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HOW TO SOLVE MODEL EQUATION OF HIERARCHICAL DIFFUSION USING SOME MATRIX ALGEBRA

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Abstract. *Problems of a random walk on a binary tree have been reformulated on any homogeneous tree. Cauchy problem of the random walk for homogeneous and nonhomogeneous equation having a Parisi matrix as a coefficient is formulated and solved with help of a special commutative ring of matrices. The ring containing the Parisi matrix is constructed. The method can be generalized on multidimensional case, for differential equations in non-Archimedean time, and for difference equations.*

Key words: *hierarchical diffusion, random walk, homogeneous tree, Parisi matrix*

1. Introduction

First problems of a random walk on a binary tree were described in [1]. In the paper [2] was stated a model equation of hierarchical diffusion and then it was reformulated to an integral differential equation in terms of p-adic analysis. Further it was solved by Fourier transform in frames of p-adic analysis and functional analysis. We consider Cauchy problem of a random walk equation

$$\frac{d\mathbf{P}(t)}{dt} = \mathbf{QP}(t), \quad \mathbf{P}(0) = (1, 0, \ldots, 0)^T \ ,$$

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where $n \in \mathbb{N}$, $(p_0,\ p_1,\dots,p_n,\dots)$, $p_k \in \mathbb{N}\backslash\{1\}$, $\mathbf{Q}$ is a $p_0 \cdot p_1 \cdots p_{n-1} \times p_0 \cdot p_1 \cdots p_{n-1}$ block diagonal Parisi matrix, $\mathbf{P}(t)$ is a vector-column of $p_0 \cdot p_1 \cdots p_{n-1}$ entries. The equation is called the dynamical or model equation of hierarchical diffusion. We describe homogeneous tree in section 2. and the random walk equation on it in section 3. In opposite to [2] we did not convert it to the integral form but left RHS of it as a product of the matrix $\mathbf{Q}$ and vector $\mathbf{P}(t)$ and applied to it a purely algebraic method. In fact, $\mathbf{Q}$ can be decomposed into a linear combination of more simple basis matrices that forms a commutative ring (this ring was constructed in [4] for finding eigenvalues of a Parisi matrix). The corresponding ring is constructed in section 4. Consequently, the solution of the problem has the form $\mathbf{P}(t) = \exp(t\mathbf{Q})\mathbf{P}(0)$ where $\exp(t\mathbf{Q})$ is again a linear combination of the basis matrices from the ring. Another advantage of this approach is that there is no necessity to utilize Fourier transform technique, integration theory of complex valued functions of p-adic argument and adjust function spaces and the equation to solve it. We may consider that the solution either complex valued or p-adic valued because it is in fact a rational valued function and it belongs to both classes. This option allows us to develop obtained results for multivariate case considering random walking on a Cartesian products of two or more different homogeneous trees or to solve the equation in non-Archimedean time.

2. Preliminary concepts and terms

We consider a tree as a graph without loops. Our tree is a finite and grows from a root in a top to down with leafs at a bottom. The bottom is ended by leafs and is a flat by form for the sake of simplicity. The root is ramified onto 2 or more edges. The ends of these edges are forming the first level of the tree. We call them vertices of the first level. Each vertex of the first level is ramified into 2 or more edges. The ramification from vertex to vertex along the level is the same. This is a condition of homogeneity. But ramification between root and the first level or between different levels can be distinguished. Therefore, we may describe our tree as a finite tree having the same number of outgoing edges coming from vertices on the same level.

Now we equip the tree by a natural metrics $\rho(\cdot,\cdot)$ given on the leafs $x,\ y$: $\rho(x,\ y) = 2^{-(n-k)}$ where n is a number of a level where leafs x and y are placed and k is a number of a level where is their closest common ancestor (or where the branches outgoing from x and y onto the root direction are join). This metrics is natural because it can be used in genealogy in order to distinguish two relatives or two employees in a hierarchical organization. Instead of 2 in the metrics we may use any other number $q > 1$ or, even more general, ramification index. For instance, if the tree has ramification indexes: $p_0, p_1, p_2, \dots, p_{n-1}$, i.e. p_0 is a ramification index between root and first level,..., p_k is that between k-th and $k+1$-th level and so on, and closest common ancestor of x and y is on the k-th level then the distance

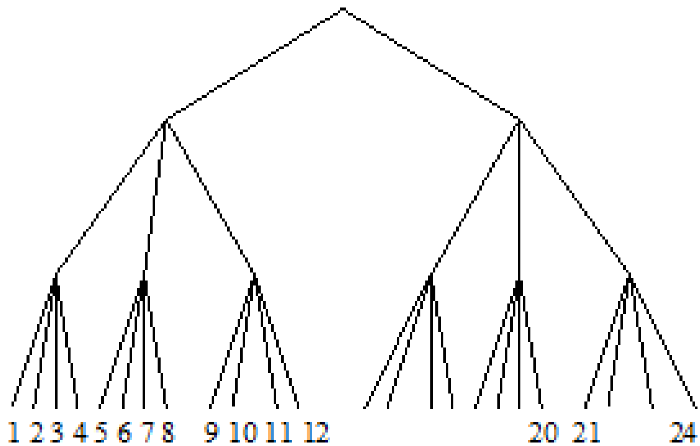

FIGURE 1: Homogeneous tree with ramification indexes $p_0 = 2$, $p_1 = 3$, $p_3 = 4$

between x and y can be product of reciprocals p_0, p_1,...,p_{k-1}:

$$\rho(x, y) = \prod_{j=0}^{k-1} p_j^{-1} \ .$$

If all $p_j = 2$ we get binary tree and corresponding metrics $\rho(x, y) = 2^{-k}$.

On Fig.1 we have $p_0 = 2$, $p_1 = 3$, $p_3 = 4$.

3. Random Walk on the Homogeneous Tree. Transition Probability Matrix

Here will be considered particle jumping from one leaf of the tree to another one with a probability proportional to the inverse value of the distance between the leafs. It is quite natural assumption and we take the probability of a particle transit from a leaf x to y, $x \neq y$ as $P(x, y) := \dfrac{C}{\rho(x, y)} = C \prod_{j=0}^{k-1} p_j$. Constant C is chosen such that law of total probability is hold. In more general situation we may take a growing function F vanishing at zero and put $P(x, y) = F\left(\dfrac{1}{\rho(x, y)}\right)$ which is bounded by law of total probability. In addition we put $P(x, x) \stackrel{\text{def}}{:=} q \in [0,\ 1)$. Therefore, the probability of random walk of a particle on the tree is given. But the random walk depends on time $t \in [0,\ T]$ and we are using the time in the model assuming

$$p_{ij} = P(A_i(t+\Delta t)|A_j(t)) = F\left(\frac{1}{\rho(i,j)}\right) ,$$

where $A_i(t)$ is an event "particle in the leaf i at moment t", $F\ :\ [0,\ +\infty) \to [0,\ 1)$ is a growing function such that $F(0) = 0$.

Therefore, by the law of total probability

$$P(A_i(t+\Delta t)) = \sum_{j=1}^{p_0 p_1 \cdots p_{n-1}} p_{ij} P(A_j(t)) \ , i = 1, 2, \ldots, p_0 \cdots p_{n-1} \ . \tag{1}$$

It is more convenient for us to rewrite (1) in the form of a matrix equation

$$\mathbf{P}(t+\Delta t) = \mathbf{A}\mathbf{P}(t),\ A_{ij} = p_{ij},\ \mathbf{P}(t) = \left(P(A_1), \ldots, P(A_{p_0 \cdots p_{n-1}})\right)^T . \tag{2}$$

The matrix $\mathbf{A}$ is called the transition probability matrix and has properties

$$A_{ij} = A_{ji},\ \sum_j A_{ij} = 1\ .$$

4. The Dynamical Equation and its Solution in an Exponential Form

From the previous section we have the equation $\mathbf{P}(t+\Delta t) = \mathbf{A}\mathbf{P}(t)$ which is equivalent to $\mathbf{P}(t+\Delta t) - \mathbf{P}(t) = \mathbf{A}\mathbf{P}(t) - \mathbf{P}(t)$ and then

$$\mathbf{P}(t+\Delta t) - \mathbf{P}(t) = (\mathbf{A} - \mathbf{I})\mathbf{P}(t) \tag{3}$$

Assuming

$$\mathbf{A} - \mathbf{I} = \mathbf{Q}\Delta t, \tag{4}$$

and dividing both sides of (3) by Δt we get $\dfrac{\mathbf{P}(t+\Delta t) - \mathbf{P}(t)}{\Delta t} = \mathbf{Q}\mathbf{P}(t)$ which after tending Δt to zero we obtain the differential equation

$$\frac{d\mathbf{P}(t)}{dt} = \mathbf{Q}\mathbf{P}(t)\ . \tag{5}$$

The solution of (5) is $\mathbf{P}(t) = \exp(t\mathbf{Q})\mathbf{P}(0)$, where $\mathbf{P}(0) = (1, 0, \ldots, 0)^T$ and $\exp(t\mathbf{Q})$ can be evaluated directly using a special form of $\mathbf{Q}$ (see Fig. 2).

Matrix $\mathbf{Q}$ has a block diagonal form related to the tree, i.e. it has entries q_0 on the diagonal, q_1 entries on $p_0 \times p_0$ diagonal blocks, q_2 entries on $p_0 p_1 \times p_0 p_1$ diagonal blocks, etc., q_k entries on $p_0 \cdot p_1 \cdots p_{k-1} \times p_0 \cdot p_1 \cdots p_{k-1}$ diagonal blocks, and finally q_n entries on $p_0 \cdots p_{n-1} \times p_0 \cdots p_{n-1}$ diagonal block that is coincide with the size of matrix $\mathbf{Q}$. Due to this form we introduce $n+1$ matrices $\mathbf{I}_0,\ \mathbf{I}_1\ , \ldots, \mathbf{I}_n$ such that each $\mathbf{I}_k$ has on its principle diagonal $p_0 \cdots p_{k-1} \times p_0 \cdots p_{k-1}$ blocks of units and all other entries are zeros. For the tree on Fig. 1 we have four basis matrices that are depicted on Fig. 3.

$$
\mathrm{Q} = \begin{pmatrix}
q_0 & q_1 & q_2 & q_2 & q_2 & q_2 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 \\
q_1 & q_0 & q_2 & q_2 & q_2 & q_2 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 \\
q_2 & q_2 & q_0 & q_1 & q_2 & q_2 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 \\
q_2 & q_2 & q_1 & q_0 & q_2 & q_2 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 \\
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q_2 & q_2 & q_2 & q_2 & q_1 & q_0 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 \\
q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_0 & q_1 & q_2 & q_2 & q_2 & q_2 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 \\
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q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_1 & q_0 & q_2 & q_2 & q_2 & q_2 \\
q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_2 & q_2 & q_0 & q_1 & q_2 & q_2 \\
q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_2 & q_2 & q_1 & q_0 & q_2 & q_2 \\
q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_2 & q_2 & q_2 & q_2 & q_0 & q_1 \\
q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_3 & q_2 & q_2 & q_2 & q_2 & q_1 & q_0
\end{pmatrix}
$$

FIGURE 2: **Q** matrix corresponding to the tree on Fig. 1

For the situation it is clear that

$$\mathbf{Q} = q_0(\mathbf{I}_1 - \mathbf{I}_0) + q_1(\mathbf{I}_2 - \mathbf{I}_1) + q_3(\mathbf{I}_3 - \mathbf{I}_2)\ .$$

In the general form we get

$$\mathbf{Q} = q_0(\mathbf{I}_1 - \mathbf{I}_0) + q_1(\mathbf{I}_2 - \mathbf{I}_1) + \cdots + q_n(\mathbf{I}_n - \mathbf{I}_{n-1})\ . \tag{6}$$

In [3, 4] was proved that the set of linear combinations of $\mathbf{I}_0,\ \mathbf{I}_1\ ,\dots,\mathbf{I}_n$ forms commutative ring with respect to matrix multiplication and summation. The ring has a unit $\mathbf{I}_0$.

Here and forth below we put by definition $I_{-1} = 0$ (zero matrix), $q_{n+1} = 0$, and $p_0 \cdot p_1 \cdots p_{k-1} = 1$, for $k = 0$. Taking this into account, (6) can be transformed into

$$\mathbf{Q} = \sum_{k=0}^{n} q_k(\mathbf{I}_k - \mathbf{I}_{k-1}) = \sum_{k=0}^{n}(q_k - q_{k+1})\mathbf{I}_k\ . \tag{7}$$

Lemma 1. *For any $k, j \in \{1, 2, \dots, n\}$ we get*

$$\mathbf{I}_k \cdot \mathbf{I}_j = \mathbf{I}_j \cdot \mathbf{I}_k = p_0 \cdot p_1 \cdots p_{\min(k,j)-1}\mathbf{I}_{\max(k,j)}\ . \tag{8}$$

Moreover, if $\mathbf{u}_j = (\overbrace{\underbrace{1, 1, \dots, 1, 0, \dots, 0}_{p_0 \cdot p_1 \cdots p_{n-1}}}^{p_0 \cdot p_1 \cdots p_{j-1}})$ *then*

$$\mathbf{I}_k \cdot \mathbf{u}_j = p_0 \cdot p_1 \cdots p_{\min\{k,j\}-1}\mathbf{u}_{\max\{k,j\}}\ . \tag{9}$$

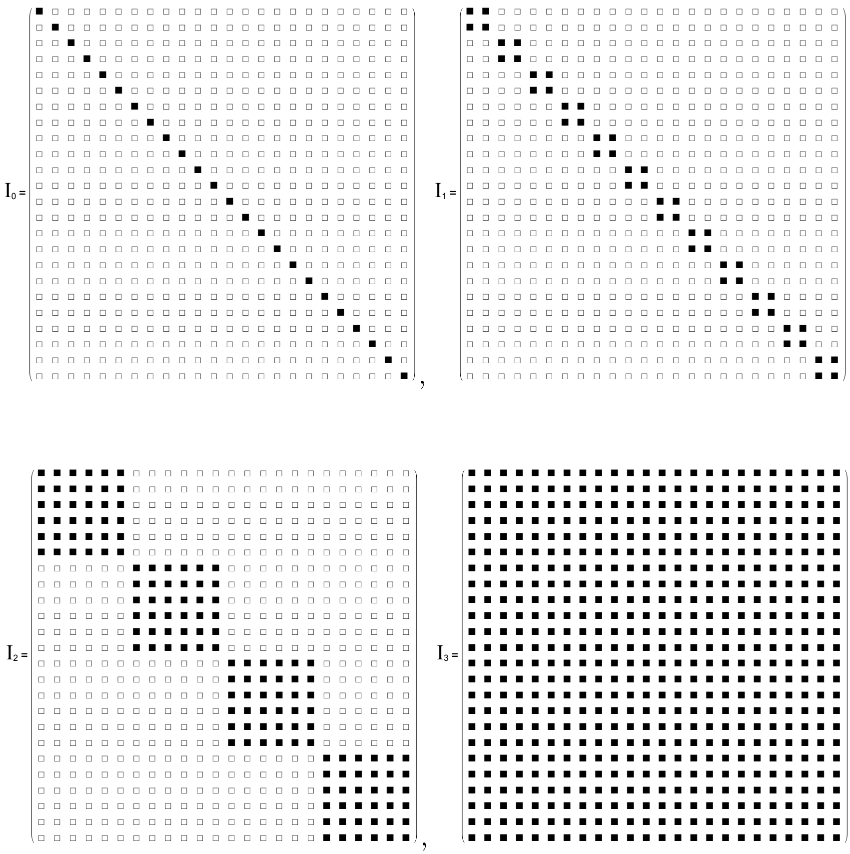

FIGURE 3: Basis elements of the matrix algebra that contains $\mathbf{Q}$, black boxes denote units, empty boxes denote zeros.

Proof. The proof of (8) and (12) is immediate from multiplication of two matrices and multiplication of a matrix $\mathbf{I}_k$ on a vector $\mathbf{u}_j$ correspondingly.

Corollary 1. *For any* $k \in \{1, 2, \ldots, n\}$ *the equality* $\mathbf{I}_k^2 = \mathbf{I}_k \cdot \mathbf{I}_k = p_0 \cdot p_1 \cdots p_{k-1} \mathbf{I}_k$ *holds true.*

Corollary 2. *For any* $k \in \{1, 2, \ldots, n\}$ *and for any* $\nu \in \mathbb{N}$ *the equality* $\mathbf{I}_k^\nu = (p_0 \cdot p_1 \cdots p_{k-1})^{\nu-1} \mathbf{I}_k$ *holds true.*

Corollary 3. *For any* $k \in \{1, 2, \ldots, n\}$ *and for any* $\alpha \in \mathbb{C}$ *one can get following* *equalities*

$$\begin{aligned} e^{\alpha \mathbf{I}_k} &= \sum_{\nu=0}^{\infty} \frac{(\alpha \mathbf{I}_k)^\nu}{\nu!} = \mathbf{I}_0 + \sum_{\nu=1}^{\infty} \frac{\alpha^\nu}{\nu!} (p_0 \cdot p_1 \cdots p_{k-1})^{\nu-1} \mathbf{I}_k \\ &= \mathbf{I}_0 + \frac{e^{\alpha\, p_0 \cdot p_1 \cdots p_{k-1}} - 1}{p_0 \cdot p_1 \cdots p_{k-1}} \mathbf{I}_k. \end{aligned} \tag{10}$$

Let us try to describe the solution of (5) in terms of matrices $\mathbf{I}_0, \mathbf{I}_1, \ldots, \mathbf{I}_n$. Indeed, $\mathbf{P}(t) = \exp(t\mathbf{Q})\mathbf{P}(0)$ where $\mathbf{P}(0) = (1, 0, \ldots, 0)^T$ or in our notations (see Lemma 1) $\mathbf{P}(0) = \mathbf{u}_0$. Using (7), (8), (9) and (10) one can show that

$$\begin{aligned} \exp(t\mathbf{Q})\mathbf{P}(0) &= \exp\left(t \sum_{k=0}^{n} (q_k - q_{k+1}) \mathbf{I}_k \right) \mathbf{u}_0 = \prod_{k=0}^{n} \exp\left(t(q_k - q_{k+1}) \mathbf{I}_k \right) \mathbf{u}_0 \\ &= e^{t(q_0 - q_1)} \prod_{k=1}^{n} \left(\mathbf{I}_0 + \frac{e^{t(q_k - q_{k+1}) p_0 \cdot p_1 \cdots p_{k-1}} - 1}{p_0 \cdot p_1 \cdots p_{k-1}} \mathbf{I}_k \right) \mathbf{u}_0 \end{aligned}$$

$$= \left(e^{t(q_0 - q_1)} \mathbf{I}_0 + \sum_{k=1}^{n} \frac{e^{t \sum_{j=0}^{k-1} (q_j - q_{j+1}) p_0 \cdot p_1 \cdots p_{j-1}} (e^{t(q_k - q_{k+1}) p_0 \cdot p_1 \cdots p_{k-1}} - 1)}{p_0 \cdot p_1 \cdots p_{k-1}} \mathbf{I}_k \right) \mathbf{u}_0$$

$$= e^{t(q_0 - q_1)} \mathbf{u}_0 + \sum_{k=1}^{n} \frac{e^{t \sum_{j=0}^{k-1} (q_j - q_{j+1}) p_0 \cdot p_1 \cdots p_{j-1}} (e^{t(q_k - q_{k+1}) p_0 \cdot p_1 \cdots p_{k-1}} - 1)}{p_0 \cdot p_1 \cdots p_{k-1}} \mathbf{u}_k$$

Therefore, we proved

Theorem 1. *The solution of* (5) *with the initial data* $\mathbf{P}(0) = \mathbf{u}_0$ *is equal to*

$$\mathbf{P}(t) = e^{t(q_0 - q_1)} \mathbf{u}_0 + \sum_{k=1}^{n} \frac{e^{t \sum_{j=0}^{k-1} (q_j - q_{j+1}) p_0 \cdot p_1 \cdots p_{j-1}} (e^{t(q_k - q_{k+1}) p_0 \cdot p_1 \cdots p_{k-1}} - 1)}{p_0 \cdot p_1 \cdots p_{k-1}} \mathbf{u}_k \ , \tag{11}$$

where $\mathbf{u}_k = (\underbrace{\overbrace{1, 1, \ldots, 1}^{p_0 \cdot p_1 \cdots p_{k-1}}, 0, \ldots, 0}_{p_0 \cdot p_1 \cdots p_{n-1}})$, $k = 0, 1, 2, \ldots, n.$

Remark 1. The solution of the nonhomogeneous equation $\frac{d\mathbf{P}(t)}{dt} = \mathbf{Q}\mathbf{P}(t) + \mathbf{f}(t)$ with the initial data $\mathbf{P}(0) = \mathbf{u}_0$ is equal to

$$\mathbf{P}_1(t) = \mathbf{P}(t) + \int_0^t e^{(t-\tau)\mathbf{Q}} \mathbf{f}(\tau) d\tau \ ,$$

where $\mathbf{P}(t)$ is the function (11).

Remark 2. Our approach can be applied for solving of the difference equation (2) with the initial data $\mathbf{P}(0) = \mathbf{u}_0$ because $\mathbf{A}$ has a similar block-diagonal form as $\mathbf{Q}$ and therefore belongs to the algebra.

Since

$$\mathbf{P}(t + \Delta t) = \mathbf{A}\mathbf{P}(t) \ , \tag{12}$$

then

$$\mathbf{P}(0 + \Delta t) = \mathbf{A}\mathbf{P}(0) \ , \tag{13}$$

$$\mathbf{P}(\Delta t + \Delta t) = \mathbf{A}\mathbf{P}(\Delta t) = \mathbf{A}\mathbf{A}\mathbf{P}(0) = \mathbf{A}^2\mathbf{P}(0) \ , \tag{14}$$

$$\mathbf{P}(n\Delta t) = \mathbf{P}((n-1)\Delta t + \Delta t) = \mathbf{A} \cdot \mathbf{A}^{n-1}\mathbf{P}(0) = \mathbf{A}^n\mathbf{P}(0) \ . \tag{15}$$

Consequently, we may get the solution at any point $n\Delta t$, $n = 0, 1, 2, \dots, N$ as $\mathbf{P}(n\Delta t) = \mathbf{A}^n\mathbf{u}_0$. Moreover, we can get the solution for negative integers n if $\mathbf{A}$ is invertible.

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KAKO REŠITI JEDNAČINU MODELA HIJERARHIJSKE DIFUZIJE POMOĆU MATRIČNE ALGEBRE

Problemi slučajnih šetnji na binarnom drvetu su reformulisani za bilo koje homogeno drvo. Košijev problem slučajne šetnje za homogenu i nehomogenu jednačinu, koja ima Parizijevu matricu kao koeficijent, je formulisana i rešena pomoću specijalnog komutativnog prstena matrica. Konstruisan je prsten koji sadrži Parizijevu matricu. Metod može biti generalisan na višedimenzionalni slučaj, za diferencijalne jednačine sa ne-Arhimedovim vremenom i za diferencne jednačine.

Ključne reči: *hijerarhijska difuzija, slučajna šetnja, homogeno drvo, Parizijeva matrica*