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GROUP VELOCITY OF LIGHT IN LADDER-TYPE SPHERICAL QUANTUM DOT WITH HIDROGENIC IMPURITY

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Abstract. *The present paper analyzes the group velocity of light in a ladder-type spherical quantum dot with on-center hydrogenic impurity. The three level ladder configuration is realized by energy levels of hydrogenic impurity, together with the probe and control laser fields. Group velocity of the probe field is then investigated as a function of the spherical quantum dot radius, probe field frequency, and control laser field intensity.*

Key words: *spherical quantum dot, group velocity, hydrogenic impurity, ladder-type configuration*

1. INTRODUCTION

The velocity of propagation of light through different atomic vapors has been investigated during the last decades. The possibility of using coherence and interference effects for the purpose of enhancement of refractive index and reducing the velocity of light was noticed by (Scully, 1991) and (Harris et al., 1990). In the paper by (Harris et al., 1992), the velocity of light was decreased 250 times by a 10 cm long Pb vapor cell. After that, using a similar technique, the velocity of light was decreased to 90 m/s (Kash et al., 1999), 17 m/s (Hau et al., 1999) and 8 m/s (Budker et al., 1999).

In recent years, developments in modern technology have made it possible to fabricate very small semiconductor structures, such as quantum wells (QWs), quantum wires (QWs), and quantum dots (QDs). All these quantum structures have properties to change their eigenvalues, their wave functions, and their dipole momenta by changing their dimensions. Therefore, they are very important for potential use in nanoelectronic and optoelectronic devices (Segal et al., 2003). These nanostructures have also been used for the realization of slow light effect. This effect is important since it has potential applications in quantum computers and information storage (Liu et al., 2001; Heinze et al., 2013). (Mirzaei et al.,

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2014) investigated the group velocity of light in cylindrical quantum dots. (Rezaei et al., 2014) studied the group velocity of light in a two-dimensional parabolic quantum dot.

To our knowledge, the slow light effect in a spherical quantum dot (SQD) with hydrogenic impurity in a three-level ladder configuration has not been investigated so far. In this paper, we investigated the effect of hydrogenic impurity, SQD radius, and laser field intensity on a light group velocity.

The paper is organized in the following way. The theoretical framework is presented in Section 2. Section 3 is devoted to the results and a brief discussion. Finally, the conclusion is given at the end of this paper, in Section 4.

2. THEORY

In this paper a spherical QD with radius R_0 is considered. The Hamiltonian of the on-center hydrogenic impurity in a spherical QD in effective atomic units is given by:

$$H = -\frac{1}{2}\nabla^2 - \frac{1}{r} + V(r), \tag{1}$$

where $V(r)$ is confining potential:

$$V(r) = \begin{cases} 0, & r \le R_0 \\ \infty, & r > R_0 \end{cases}. \tag{2}$$

In atomic units, distance and energy are measured in effective Bohr radius ($a_0^* =$ ($4\pi\varepsilon\hbar^2/(m^* e^2)$) and energy units ($E^* = m^* e^4/(4\pi\varepsilon\hbar^2)^2$), where m^* is the electron effective mass, e is the electron charge, ε is the dielectric constant of the used material and $\hbar$ is Planck constant. Wave function of this system can be written as a product of the radial part $R_{nl}(r)$ and spherical harmonics $Y_{lm}(\theta,\ \phi)$:

$$\Psi_{nlm}(\vec{r}) = C_{nl} R_{nl}(r) Y_{lm}(\theta,\ \phi), \tag{3}$$

where C_{nl} are the normalization constants. Radial wave function is given by:

$$R_{nl}(r) = r^l e^{-r\sqrt{-2E_{nl}}} F\left(-\frac{1}{\sqrt{-2E_{nl}}} + l + 1,\ 2l+2,\ 2r\sqrt{-2E_{nl}}\right), \tag{4}$$

where F is confluent hypergeometric (Kummer) function. Eigenenergies E_{nl} are found from the Dirichlet boundary condition, for which $R_{nl}(r = R_0) = 0$ is fulfilled.

Here, we take into consideration the states: ls_0, $2p_{-1}$, and $3d_{-2}$, and their interaction with the two left-circularly polarized laser fields. The transitions l$s_0 \leftrightarrow 2p_{-1}$ and $2p_{-1} \leftrightarrow 3d_{-2}$ are two dipole allowed transitions and l$s_0 \leftrightarrow 3d_{-2}$ is a dipole forbidden transition, thus, making a ladder-type configuration. The corresponding transition dipole momenta, transition frequencies, and decay rates are d_{21}, d_{32}, ω_{21}, ω_{32}, γ_{21}, and γ_{32}, respectively. A weak probe laser field (with intensity I_p, central frequency ω_p, and transition detuning $\Delta_p = \omega_{21} - \omega_p$) and a strong control laser field (with intensity I_c, central frequency ω_c, and transition detuning $\Delta_c = \omega_{32} - \omega_c$) drive the transitions l$s_0 \leftrightarrow 2p_{-1}$ and $2p_{-1} \leftrightarrow 3d_{-2}$, respectively.

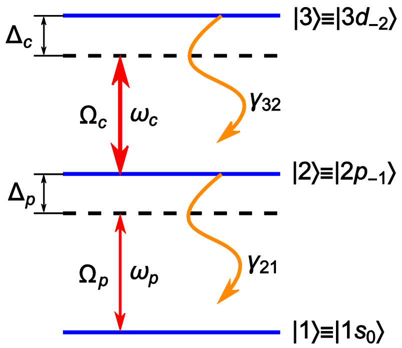

Fig. 1 Schematic diagram of a three-level ladder system.

The Hamiltonian of this system, in the interaction picture under the dipole and rotating wave approximations, can be written as:

$$H = \hbar\Delta_p |2\rangle\langle 2| + \hbar(\Delta_p + \Delta_c)|3\rangle\langle 3| - \hbar\Omega_p(|1\rangle\langle 2| + |2\rangle\langle 1|) - \hbar\Omega_c(|2\rangle\langle 3| + |3\rangle\langle 2|) \tag{5}$$

where $\Omega_p = \sqrt{2I_p d_{21}{}^2 / (\varepsilon_0 n_r c\hbar^2)}$ and $\Omega_c = \sqrt{2I_c d_{32}{}^2 / (\varepsilon_0 n_r c\hbar^2)}$ are the Rabi frequencies. The evolution of the system is described by the quantum Liouville equation:

$$\dot{\rho} = -\frac{i}{\hbar}[H,\rho] + L\rho \tag{6}$$

where ρ is the density operator, and $L\rho$ is the damping term. The explicit form of the master equations is therefore:

$$\begin{aligned}
\dot{\rho}_{11} &= 2\gamma_{21}\rho_{22} + i(\rho_{21} - \rho_{12})\Omega_p \\
\dot{\rho}_{33} &= -2\gamma_{32}\rho_{33} + i(\rho_{23} - \rho_{32})\Omega_p \\
\dot{\rho}_{21} &= -(\gamma_{21} + i\Delta_p)\rho_{21} + i\Omega_c\rho_{31} + i(\rho_{11} - \rho_{22})\Omega_p \\
\dot{\rho}_{32} &= -(\gamma_{21} + \gamma_{32} + i\Delta_c)\rho_{32} + i\Omega_c(\rho_{22} - \rho_{22}) - i\Omega_p\rho_{31} \\
\dot{\rho}_{31} &= -(\gamma_{32} + i(\Delta_c + \Delta_p))\rho_{31} + i\Omega_c\rho_{21} - i\Omega_p\rho_{32}
\end{aligned} \tag{7}$$

In the steady state regime, and using the closure relation $\rho_{11} + \rho_{22} + \rho_{33} = 1$, this system of equations can be salved analytically.

The matrix dipole elements, that appear in equations for calculating the Rabi frequencies, for σ^- transitions are calculated by using the following equation (Weissbluth, 1978):

$$d_{i,i-1} = \langle \Psi_{n_i l_i m_i} | r_{-1} | \Psi_{n_{i-1} l_{i-1} m_{i-1}} \rangle = C^*_{n_i l_i} C_{n_{i-1} l_{i-1}} \sqrt{\frac{i-1}{2i-1}} r_{i,i-1} \tag{8}$$

where $i = 2,3$ and $r_{i,i-1}$ stands for:

$$r_{i,i-1} = \left\langle R_{n_i l_i} \middle| r \middle| R_{n_{i-1} l_{i-1}} \right\rangle . \tag{9}$$

The quantity r_{-1} is equal to $r_{-1} = x - iy$, and is included in the calculations since the left-circularly polarized laser fields are applied. The susceptibility of the system related to the probe field is given through the matrix element ρ_{21}:

$$\chi = \frac{2Nd_{21}^{\ 2}}{\varepsilon_0 \hbar \Omega_p} \rho_{21} \tag{10}$$

where N is the density of the three-level systems. The refractive index can be written as:

$$n = 1 + \frac{1}{2} \operatorname{Re} \chi , \tag{11}$$

and the group velocity of the probe light as:

$$v_g = \frac{c}{n(\omega_p) + \omega_p \dfrac{dn}{d\omega_p}} . \tag{12}$$

3. Results and Discussion

In this paper group velocity of light in a ladder-type three level GaAs spherical QDs with hydrogenic impurity is investigated. We analyzed the effects of control laser intensity and radius on the light group velocity.

The relative dielectric constant of GaAs is $\varepsilon = 12.9$, and the electron effective mass is $m^* = 0.067m_e$, where m_e is the mass of the free electron (Adachi, 1985; Lien et al. 2001). Therefore, we obtained $a^* = 10.4$ *nm* and $E^* = 10.5$ *meV* for the effective Bohr radius and atomic energy units, respectively. Decoherence in semiconductors originates from radiative decay and electron-phonon interactions. At very low temperatures the radiative decay dominates over electron-phonon interactions. Therefore, only this term for decoherence needs to be included. In Table 1 the energy values for the states: $1s_0$, $2p_{-1}$, and $3d_{-2}$ as well the transition dipole moment elements d_{21} and d_{32} are given. Probe field intensity is set to $I_p = 10^4$ W/m^2.

Table 1 Energy values and the transition dipole moment elements.

R [a_0^*]	ω_{1s} [10^{10} Hz]	ω_{2p} [10^{10} Hz]	ω_{3d} [10^{10} Hz]	d_{21} [10^{-27} Cm]	d_{32} [10^{-27} Cm]
1.0	3929.84	13612.4	24776.7	0.474471	0.622274
1.2	2101.19	9016.35	16826.5	0.562158	0.744400
1.4	1071.20	6301.81	12083.0	0.647028	0.865696
1.6	449.123	4576.68	9036.67	0.728884	0.986137

3.1. Effect of control laser field

Fig. 2 displays the group velocity in terms of the probe laser frequency for a QD radius $R_0 = 1.0\, a_0^*$. The group velocity is minimal for a resonant frequency, when control field is off – the black line in Fig. 2. If control field is turned on, the slow light is created at a range of frequencies – the range between the two velocity minima. This range is called slow light frequency range. The slow light frequency range becomes enlarged by increasing the control laser field intensity. The maximum of the group velocity of the probe field inside the slow light frequency range occurs at resonant frequency, and increases by increasing the control field intensity.

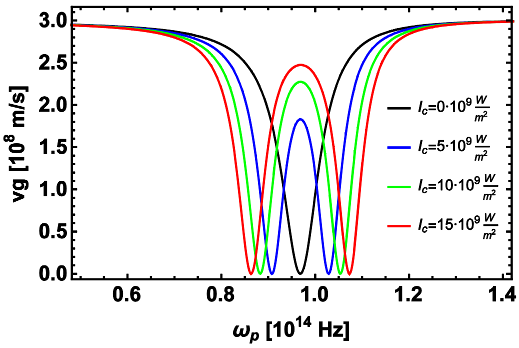

Fig. 2 Light group velocity in spherical QDs with radius $R_0 = 1.0\, a_0^*$ at different control laser field intensities in terms of the probe field frequency.

3.2. Effect of QD radius

In Fig. 3 group light velocity is shown for different SQDs radii, when control laser field is turned off. It is apparent that the frequency at which the minimal group velocity is achieved, decreases by increasing the SQDs size.

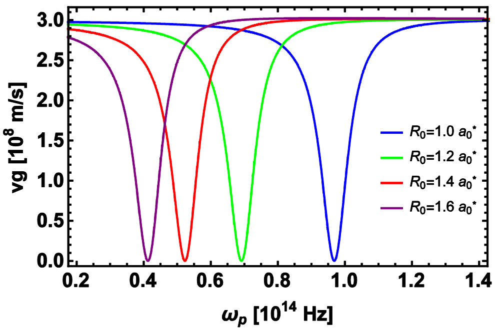

Fig. 3 Light group velocity in spherical QDs at different QDs radii in terms of the probe field frequency.

In Fig. 4 group light velocity is shown for different SQDs radii, when control laser field is turned on. It is apparent that the slow light frequency range increases with increasing SQDs radii. Also, the maximal group velocity inside the slow light frequency range increases by increasing the SQDs size.

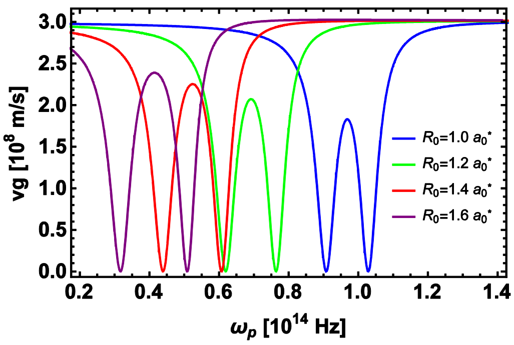

Fig. 4 Light group velocity in spherical QDs with $I_c = 5{\cdot}10^9\ W/m^2$ at different QDs radii in terms of the probe field frequency.

4. Conclusion

In summary, in this paper we investigated the energy eigenvalues and the corresponding wave functions of the hydrogenic impurity confined by SQD. The obtained results are used to further investigate the effects of hydrogenic impurity, SQDs radius, and laser field intensity on a light group velocity. To summarize, we have concluded that: (i) the slow light frequency range becomes enlarged by increasing the control laser field intensity, (ii) the maximum of the group velocity of the probe field inside the slow light frequency range occurs at resonant frequency, (iii) the maximum of the group velocity of the probe field inside the slow light frequency range increases by increasing the control field intensity, (iv) the frequency at which minimal group velocity is achieved decreases by increasing the SQDs size, (v) the slow light frequency range increases with increasing SQDs radii, and (vi) the maximal group velocity inside the slow light frequency range increase by increasing the SQDs size.

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GRUPNA BRZINA SVETLOSTI U SFERNOJ KVANTNOJ TAČKI SA VODONIČNOM NEČISTOĆOM U KASKADNOJ KONFIGURACIJI

U ovom radu je analizirana grupna brzina svetlosti u sfernoj kvantnoj tački sa vodoničnom nečistoćom u centru, u kaskadnoj konfiguraciji. Kaskadna konfiguracija sa tri nivoa je realizovana pomoću energijskih nivoa vodonične nečistoće i dva laserska polja, probnim i kontrolnim. Proučavan je uticaj poluprečnika kvantne tačke, frekvence probnog i intenziteta kontrolnog polja na grupnu brzinu probnog polja.

Ključne reči: *Sferna kvantna tačka, Vodonična nečistoća, Grupa brzina, Kaskadna konfiguracija*