends, only the first component, due to the contact force in the middle, is present:

$$W_{II,z}(l) = \frac{F_z l^3}{24EI}. \tag{5}$$

The difference between the two is equal to indentation depth u_z in the contact:

$$u_z = W_{I,z}(l) - W_{II,z}(l) = -\frac{F_z l^3}{12EI} + \frac{W_{0,z}}{2}. \tag{6}$$

As noted above, we assume that the indentation depth is small compared to the deflection of the beam, and apply non-penetration condition, $u_z = 0$, which leads to a linear relationship between contact force and deflection of the free end:

$$F_z = 6W_{0,z}\frac{EI}{l^3}. \tag{7}$$

For tangential loading, the lower beam is stressed length-wise; its deformation therefore can be neglected. The equations in this case become

$$W_{I,x}(l) = -\frac{F_x l^3}{24EI} + \frac{W_{0,x}}{2}, \tag{8}$$

$$W_{II,x} = 0. \tag{9}$$

Proceeding as above, we obtain

$$F_x = 12W_{0,x}\frac{EI}{l^3}. \tag{10}$$

The force is thus linearly proportional to the deflection of the end of the movable beam. If the latter is now oscillating according to $W_z = W_{z,0} + \Delta W_z \sin \omega t$ and $W_x = \Delta W_x \sin(\omega t + \phi_0)$, then the amplitudes of the force oscillations in the contact are

$$\Delta F_z = 6\Delta W_z \frac{EI}{l^3}, \tag{11}$$

$$\Delta F_x = 12\Delta W_x \frac{EI}{l^3}. \tag{12}$$

Now, when the oscillation of contact forces is known, the corresponding components of relative displacement of contacting bodies, u_x and u_z can be found by dividing the force increments by contact stiffness k_x in the tangential direction or k_z in normal direction. The latter are known to be

$$k_x = 2G^* a, \tag{13}$$

$$k_z = 2E^* a, \tag{14}$$

where a, the contact radius, is equal to $\sqrt{Ru_z}$ in the contact of a sphere with a plane or the contact of two crossed cylinders [6]. The derivative of the normal contact stiffness with respect to indentation depth u_z is given by

$$\frac{\partial^2 F_n}{\partial u_z{}^2} = \frac{\partial k_z}{\partial u_z} = E^* \sqrt{\frac{R}{u_z}}. \tag{15}$$

One last step is necessary to tie all equations together: indentation depth u_z, which, for a spherical contact, is given by [6]

$$u_z = \left(\frac{3F^{(0)}}{4E^* \sqrt{R}} \right)^{2/3}, \tag{16}$$

where $F^{(0)} = 6W_{z,0} \frac{EI}{l^3}$ is the initial loading determined with Eq. (7). Substituting all factors into the relaxation-damping Eq. (1) and simplifying, gives the following result:

$$Q = 4\left(\frac{2}{3}\right)^{2/3} \pi^{5/3} \frac{(1+\nu)(2-\nu)}{(1-\nu^2)^{1/3}} E \left(\frac{R}{l}\right)^5 \left(\frac{R}{W_{z,0}}\right)^{4/3} \Delta W_x^2 \left|\Delta W_z\right| \sin^2 \phi_0 . \tag{17}$$

By introducing fiber aspect ratio $\alpha = l/R$, the normalized initial displacement $\tilde{W}_0 = \Delta W_z / R$ and grouping some of the factors under

$$q = 4\left(\frac{2}{3}\right)^{2/3} \pi^{5/3} \frac{(1+\nu)(2-\nu)}{(1-\nu^2)^{1/3}}, \tag{18}$$

we can write the result more compactly as

$$Q = q\alpha^{-5} \tilde{W}_0^{-4/3} E \left(\Delta W_x\right)^2 \left|\Delta W_z\right| \sin^2 \phi_0 . \tag{19}$$

Note that q varies only slightly for typical values of ν. Its numerical value is approximately 45 for $\nu = 0.2$, 47 for $\nu = 0.3$ and 51 for $\nu = 0.5$).

3. Discussion

The obtained result for a system of two crossed fibers is similar to Eq. (1) that describes relaxation damping when oscillations are applied to the contact directly. In particular, the proportionality to the square of the tangential oscillation amplitude, and the modulus of the normal oscillation amplitude is preserved, which is, of course, not surprising, since linearity is assumed in the derivation. More interesting is the inverse proportionality to the fifth power of the aspect ratio of the fibers, which means that the effect will be much more pronounced in densely woven fabrics than in sparse ones.

The obtained result is only valid for an infinite coefficient of friction. An interesting avenue for future work would be to consider realistic coefficients of friction and to determine the relative importance of frictional and structural damping. Also, although a physical interpretation of relaxation damping in perfect stick conditions is given in [4], the underlying mechanism in the presence of sliding is yet to be determined. Other unexplored possibilities involve other boundary conditions for the mesh cell, embedding the cell in a viscous medium (which would extend the results to woven composites), as well as experimental verification.

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