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# TACHYONIC INFLATION ON (NON-)ARCHIMEDEAN SPACES †

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**Abstract.** *The relevance of quantum rolling tachyons and corresponding inflation scenario in the frame of the standard, p-adic and adelic cosmology are reviewed. The field theory of tachyon matter proposed by Sen in a zero-dimensional version leads to a number of models with a particle moving under different potentials. We consider quantum propagators of the models, as well as, the vacuum states and conditions necessary to construct an adelic generalization. In addition we present inflationary scenarios for some interesting models based on analytic and numeric calculations. A brief overview of the state of art in the field and suggestions for further consideration close the paper.*

**Key words:** *tachyons, DBI scalar field, inflation, quantum cosmology, nonarchimedean spaces*

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## 1. Introduction

The main task of quantum cosmology [1] is to describe the evolution of the universe in the very early stage. Usually one takes that the universe is described by a complex wave function. Since quantum cosmology is related to the Planck scale phenomena it is logical to consider various geometries (in particular the nonarchimedean [2] and noncommutative [3] one) and parametrization of the space-time coordinates: real, $p$-adic, or adelic [4].

It is quite natural to consider that in the very early stage of its evolution the universe was in a quantum state, which is described by a wave function. Concerning the wave function, we will here maintain the standard point of view: the wave function takes complex values, but space-time coordinates and matter fields will be mainly treated in a more complete way to be adelic, i.e. they have real as well as $p$-adic properties simultaneously.

There is a quantum gravity uncertainty [5] $\Delta x$ while measuring distances around the Planck length $\ell_0 \sim 10^{-33} cm$, which restricts priority of archimedean geometry based on real numbers and gives rise to employment of nonarchimedean geometry related to $p$-adic numbers [6].

Adelic quantum mechanics (AQM) [7] applied to quantum cosmology provides realization of all the above statements. The successful application of $p$-adic numbers and adeles in modern physics started in 1987, in the context of string amplitudes [6, 8], however for a systematic research in this field it was necessary to formulate $p$-adic quantum mechanics [9] and AQM [7].

$p$-Adic gravity and the wave function of the universe were considered in [10]. An idea of the fluctuating number fields at the Planck scale was introduced and it was suggested to restrict the Hartle-Hawking [11] proposal to summation only over algebraic manifolds. Like in AQM, adelic eigenfunction of the universe is a product of the corresponding eigenfunctions of real and all $p$-adic cases. $p$-Adic wave functions are defined by $p$-adic generalization of the Hartle-Hawking path integral proposal.

Supernova Ia observations [13] and cosmic microwave background (CMB) radiation data are suggesting that the expansion of our Universe seems to be in an accelerated state which is referred to as the "dark energy" effect. A need for understanding these new and rather surprising facts, including (cold) "dark matter", has motivated numerous authors to reconsider different inflation scenarios. Despite some evident problems [14] such as insufficiently long period of inflation, tachyon-driven scenarios [15, 16] remain highly interesting for study.

There have been a number of attempts to understand this description of the early Universe via (classical) nonlocal cosmological models, first of all via $p$-adic inflation models [17, 18], which are represented by nonlocal $p$-adic string theory coupled to gravity. Another direction is the investigation of the $p$-adic inflation near a maximum of the nonlocal potential. It was found that higher-order derivative operators can support a (sufficiently) prolonged phase of slow-roll inflation [19].

The AQM contains ordinary and all $p$-adic quantum mechanics. As there is not an appropriate $p$-adic Schrödinger equation, there is also no $p$-adic generalization of the Wheeler-De Witt equation. Instead of the differential approach, Feynman's path integral method is exploited [20, 21, 22].

There was a problem with the Hartle-Hawking approach when matter fields were included into consideration. The solution of this problem was proposed. It was found that consideration was much more successful when minisuperspace cosmological models were treated as models of adelic quantum mechanics [12]. For the review and detailed discussion see [2, 23]. The nonarchimedean and noncommutative cosmological quantum models with extra dimensions and an accelerating phase have been considered, as well as the relevant models in a pure quantum mechanical context [24].

This review is organized as follows: after the Introduction, in Chapter 2 we give basic information on "$p$-adics" and adeles. Chapter 3 is devoted to $p$-adic and AQM as an underlying formalism for the corresponding approach to quantum cosmology, which is briefly explained in Chapter 4. The next two Chapters are reserved for actual problems in HEP and cosmology: tachyon dynamics and related inflation. Following S. Kar's idea on the possibility of the examination of zero dimensional theory of the field theory of (real) tachyon matter [25], we consider real and $p$-adic aspects of a few relevant models in Chapter 5. The corresponding propagators and vacuum states for $p$-adic and adelic tachyons are considered. A recent generalization of tachyon field dynamics is also briefly reviewed. Classical inflationary processes for these models are calculated and discussed in Chapter 7. Conclusion and a few ideas for future research ending the paper. A list of references is quite subjective, it is neither exhaustive nor complete one, but could be useful for a reader interested in having a better insight in this field of investigation.

## 2. $p$-Adic Numbers and Adeles

The completion of (rational numbers) $Q$ with respect to the standard absolute value ($|\cdot|_\infty$) gives (real numbers) $R$, and an algebraic extension of $R$ makes (complex numbers) $C$. According to the Ostrowski theorem [4] any non-trivial norm on the field of rational numbers $Q$ is equivalent to the absolute value $|\cdot|_\infty$ or to the $p$-adic norm $|\cdot|_p$, where $p$ is a prime number. $p$-Adic norm is the nonarchimedean (ultrametric) one and for a rational number, $0 \neq x \in Q,\ x = p^\nu \frac{m}{n},\ 0 \neq n, \nu, m \in Z$, has a value $|x|_p = p^{-\nu}$. The completion of $Q$ with respect to the $p$-adic norm for a fixed $p$ leads to the corresponding field of $p$-adic numbers $Q_p$. The completions of $Q$ with respect to $|\cdot|_\infty$ and all $|\cdot|_p$ exhaust all possible completions of $Q$. $p$-Adic number $x \in Q_p$, in the canonical form, is an infinite expansion

$$x = p^\nu \sum_{i=0}^{\infty} x_i p^i, \qquad x_0 \neq 0, \quad 0 \leq x_i \leq p-1. \tag{1}$$

The norm of $p$-adic number $x$ in (1) is $|x|_p = p^{-\nu}$ and satisfies not only the triangle inequality, but also the stronger one

$$|x + y|_p \leq \max(|x|_p, |y|_p). \tag{2}$$

Metric on $Q_p$ is defined by $d_p(x, y) = |x - y|_p$. This metric is the nonarchimedean one and the pair $(Q_p, d_p)$ presents locally compact, topologically complete, separable and totally disconnected $p$-adic metric space. $p$-Adic ball $B_\nu(a)$ with the centre at the point $a$ and the radius $p^\nu$ is the set

$$B_\nu(a) = \{x \in Q_p : \ |x - a|_p \leq p^\nu, \ \nu \in Z\}. \tag{3}$$

Elementary $p$-adic functions [26] are given by the series of the same form as in the real case, with the corresponding domain of convergence.

Real and $p$-adic numbers are unified in the form of the adeles [27]. An adele is an infinite sequence

$$a = (a_\infty, a_2, ..., a_p, ...), \tag{4}$$

where $a_\infty \in Q_\infty$, and $a_p \in Q_p$, with restriction to $a_p \in Z_p$ ( $Z_p = \{x \in Q_p : |x|_p \leq 1\}$) for all but a finite set $S$ of primes $p$. If we introduce $\mathcal{A}(S) = Q_\infty \times \prod_{p \in S} Q_p \times \prod_{p \notin S} Z_p$ then the space of all adeles is $\mathcal{A} = \bigcup_S \mathcal{A}(S)$, which is a topological ring. An important function on $\mathcal{A}$ is the additive character $\chi(x), \ x \in \mathcal{A}$, which is a continuous and complex-valued function with basic properties:

$$|\chi(x)|_\infty = 1, \quad \chi(x + y) = \chi(x)\chi(y). \tag{5}$$

This additive character may be presented as

$$\chi(x) = \prod_\upsilon \chi_\upsilon(x_\upsilon) = \exp(-2\pi i x_\infty) \prod_p \exp(2\pi i \{x_p\}_p), \tag{6}$$

where $\upsilon = \infty, 2, \cdots, p, \cdots$, and $\{x\}_p$ is the fractional part of the $p$-adic number $x$. Map $\varphi : \mathcal{A} \to C$, which has the form

$$\varphi(x) = \varphi_\infty(x_\infty) \prod_{p \in S} \varphi_p(x_p) \prod_{p \notin S} \Omega(| \ x_p \ |_p), \tag{7}$$

where $\varphi_\infty(x_\infty) \in D(Q_\infty)$ is an infinitely differentiable function on $Q_\infty$ and falls to zero faster than any power of $| \ x_\infty \ |_\infty$ as $| \ x_\infty \ |_\infty \to \infty, \ \ \varphi_p(x_p) \in D(Q_p)$ is a locally constant function with compact support, and

$$\Omega(|x|_p) = \begin{cases} 1, & |x|_p \leq 1, \\ 0, & |x|_p > 1, \end{cases} \tag{8}$$

is called an elementary function on $\mathcal{A}$. The existence of $\Omega$-function is unavoidable for a construction of any quantum adelic model. The Fourier transform is

$$\tilde{\varphi}(\xi) = \int_{\mathcal{A}} \varphi(x)\chi(\xi x) dx \tag{9}$$

and it maps one-to-one $D(\mathcal{A})$ onto $D'(\mathcal{A})$. It is worth noting that $\Omega$-function is a counterpart of the Gaussian in the real case, since it is invariant with respect to the Fourier transform. It is also an important issue for consideration of the ground state(s) of quantum mechanical systems at high energies, where the use of $p$-adic numbers and nonarchimedean geometry in "modelling" should be fully justified. About the integrals of the Gauss type over the $p$-adics and $\lambda_p$ arithmetic function see, for instance, [4].

## 3. $p$-Adic and Adelic Quantum Mechanics

In the foundations of standard quantum mechanics (over $R$) one usually starts with a representation of the canonical commutation relation

$$[\hat{q}, \hat{k}] = i\hbar, \tag{10}$$

where $q$ is a spatial coordinate and $k$ is the corresponding momentum. The canonical commutation relation in $p$-adic case can be represented by the Weyl operators ($h = 1$)

$$\hat{Q}_p(\alpha)\psi_p(x) = \chi_p(\alpha x)\psi_p(x), \quad \hat{K}_p(\beta)\psi(x) = \psi_p(x+\beta), \tag{11}$$

$$\hat{Q}_p(\alpha)\hat{K}_p(\beta) = \chi_p(\alpha\beta)\hat{K}_p(\beta)\hat{Q}_p(\alpha) \tag{12}$$

so that (12) states instead of (10) in the $p$-adic one.

Dynamics of a $p$-adic quantum model is described by a unitary operator of evolution $U(t)$ without using the Hamiltonian. Instead of that, the evolution operator has been formulated in terms of its kernel $\mathcal{K}_t(x, y)$

$$U_p(t)\psi(x) = \int_{Q_p} \mathcal{K}_t(x, y)\psi(y)dy. \tag{13}$$

In this way [9] $p$-adic quantum mechanics is given by a triple

$$(L_2(Q_p), W_p(z_p), U_p(t_p)). \tag{14}$$

Keeping in mind that standard quantum mechanics can be also given as the corresponding triple, ordinary and $p$-adic quantum mechanics can be unified in the form of AQM [7]

$$(L_2(\mathcal{A}), W(z), U(t)). \tag{15}$$

$L_2(\mathcal{A})$ is the Hilbert space on $\mathcal{A}$, $W(z)$ is a unitary representation of the Heisenberg-Weyl group on $L_2(\mathcal{A})$ and $U(t)$ is a unitary representation of the evolution operator on $L_2(\mathcal{A})$. The evolution operator $U(t)$ is defined by

$$U(t)\psi(x) = \int_{\mathcal{A}} \mathcal{K}_t(x, y)\psi(y)dy = \prod_v \int_{Q_v} \mathcal{K}_t^{(v)}(x_v, y_v)\psi^{(v)}(y_v)dy_v. \tag{16}$$

About the eigenvalue problem for $U(t)$ see [7].

Note, in accordance with (7) that any adelic eigenfunction has the form

$$\Psi(x)=\Psi_\infty(x_\infty)\prod_{p\in S}\Psi_p(x_p)\prod_{p\notin S}\Omega(\mid x_p\mid_p),\quad x\in\mathcal{A}, \tag{17}$$

where $\Psi_\infty\in L_2(R)$, $\Psi_p\in L_2(Q_p)$. A suitable way to calculate $p$-adic propagator $\mathcal{K}_p(x'',t'';x',t')$ is to use Feynman's path integral method, i.e.

$$\mathcal{K}(x'',t'';x',t')=\int_{x',t'}^{x'',t''}\chi_p\left(-\frac{1}{h}\int_{t'}^{t''}L(\dot q,q,t)dt\right)\mathcal{D}q. \tag{18}$$

It has been evaluated [20, 28] for quadratic Lagrangians in the same way for real and $p$-adic case and the following exact general expression is obtained:

$$\mathcal{K}_v(x'',t'';x',t')=\lambda_v\left(-\frac{1}{2h}\frac{\partial^2\bar S}{\partial x''\partial x'}\right)\left|\frac{1}{h}\frac{\partial^2\bar S}{\partial x''\partial x'}\right|_v^{\frac12}\chi_v(-\frac{1}{h}\bar S(x'',t'';x',t')), \tag{19}$$

When one has a system with more than one dimension with uncoupled spatial coordinates, then the total propagator is the product of the corresponding one-dimensional propagators. AQM may be regarded as a starting point for construction of a more complete theory. In the low-energy limit adelic quantum mechanics becomes the ordinary one [21].

## 4. Quantum Cosmology and Adelic Minisuperspace Models

According to the so-called standard cosmological model, in the very beginning the universe was very small, dense, hot and started to expand very fast. This initial period of evolution and beginning of inflation should be unavoidably described by quantum theory. In the path integral approach to quantum cosmology over the field of real numbers $R$, the starting point is the idea that the amplitude to go from one state with intrinsic metric $h'_{ij}$, and matter configuration $\phi'$ on an initial hypersurface $\Sigma'$, to another state with metric $h''_{ij}$, and matter configuration $\phi''$ on a final hypersurface $\Sigma''$, is given by a functional integral of the form

$$\langle h''_{ij},\phi'',\Sigma''|h'_{ij},\phi',\Sigma'\rangle=\int\mathcal{D}g_{\mu\nu}\mathcal{D}\Phi e^{-S[g_{\mu\nu},\Phi]}, \tag{20}$$

over all four-geometries $g_{\mu\nu}$, and matter configurations $\Phi$, which interpolate between the initial and final configurations. In this expression $S[g_{\mu\nu},\Phi]$ is an Einstein-Hilbert action for the gravitational and matter fields. This expression stays valid in the $p$-adic case too.

Only as an useful illustration let us present a de Sitter model. In general, the de Sitter models are the models with the cosmological constant $\Lambda$ and without matter fields. The corresponding Einstein-Hilbert action is [29]

$$S=\frac{1}{16\pi G}\int_M d^Dx\sqrt{-g}(R-2\Lambda)+\frac{1}{8\pi G}\int_{\partial M}d^{D-1}x\sqrt{h}K, \tag{21}$$

where R is the scalar curvature of $D$-manifold $M$, $K$ is the trace of the extrinsic curvature $K_{ij}$ of the boundary $\partial M$ of the $D$-manifold $M$. The general form of the metric for these models is

$$ds^2 = \sigma^2[-N^2dt^2 + a^2(t)d\Omega^2_{D-1}], \tag{22}$$

where $d\Omega^2_{D-1}$ denotes the metric on the unit $(D-1)$-sphere

$$\sigma^{D-2} = \frac{8\pi G}{V^{D-1}(D-1)(D-2)},$$

and $V^{D-1}$ is the volume of the unit $(D-1)$-sphere. In the $D=3$ case, this model is related to the multiple sphere configuration and wormhole solutions. $\upsilon$-Adic classical action for this model is [2]

$$\bar{S}_\upsilon(a'', N; a', 0) = \frac{1}{2\sqrt{\lambda}}\left[N\sqrt{\lambda} + \lambda\left(\frac{2a''a'}{\sinh(N\sqrt{\lambda})} - \frac{a'^2 + a''^2}{\tanh(N\sqrt{\lambda})}\right)\right]. \tag{23}$$

Let us note that $a$ denotes a scale factor and $\lambda$ denotes here the appropriately rescaled cosmological constant $\Lambda$, i.e. $\lambda = \sigma^2\Lambda$. This model was investigated in details [23, 30]. For this model, the adelic wave function is in the form

$$\Psi(a) = \Psi_\infty(a)\prod_p \Psi_p(a_p), \tag{24}$$

where again $\Psi_\infty(a)$ is a standard wave function and $\Psi_p(a_p)$ are $p$-adic wave functions. It is very important that only for finite numbers of $p$, $p$-adic wave functions can be different from $\Omega$ function which is defined by equation (8). At this place we indicate a considerable similarity between the action (23) for the de Sitter model in 2+1 dimensions and the action (54) for the tachyon field in the zero dimensional model!

It is well known that finding conditions under which quantum-mechanical $p$-adic ground state exists in the form of $\Omega$-function and some other eigenfunctions leads to the desired result and it enables adelization of all exactly soluble minisuperspace cosmological models. As usual it provides some restrictions on the parameters of the models. One can suppose that nonarchimedean geometry or "nonarchimedean phase" in evolution of the Universe restricts a set of initial conditions and a set of Lagrangians related to a realistic dynamics of our Universe [31]. The necessary condition for the existence of an adelic model is existence of $p$-adic quantum-mechanical ground state $\Omega(|q_\alpha|_p)$, i.e.

$$\int_{|q_\alpha'|_p\le 1} \mathcal{K}_p(q_\alpha'', N; q_\alpha', 0)dq_\alpha' = \Omega(|q_\alpha''|_p), \tag{25}$$

and, analogously, if a system is in the state $\Omega(p^\nu|q_\alpha|_p)$. If $p$-adic ground state is of the form of the $\delta$-function, one can investigate conditions under which the corresponding kernel of the model satisfies equation

$$\int_{Q_p} \mathcal{K}_p(q''_\alpha, T; q', 0)\delta(p^\nu - |q'_\alpha|_p)dq'_\alpha = \chi_p(ET)\delta(p^\nu - |q''_\alpha|_p), \tag{26}$$

with zero energy $E = 0$. Tachyon fields, inflation and their classical and quantum aspects are discussed in the next Chapters.

## 5. Classical and quantum tachyons

As a central part of the paper we consider a model of tachyons based on a DBI action. This model of non-standard Lagrangian and tachyon-like "matter" was proposed by A. Sen [32]. Here the action is given as:

$$S = -\int d^{D+1}xV(T)\sqrt{1+\eta^{ij}\partial_i T\partial_j T} \tag{27}$$

where $\eta_{00} = -1$ and $\eta_{\alpha\beta} = \delta_{\alpha\beta}$, $\alpha, \beta = 1, 2, 3, ..., D$, $T(x)$ is the scalar tachyon field and $V(T)$ is a tachyon potential. Let conisder a lower dimensional analogues of this tachyon field theory. The corresponding zero dimensional analogue of a tachyon field can be obtained by the correspondence: $x^i \to t, T \to x, V(T) \to V(x)$. The Lagrangian and the action are

$$L_{tach} = -V(x)\sqrt{1-\dot{x}^2}. \tag{28}$$

$$S = \int L_{tach} = -\int dtV(x)\sqrt{1-\dot{x}^2}. \tag{29}$$

In the flat space-time background the equation of motion is [33]

$$\ddot{x}(t) - \frac{1}{V(x)}\frac{dV}{dx}\dot{x}^2(t) = -\frac{1}{V(x)}\frac{dV}{dx}. \tag{30}$$

There is a "mathematical" method of transforming a class of non-standard Lagrangians to a canonical form [34, 35]. An improved method based on the classical canonical transformation (CCT), we present here, with a potential to be very useful in quantization of the models based on DBI Lagrangians.

### 5.1. Classical Canonical Transformation

It is well known that the CCT is a change of the phase space variables $(x, k)$ to the new $(\tilde{x}, \tilde{k})$ ones, which preserves the Poisson bracket

$$\{x, k\}_{P.B.} = 1 = \{\tilde{x}, \tilde{k}\}_{P.B.}. \tag{31}$$

It was shown that a unitary transformations of coordinate (field) $x$ and conjugate momenta $k$ at the classical level is

$$x, k \quad \mapsto \quad \tilde{x}, \tilde{k}, \tag{32}$$

$$\mathcal{H}_{tach}(x, k) \quad \mapsto \quad \tilde{\mathcal{H}}_{tach}(\tilde{x}, \tilde{k}), \tag{33}$$

which also preserves form of Hamilton's equations [33].

Note the conjugate momenta and (conserved) Hamiltionian are [15]:

$$\begin{aligned} k = \frac{\partial L_{tach}}{\partial \dot{x}} &= \frac{\dot{x}}{\sqrt{1-\dot{x}^2}}V(x), \\ \mathcal{H}_{tach}(x,k) &= \sqrt{k^2+V^2(x)}. \end{aligned} \tag{34}$$

A generating function $G$, which specifies point canonical transformation is constructed

$$G(\tilde{x},k) = -kF(\tilde{x}), \tag{35}$$

where $F(\tilde{x})$ is an arbitrary function of a new field. The new coordinate and old momenta can be expressed as

$$\tilde{x} = F^{-1}(x), \quad k = \frac{1}{F'(\tilde{x})}\tilde{k}, \tag{36}$$

where $F^{-1}(x)$ is an inverse function of $F(\tilde{x})$ and $F' = dF/d\tilde{x}$.

Hamilton's equations become

$$\dot{\tilde{x}} = \frac{1}{F'}\frac{\tilde{k}}{\sqrt{\tilde{k}^2+F'^2V^2}}, \tag{37}$$

$$\dot{\tilde{k}} = -\frac{1}{F'^2}\frac{1}{\sqrt{\tilde{k}^2+F'^2V^2}}\left[(F')^2V\frac{dV(F)}{dF} - F''k^2\right], \tag{38}$$

while the equation of motion transforms to

$$\ddot{\tilde{x}} + \left(\frac{F''}{F'} - F'\frac{d\ln V(F)}{dF}\right)\dot{\tilde{x}}^2 + \frac{1}{F'}\frac{d\ln V(F)}{dF} = 0. \tag{39}$$

Note that equation (39) still contains term quadratic with respect to time derivative of a new coordinate (field) $\dot{\tilde{x}}$ (like (30)). Let us stress that this procedure, at the classical level, is formally invariant with respect to the choice of the background number fields $R$ or $Q_p$!

Up to now function $F(\tilde{x})$ was an arbitrary one. If a function $1/V(x)$ is integrable, the function $F(\tilde{x})$ can be defined in such a way that the second term in (39) vanishes. Then, the equation of motion (39) is reduced to

$$\ddot{\tilde{x}} + \frac{1}{F'}\frac{d\ln V(F)}{dF} = 0. \tag{40}$$

Note that equation (40) now stands for the system without term quadratic with respect to $\dot{\tilde{x}}$ (unlike (30) and (39)).

### 5.2. Tachyon Potentials

There are several tachyon potentials motivated by string theory and DBI action. Let us first consider the potential of the form ($\lambda > 0, \beta > 0 - const$)

$$V(x) = \frac{\lambda}{\beta \cosh x}, \qquad \mathcal{L}_{tach} = \frac{\lambda}{\beta \cosh x}\sqrt{1 - \dot{x}^2}. \tag{41}$$

The equation of motion (30) for this potential becomes

$$\ddot{x}(t) + \beta \tanh(\beta x)\dot{x}(t)^2 = \beta \tanh(\beta x), \tag{42}$$

and its general solution is

$$x(t) = \frac{1}{\beta}\text{arcsinh}\left[\pm\sqrt{1 - \frac{1}{C_1^2}}\sinh(\beta C_2 \pm \beta t)\right], \tag{43}$$

where constants $C_1$ and $C_2$ can be determined from the initial and the final conditions $x(0) = x_1$ and $x(T) = x_2$.

The tachyonic Lagrangian (28) for the potential (41) is unsuitable to be quantized, in particular, by the path integral method. However, we can apply CCT to find a locally equivalent canonical Lagrangian. If choose

$$F^{-1}(x) = \int^{x} \frac{dx}{V(x)} = \frac{1}{\lambda\beta}\sinh(\beta x), \tag{44}$$

it leads to the full generating function of the form

$$G(\tilde{x}, P) = -kF(\tilde{x}) = -\frac{k}{\beta}\text{arcsinh}(\lambda\tilde{\beta x}), \tag{45}$$

The equation of motion (42) takes a very simple form

$$\ddot{\tilde{x}} - \beta^2\tilde{x} = 0 \tag{46}$$

and the Lagrangian (41) transforms to quadratic Lagrangian [34, 35, 36]

$$\mathcal{L}_{quad}(\tilde{x}, \dot{\tilde{x}}) = \frac{1}{2}\dot{\tilde{x}}^2 + \frac{1}{2}\beta^2\tilde{x}^2. \tag{47}$$

The corresponding classical action is

$$S_{cl}(\tilde{x}_2, T; \tilde{x}_1, 0) = \int_0^T L_{quad}dt = \frac{\beta\left(\tilde{x}_1^2 + \tilde{x}_2^2\right)\cosh(\beta T) - 2\tilde{x}_1\tilde{x}_2}{2\sinh(\beta T)}, \tag{48}$$

The second interesting example is an exponential potential

$$V(T) = e^{-\alpha x}, \quad \alpha > 0 - const, \tag{49}$$

The functions $F^{-1}(x)$ and $F(\tilde{x})$ become

$$F^{-1}(x) = \frac{1}{\alpha}e^{\alpha x}, \quad F(\tilde{x}) = \frac{1}{\alpha}\ln(\alpha\tilde{x}). \tag{50}$$

The full generating function (35)

$$G(\tilde{x}, k) = -kF(\tilde{x}) = -\frac{k}{\alpha}\ln(\alpha\tilde{x}), \tag{51}$$

reduces equation of motion to again a simple form

$$\ddot{\tilde{x}} - \alpha^2\tilde{x} = 0. \tag{52}$$

Obviously, this equation of motion can be delivered from

$$\mathcal{L}_{quad}(\tilde{x}, \dot{\tilde{x}}) = \frac{1}{2}\dot{\tilde{x}}^2 + \frac{1}{2}\alpha^2\tilde{x}^2. \tag{53}$$

The corresponding classical action is

$$S_{cl}(\tilde{x}_2, T; \tilde{x}_1, 0) = \frac{\alpha\left(\left(\tilde{x}_1^2 + \tilde{x}_2^2\right)\cosh(\alpha T) - 2\tilde{x}_1\tilde{x}_2\right)}{2\sinh(\alpha T)}, \tag{54}$$

and it has the same form as the action (48).

It can easily be seen that the quadratic actions (48) and (54) can be quantized directly using the path integral method. In the real case the transition amplitude for the quadratic action (48) is [20, 28, 36]:

$$\begin{aligned} K_\infty(\tilde{x}_2, T; \tilde{x}_1, 0) &= \lambda_\infty\left(-\frac{1}{2h}\frac{\partial^2 S_{cl}}{\partial\tilde{x}_1\partial\tilde{x}_2}\right)\left|\frac{1}{h}\frac{\partial^2 S_{cl}}{\partial\tilde{x}_1\partial\tilde{x}_2}\right|_\infty^{1/2}\chi_\infty\left(-\frac{1}{h}S_{cl}\right) \\ &= \sqrt{\frac{-i\beta h^{-1}}{\sinh(\beta T)}}\exp\left(i\frac{\left(\tilde{x}_1^2 + \tilde{x}_2^2\right)\cosh(\beta T) - 2\tilde{x}_1\tilde{x}_2}{4\hbar\beta^{-1}\sinh(\beta T)}\right). \end{aligned} \tag{55}$$

The transition amplitudes allow us, at least in principle, to describe quantum dynamics of a tachyonic system with potentials (41) and (49), respectively, in the nonrelativistic quantum limit. We note that the actions (48) and (54) are different from the action (23) only in one constant term.

Because the action (48) is quadratic one, the corresponding kernel has an adelic like form [20]

$$\begin{aligned} \mathcal{K}_v(\tilde{x}_2, T; \tilde{x}_1, 0) &= \lambda_v\left(\frac{\beta}{2\sinh(\beta T)}\right)\left|\frac{\beta}{\sinh(\beta T)}\right|_v^{1/2} \\ &\times \chi_v\left(-\beta\frac{\left(\tilde{x}_1^2 + \tilde{x}_2^2\right)\cosh(\beta T) - 2\tilde{x}_1\tilde{x}_2}{2\sinh(\beta T)}\right). \end{aligned} \tag{56}$$

For the $p$-adic wave functions (in the case $p \neq 2$) we get [36]

$$\Psi_p(\tilde{x}) = \Omega(|\tilde{x}|_p), \quad \text{for} \begin{cases} |T|_p = 1, & \\ |T|_p < 1, & |\beta \tilde{x}_2^2 T|_p \leq 1. \end{cases} \tag{57}$$

The above condition is in accordance with the conditions for the convergence of the $p$-adic analytical functions which appear in the solution of the equation of motion (43) and the classical action (48). A relevant physical conclusion served from these relations still needs a more realistic model with tachyon matter and a precise form of metrics.

## 6. $p$-Adic Inflation

Cosmological inflation has become an integral part of the standard model of the universe. It provides important clues for structure formation in the universe and is capable of removing the shortcomings of standard cosmology.

Many string theorists and cosmologists have tried to find natural realizations of inflation within string theory, and novel features which would help to distinguish the string-based models from their more conventional field theory counterparts. In most examples to date, string theory has been used to derive an effective 4D field theory operating at energies below the string scale and all the inflationary predictions are made on a low energy effective field theory. However, a few problems still exist. For instance it is often difficult to identify features of string theory inflation that cannot be reproduced in the more conventional models. Thus, there is motivation to consider models in which inflation takes place at higher energy scales where stringy corrections to the low energy effective action are playing an important role. Also, it is worth to study nonlocality (connected with nonarchimedean aspects) [37], as well as a broad class of nonlocal inflationary models.

Gibbons has emphasized the cosmological implication of tachyonic condensate rolling towards its ground state [15]. The tachyonic matter might provide an explanation for inflation at the early epochs and could contribute to a new form of dark matter. There are hopes [17] that nonlocal inflation can succeed where the real string theory fails. $p$-Adic string theory, initiated by Volovich [6] and developed by Arefeva, Dragovich, Goshal, Frampton, Freund, Sen, Witten and many other, despite some open and serious problems is an interesting and wide field of research [38].

Let us remind that starting from the action of the $p$-adic string, with $m_s$ the string mass scale and $g_s$ the open string coupling constant,

$$S = \frac{m_s^4}{g_p^2} \int d^4x \left( -\frac{1}{2} \phi p^{-\frac{-\partial_t^2 + \bigtriangledown^2}{2m_s^2}} \phi + \frac{1}{p+1} \phi^{p+1} \right), \quad \frac{1}{g_p^2} = \frac{1}{g_s^2} \frac{p^2}{p-1}, \tag{58}$$

for the open string tachyon scalar field $\phi(x)$, it has been shown that a $p$-adic tachyon drives a sufficiently long period of inflation while rolling away from the maximum of its potential. Even though this result is constrained by $p \gg 1$ and obtained by an approximation, it is a good motivation to consider $p$-adic inflation for different tachyonic potentials. In particular, it would be interesting to study $p$-adic inflation in quantum regime and in adelic framework to overcome the constraint $p \gg 1$, with a still unclear physical meaning [18].

## 7. Tachyon Potential in the FRW metrics

In this subsection we will briefly discuss tachyon potentials on a real spaces in a more general case and calculate the slow-roll and observational parameters. Let us consider the DBI action (27) in homogeneous and isotropic space with the Friedmann-Robertson-Walker metrics $\eta_{00} = -1$ and $\eta_{11} = \eta_{22} = \eta_{33} = a(t)$, where $a(t)$ is the scale factor of the universe [39]. The tachyon field can be split into a homogeneous time dependent contribution $x(t)$ and a small $\mathbf{x}$-dependent perturbation $\delta x(t, \mathbf{x})$ [40]. In the following discussion we will focus only on the homogeneous (time dependent) contribution.

The Friedman equation takes the standard form

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_{Pl}^2} \frac{V}{(1-\dot{x}^2)^{1/2}}, \tag{59}$$

where $H$ is the Hubble parameter and $M_{Pl} = (8\pi G)^{-1/2}$ is the reduced Planck mass. We assume that the space is spatially flat and the cosmological constant is equal to zero. The energy-momentum conservation equation is

$$\dot{\rho} = -3H(P + \rho), \tag{60}$$

where a pressure ($P$) and an energy density ($\rho$) are

$$P = -V(x)\sqrt{1-\dot{x}^2}, \quad \rho = \frac{V(x)}{\sqrt{1-\dot{x}^2}}. \tag{61}$$

Like in the case of a scalar field with a standard-type Lagrangian, the pressure and the energy density are equal to the Lagrangian ($P = \mathcal{L}$) and to the Hamiltonian density ($\rho = \mathcal{H}$), respectively. Equation (60) is transformed into a second order differential equation

$$\frac{\ddot{x}}{1-\dot{x}^2} + 3H\dot{x} + \frac{V'}{V} = 0. \tag{62}$$

It is convenient to rescale the field $x$ and the Friedmann equations (59) and (62) by introducing a constant $x_0$. Additionally, the cosmic time is rescaled as $\tau = tx_0$ and we introduce the dimensionless quantities [39]

$$\tilde{x} = \frac{x}{x_0}, \quad U(\tilde{x}) = \frac{V(x)}{\lambda}, \quad \tilde{H} = \frac{H}{x_0}. \tag{63}$$

The system of equations (59-62) can be written as

$$\tilde{H}^2 = \frac{X_0^2}{3}\frac{U(\tilde{x})}{\sqrt{1-\dot{\tilde{x}}^2}}. \tag{64}$$

$$\ddot{\tilde{x}} + X_0\sqrt{3U(\tilde{x})(1-\dot{\tilde{x}}^2)^{3/2}}\dot{\tilde{x}} + \frac{(1-\dot{\tilde{x}}^2)}{U(\tilde{x})}\frac{dU(\tilde{x})}{d\tilde{x}} = 0, \tag{65}$$

In addition, the dimensionless Friedman acceleration equation is

$$\dot{\tilde{H}} = -\frac{X_0^2}{2}(\tilde{P}+\tilde{\rho}), \tag{66}$$

where the dot denotes a derivative with respect to $\tau$, and

$$\tilde{\rho} = \frac{U(\tilde{x})}{\sqrt{1-\dot{\tilde{x}}^2}}, \quad \tilde{P} = -U(\tilde{x})\sqrt{1-\dot{\tilde{x}}^2}. \tag{67}$$

Beside, $X_0$ is a dimensionless ratio which characterizes flatness of the potential $V(x)$ close to its peak [39, 41]

$$X_0 = \frac{\lambda x_0^2}{M_{Pl}^2}, \tag{68}$$

where $\lambda = M_s^4/g_s/(2\pi)^3$ is a constant motivated by string theory, $M_s$ is the string mass and $g_s$ is the string coupling.

### 7.1. Conditions for tachyon inflation

The slow-roll parameters can be defined in a few different ways. In this paper so-called Hubble hierarchy parameters are used as derivatives of the Hubble parameter ($H$) with respect to the number of $e$-foldings ($N$). [40, 42]

$$\epsilon_{i+1} \equiv \frac{d\ln|\epsilon_i|}{dN}, \quad i \geq 0, \quad \epsilon_0 \equiv \frac{H_*}{H}, \tag{69}$$

where $H_*$ is the value of $H$ at some chosen time. The number of $e$-folds is

$$N(t) = \int_{t_s}^{t_e} H(t)dt, \tag{70}$$

where $t_s$ is the time when counting of $e$-folds began and $t_e$ is the time at the end of inflation. The conditions for inflation are satisfied when $\epsilon_i < 1$ ($i = 1, 2, 3...$) and inflation ends when any of them exceeds unity.

Unfortunately, the slow-roll parameters of inflation cannot be directly measured. However, the results of Planck Collaboration [42] and previous missions provided limits on other parameters that can be both measured and calculated in the models. The most important observable parameters are the scalar spectral index ($n_s$) and the tensor-to-scalar ratio ($r$). In terms of the slow-roll parameters (69) $r$ and the $n_s$, to the lowest order, can be written as

$$r = 16\epsilon_1(x_s), \quad n_s = 1 - 2\epsilon_1(x_s) - \epsilon_2(x_s), \tag{71}$$

where $\epsilon_1$ and $\epsilon_2$ are the slow-roll parameters at some particular moment at the beginning of inflation (i.e. $x_s = x(t_s)$).

### 7.2. Dynamics of inflation

In this section we can review some numerical results for previously mentioned potentials (41) and (49). The slow-roll parameters, $n_s$ and $r$ are calculated and compared with the observational data.

The potentials could be written in more suitable, dimensionless, form for numerical calculations

$$U(\tilde{x}) = \frac{1}{\cosh \tilde{x}} \quad \text{and} \quad U(\tilde{x}) = \exp(-\tilde{x}), \tag{72}$$

where the potentials (41) and (49) are rescaled in accordance with (63). Equations (64)-(65) are solved numerically for the initial conditions $\tilde{x}(0) = x_i$ (where $x_i$ is calculated from the potentials (72) from the standard slow-roll conditions [39, 40, 42]), and $\dot{\tilde{x}}(0)$ is limited to a very small value ($\dot{\tilde{x}}(0) \to 0$) [39]. The solutions $\tilde{x}(\tau)$ and $\tilde{H}(\tau)$ of these equations are used to calculate the slow-roll parameters (69). Due to the fact that the solution $\tilde{x}(\tau)$ is time dependent we have to use the number of $e$-foldings given by equation (70). The results are calculated for a sample of sets $(N, X_0)$ where $45 \leq N \leq 120$ and $5 \leq X_0 \leq 25$.

Finally, the tensor-scalar ratio $r(x_s)$ and spectral index $n_s(x_s)$ are calculated from equation (71) for the corrected number of $e$-foldings. Results for the potential $U(\tilde{x}) = 1/\cosh(\tilde{x})$ are shown in Fig. 1.

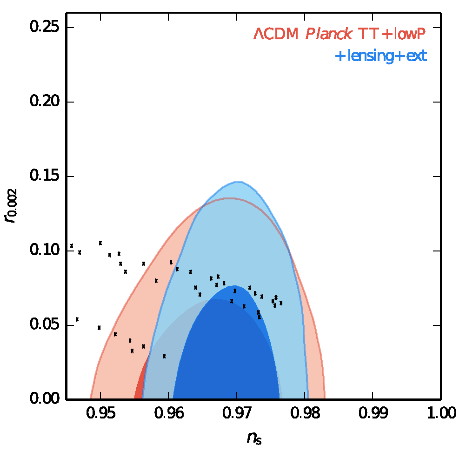


FIG. 1: $r$ - $n_s$ diagram with observational constraints [42] for potential $U(\tilde{x}) = 1/\cosh(\tilde{x})$. The dots represent the observationa parameters for the model for various $N$ and $X_0$: $45 \leq N \leq 120$ and $5 \leq X_0 \leq 25$. As it is wxpected agreement with observation is not very good, anyway the best results are obtained for $N \geq 85$ and $15 \leq X_0 \leq 25$.

## 8. Conclusion

Applications of $p$-adic numbers in quantum cosmology give new possibilities to investigate the structure of space-time at the Planck scale. In the Hartle-Hawking approach the wave function of a spatially closed universe is defined by Feynman's path integral method. However, it does not lead to the adequate adelic picture. On the other hand, the consideration of minisuperspace models in the framework of AQM gives the appropriate adelic generalization. Moreover, all these models lead to the picture of space-time as a discrete one. For all these models there exists the adelic wave function

$$\Psi(q^1,...,q^n) = \prod_{\alpha=1}^{n} \Psi_\infty(q^\alpha_\infty) \prod_p \prod_{\alpha=1}^{n} \Omega(|q^\alpha_p|_p), \tag{73}$$

Adopting the usual probability interpretation of the wave function (73) in rational points of $q^\alpha$, and because $(\Omega(|q^\alpha|_p))^2 = \Omega(|q^\alpha|_p)$ we have

$$\left|\Psi(q^1,\ldots,q^n)\right|^2_\infty = \begin{cases} |\Psi_\infty(q^\alpha)|^2_\infty, & q^\alpha \in Z, \\ 0, & q^\alpha \in Q\backslash Z. \end{cases} \tag{74}$$

This result leads to the discretization of minisuperspace coordinates $q^\alpha$. Note that this kind of discreteness depends on adelic quantum state of the universe. When the system is in an excited state, then the sharp discrete structure disappears, and minisuperspace, as well as configuration space in quantum mechanics, demonstrates usual properties of real space.

Tachyon fields and dynamics remain a promising direction of investigation in string theory and cosmology, in particular for the inflationary scenario. The tachyonic inflation approach on real spaces faces difficulties such as reheating [14] and duration. Nonlocal ($p$-adic) tachyon inflation [17, 18], in which a $p$-adic tachyon drives a sufficiently long period of inflation while rolling away from the maximum of its potential, deserves much attention.

In this paper we presented quantum propagators for the $p$-adic and adelic tachyons in a DBI context, as a natural set up for string theory - (quantum) cosmology "interaction". Conditions for the existence of the vacuum state of $p$-adic and adelic tachyons are presented. An interesting relation with the minisuperspace closed homogenous isotropic model in $(2+1)$ dimensions using Einstein gravity with a cosmological constant and an antisymmetric tensor field matter source [23, 29] is noted. A general method for for finding locally equivalent canonical tachyonic Lagrangians is demonstrated. It is very interesting to calculate the slow-roll parameters starting with locally equivalent Lagrangians (47) and (53) in a proper metric.

Finally we calculated observable inflation parameters for a set of relevant real tachyonic potentials and presented results, for one of them, comparable with the current data. However a lack of mechanism for transition from adelic to a real phase of inflation still prevents full employment of obtained adelic quantum models, in particular when a quadratic locally equivalent Lagrangian model has been built. Further investigation, including different adelic space-time backgrounds, should contribute to the better understanding of quantum rolling tachyon scenario and the origin of inflation.

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## TAHIONSKA INFLACIJA NA (NE-)ARHIMEDOVIM PROSTORIMA

*U ovom radu razmatra se značaj kvantnih kotrljajućih tahiona i odgovarajući inflatorni scenario u okviru standardne, p-adične i adelične kosmologije. Teorija polja za tahionsku materiju, koju je predložio Sen, u nula-dimenzionalnoj verziji dovodi do brojnih modela čestice koja se kreće u različitim potencijalima. U radu razmatramo kvantne propagatore za različite modele, kao i vakumska stanja i uslove koji su neophodni za konstruisanje adelične generalizacije. Osim ovoga prikazan je i inflatorni scenario za neke interesantne modele zasnovane na analitičkim i numeričkim izračunavanjima. Katak pregled stanja u ovoj oblasti i ideje za dalja istraživanja dati su na kraju ovog rada.*

**Ključne reči:** *tahioni, DBI skalarno polje, inflacija, kvantna kosmologija, nearhimedovski prostori*