ON THE CONCEPT OF LOCAL TIME[†]

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Abstract. *We briefly review the conceptual basis of the recently introduced novel concept of the so-called Local Time in quantum theory. An emphasis is placed on certain cosmological aspects.*

Key words: *quantum foundations, quantum scattering, open quantum systems, cosmology*

1. INTRODUCTION

The concept of time in physics is typically regarded as a "category"—a concept of the highest rank of abstraction that serves the axiomatization of physical theories. Then the questions like "What is Time?" seem not to make much sense. Hence the rare attempts of questioning this attitude—a comfortable zone in doing physics. Intuitively, it may seem that questioning the physical nature and theoretical position of Time may lead to the infinite regression of the kind "What is older, a hen or the egg?".

Essentially, Time in physics is introduced already by Newton in his *Principia*:

(A) Time is universal and equally flowing for every observer in the physical Universe. This idea is directly applied in quantum theory and also applies to the theory of relativity, with the completion for the latter: "*every observer in a given (fixed) reference frame*".

Nevertheless, quantum foundations offer an alternative route to making sense of the question of "What is Time?". In this paper, we briefly present, emphasize and discuss the basics of a new, recently proposed approach to the concept of time in quantum physics that provided a novel paradigm of Local Time (Kitada, 1994; Jeknić-Dugić et al., 2014; Jeknić-Dugić et al., 2016; Kitada et al., 2016). To this end, this paper essentially complements the original papers (Jeknić-Dugić et al., 2014; Jeknić-Dugić et al., 2016; Kitada et al., 2016), which are mainly devoted to the mathematical foundations and consistency of the concept of Local Time (LT).

2. HISTORICAL PERSPECTIVE

The original "derivation" of the Schrödinger's equations starts from the time-independent Schrödinger equation—in the Schrödinger picture and in the position representation (Schrödinger, 1926a; 1926b; 1926c; 1926d; 1926e). It is all appearance that "derivation" of the *time-dependent* Schrödinger equation (the same picture and the same representation) aimed at targeting the non-conservative systems hopefully being able to counterpart the classical dissipative dynamics. While the development of the quantum-mechanical counterpart of classical dissipation had to wait for another 50 years (Caldeira and Leggett, 1983; Hu et al., 1992; Breuer and Petruccione,2002; Ferialdi, 2017), Schrödinger's introduction of physical time and its role in the time-dependent equation are both an automatic as well as an *ad hoc* procedure. That is, "time" is postulated to be unique and universally valid throughout the Universe in the Newtonian sense (A). Certainly, this *ad hoc* introduction and prejudice about physical time do not follow from the derivation of the time-dependent equation. Rather, Time is non-critically taken over with its full classical "load" to apply in the quantum-mechanical context.

On the other hand, the asymmetry between the spatial degrees of freedom and time, which appears as a single parameter in the Schrödinger time-dependent equation may elevate to the "problem of time", even in the non-relativistic context. That is, while the spatial degrees of freedom are quantized (hence allowing, for e.g., the position representation), the time parameter t remains intact, i.e. non-quantized. In order to complete the desired space-time symmetry, sometimes the classical "definition" of time:

$$t = m\frac{x}{p} \tag{1}$$

is used for time-quantization (Aharonov and Bohm, 1961):

$$\hat{t} = m(\hat{x}\hat{p}^{-1} + \hat{p}^{-1}\hat{x}). \tag{2}$$

Above, the m stands for the (single) particle's mass, x for the particle's (relative) position and p for the conjugate momentum. However, this approach to Time seems to open more questions than offering answers to the already open questions.

In certain interpretations of quantum theory (Petrat and Tumulka, 2014a), a separate time instant is introduced for every particle of a composite system—a concept of local time (lt). Then, instead of the composite system's state, $\psi(x_1, x_2,...,t)$, the state takes the form of $\psi(x_1, t_1, x_2, t_2,...)$. The time-instants' *labels are assumed to be fixed* throughout the composite system's dynamics. However, this procedure (somewhat analogous to the space-time foliation in General Relativity) may safely be applied only to the systems of non-interacting particles. Certain relativistic extensions may still be free of those obstacles (see e.g. Petrat and Tumulka, 2014b).

We may say that by far the most of the historically relevant approaches to *physical* time in quantum theory are framed by the concept of the Newtonian time (or of its variations like (lt)), on one side, and by the "problem of time", on the other side. Nevertheless, the concept of Local Time goes beyond this framework as we shall see below.

3. AN ALTERNATIVE APPROACH TO THE CONCEPT OF TIME: INTUITION

First, we want strongly to stress:

(Conjecture) *It is not possible to provide a rigorous theorem on the fundamental physical nature of Time*.

Then it may be only possible to provide plausible arguments of the interpretive nature that may make a shift or an alternative for the physical paradigm of Time.

Getting back to Eq.(1) directly gives rise to the following observation (analogously for the equation (2)): if regarded as quantum mechanical observables, the position and the momentum variables *x* and *p* are *time-less*. Indeed, in the position representation, the position variable *x* is just a multiplicative operator, while the momentum, $\hat{p} \rightarrow -i\hbar \partial / \partial x$, becomes a derivative operator for the functional Hilbert state space of the system. This is indicative since the classical expression for the momentum, $p = mdx/dt$, carries the time-variable via the time-derivation. Then the question implicit to the equation (2) is unavoidable with its foundational (even metaphysical) loads:

(QT) *Is it possible that "quantum time" maybe somehow "built" from the more fundamental position and momentum quantum observables*?

Interestingly enough, quantum theory offers a basis (of course, non-rigorous) for answering the (QT) in the *affirmative* yet in a specific sense. To this end, we have in mind the profound Enss' theorem backing the modern quantum many-body scattering theory. In a simplified form and with details that can be found elsewhere (Kitada 1994; Jeknić-Dugić et al., 2014; Kitada et al., 2016), the Enss' theorem (Enss, 1978) can be presented as:

$$\left\| \left(\frac{\hat{x}}{t_\mu} - \frac{\hat{p}}{m} \right) e^{-i\hat{H}t_\mu/\hbar} \, |\psi\rangle \right\| \rightarrow 0, \ \mu = 0, \pm 1, \pm 2, ..., \tag{3}$$

as $\mu \rightarrow \pm\infty$ (i.e. $t_\mu \rightarrow \pm\infty$). Without loss of generality, further on we take, $\mu, t_\mu \geq 0$.

In Eq.(3) $\hat{x}, \hat{p}, \hat{H}$ are the system's position, momentum and the (t_μ-independent) Hamiltonian, m states for the system's mass, while t_μ represents an instant of time for the unitary Schrödinger operator, $\hat{U}(t) = e^{-i\hat{H}t_\mu/\hbar}$, for an isolated quantum system—the standard unitary Schrödinger law in the non-relativistic quantum theory. The approach to zero on the r.h.s. of Eq.(3) is monotonic, i.e. without recurrence of the initial value for arbitrary pure state $|\psi\rangle$; generalization to the arbitrary mixed quantum states is rather obvious. The theorem applies to a rather wide class of the pair interactions of the constituent particles of the isolated system subjected to the unitary Schrödinger dynamics.

From the Enss' theorem, Eq.(3), we can write the operator relation (Kitada et al., 2016):

$$\frac{\hat{x}}{t_\mu} \sim \frac{\hat{p}}{m} \tag{4}$$

as $\mu \rightarrow \infty$, where the symbol "$\sim$" emphasizes similarity in the *statistical* sense (statistics over an ensemble of quantum systems in a state $|\psi_\mu\rangle$; $|\psi_\mu\rangle = e^{-i\hat{H}t_\mu/\hbar} \, |\psi\rangle$). That is, $(\hat{x}/t_\mu)|\psi_\mu\rangle \xrightarrow{\mu \rightarrow \infty} (\hat{p}/m)|\psi_\mu\rangle$ for the *arbitrary* initial state $|\psi\rangle$. As the index μ grows, the Enss' theorem, the equation (3), guarantees that the relation Eq.(4) is better satisfied. Therefore, we have at least schematically (Kitada, 1994) that the relation,

$$t \sim m \frac{|\hat{x}|}{|\hat{p}|}, \tag{5}$$

gets better satisfied with the increase of the index μ.

Then the "time instant" t_μ *may possibly not-in-advance be given any particular physical meaning*. That is, the equations (3) and (4) may be formally considered without any physical prejudice on the parameter t_μ (Kitada, 1994; Kitada et al., 2016; Jeknić-Dugić et al., 2014).

Specifically, Eq.(5) may be alternatively understood to provide that *dynamical change of the system's state* determined by the increase of the index μ for the arbitrary initial state $|\psi\rangle$] *gives rise to the "birth"* of what is commonly known as the time in physics—due to the similarity of Eq.(5) and Eq.(1). In defense of this formal similarity, we want to stress that a proper quantum theory (including the hypothetical, more general theory, cf. e.g. Popescu and Rohrlich 1994) should somehow provide the known classical limits, including Eq.(1). In the absence of a rigorous procedure, we perform mathematically non-rigorous steps (in accordance with the above Conjecture) led by some intuition that is here hopefully clearly presented.

Intuitively, *we propose* to read Eq.(5) as follows: *down the dynamical chain* for the unitary dynamics:

$$\ldots \to |\psi_{\mu-1}\rangle \to |\psi_\mu\rangle \to |\psi_{\mu+1}\rangle \to \cdots \tag{6}$$

the parameter t_μ *is getting* quantitatively *closer* to what is "defined" by Eq.(1) as "physical time". Of course, due to the Enss' theorem, the equation (5) may become "exactly" satisfied only asymptotically, i.e. in the asymptotic limit, $\mu \to \infty$. For the finite values of μ (i.e. for the finite values of t_μ), the equation (5), i.e. Eq.(1), can only be approximately satisfied.

The physical picture becomes even more appealing after observing that the dynamics, Eq.(6), i.e. Eq.(3), is *generated* (for an isolated system) by the system's Hamiltonian $\hat{H}$. That is, it is rather obvious that, even for the same "initial" state, different Hamiltonians give rise to quantitatively different norms appearing in Eq.(3), i.e. to different quantitative levels of satisfying the equations (4) and (5). In other words, *the (isolated) system's Hamiltonian is responsible for the* dynamical *birth of time of an isolated quantum system*, which can be schematically presented as:

(LT) One Hamiltonian⟷One local time.

4. AXIOMATIC APPROACH

Following the intuition of Section 3, we can formulate a new axiomatic approach to the concept of physical time in quantum theory (elaborated in Jeknić-Dugić et al., 2014). Not surprisingly, our starting point and the *primitive* is the concept of *dynamics*, the equation (6), which simply appears as a dynamical map on the Hilbert state space of "pure" quantum states that is readily generalized for the Banach state space of the trace-norm semidefinite-positive operators—of the "statistical operators" also known as the "density matrices".

As long as the quantum system is *isolated*, its dynamics is postulated to be unitary and hence the subject of the Enss' theorem. In the asymptotic limit, $\mu \to \infty$ ($t_\mu \to \infty$), the Enss' theorem guarantees the exact fulfillment of

$$\left\| \frac{\hat{x}}{t_\mu} - \frac{\hat{p}}{m} \right\|_\mu \to 0, \tag{7}$$

for virtually every Hamiltonian (i.e. for a very large class of the pair interactions of the system's constituent particles) and for every initial state. Then we formulate the main interpretational rule for LT as follows:

> (MIRLT) *Unitary dynamics generated by the system's Hamiltonian* dynamically *gives* rise to the "birth" of local time *for that single isolated quantum system, with the classical limit Eq.(1).*

A word of *caution*: if the isolated system also contains some *isolated subsystems*, then an amendment to MIRLT is needed—see Section 6 and Section 8.

In this new view to the (isolated) quantum-systems' dynamics, the concept of the standard Newtonian time appears essentially as an *additional assumption*, rather than a necessary part of physical theories. Indeed, just by an alternative reading of the Enss' theorem, *starting minimalisticly from the quantum dynamics*, we came at the new paradigm of Local Time that appears as a "local" (isolated) system's *emergent property*, i.e. a *by-product of unitary dynamics*. That is, the notion of time is *not*, i.e. not necessarily, *any fundamental notion of universal importance*.

In order to ease acceptance of this new position, we suggest approaching the concept of local time in analogy with the concept of (local) temperature in classical thermodynamics. There, Temperature is an emergent property of a (large) collection of constituent particles that can be defined *only* for the systems in the thermodynamic equilibrium or close to the thermodynamic equilibrium (so-called quasi-static processes).

It is essential to note that the relation (LT) can be established with the *full mathematical rigor* (Kitada et al 2016). In one direction, the system's Hamiltonian uniquely determines the system's local time, while in the opposite direction, the unitary dynamics parameterized by t uniquely determines the system's Hamiltonian. The only "ingredients" in the axiomatic sense are the concepts that are unavoidable in every known-to-us fundamentals of quantum theory: the *state space* and the *unitary dynamical map* on that space. Then the description of open (non-isolated) quantum systems (Breuer and Petruccione, 2002; Rivas and Huelga, 2011) goes smoothly and without any conceptual problems (Jeknić-Dugić et al., 2016). The only possible exception might be quantum dynamics that, in principle, cannot be derived from the unitary Schrödinger dynamics for an extended isolated system—the cases not known to us yet.

Therefore, the axiomatic, *bottom-up*, approach to LT goes *algorithmically*:

(i)   define the degrees of freedom and, therefore, the state space and the dynamical chains for an (approximately) isolated quantum (sub)system,

(ii) postulate unitary dynamics for the isolated quantum systems, and

(iii) follow the two-directional relation (LT) while keeping in mind that every state in a dynamical chain uniquely determines the relative strength of every term of the system's Hamiltonian, thus giving rise

(iv) to define Local Time for the isolated system as well as for each of its subsystems (that may be open systems not describable by the unitary Schrödinger law).

Now the following questions naturally appear: (a) Is (LT) essentially not the same as (lt)?; (b) Why has the stochastic nature of Time apparently never been experimentally even suspected?; (c) In which situations may the local time be ill-defined, if in any?;

(d) What is the status of the "initial" state and the initial "instant of time"?; (e) What maybe a description of the standard unitary dynamics in the context of the local time?; (f) Does the Local Time paradigm offer something new or any alternative to the standard quantum mechanical theory based on the Newtonian universal time?

The answers to these questions are briefly presented in the next sections.

5. THE NEW FUNDAMENTAL QUANTUM MECHANICAL LAW

The one-to-one relationship between the Hamiltonian and the local time encourages the application of the algorithm but also emphasizes that Eq.(3) may have a full physical sense in this context *only* in the asymptotic limit $t_\mu \to \infty$. That is, for some smaller t_μs, *the local system's time may be not well defined*—which answers the above question (c).

On the other hand, in realistic physical situations (that cannot attain the asymptotic limit) *there may be more than one, sufficiently large*, t_μ for which the limit in Eq.(3) is *approximately* fulfilled. Hence the fundamental physical picture for LT: dynamics of an isolated system neither provides Eq.(3) exactly satisfied nor does it provide a unique (large) value of t_μ (which is assumed to have the asymptotic classical limit Eq. (1)).

Now, a non-unique value of "sufficiently-large t_μ" implies a non-unique quantum state obtained via the unitary dynamics for the system. That is, instead of the standard Schrödinger law (given in the Schrödinger picture without any representation) for a mixed state:

$$\hat{\sigma}(t_0) = \hat{U}(t_0)\hat{\sigma}(0)\hat{U}^+(t_0), \tag{8}$$

the time-instant-uncertainty introduces the following *new fundamental quantum mechanical law*:

$$\hat{\sigma}(t_0) = \int_{t_0-\Delta t}^{t_0+\Delta t} dt\, \rho(t)\hat{U}(t)\hat{\sigma}(0)\hat{U}^+(t), \tag{9}$$

where $\rho(t)$ is a probability distribution ($\mathbb{R} \ni \rho(t) \geq 0, \forall t, \int \rho(t)\, dt = 1$), and Δt determines the *uncertainty* of the "*sufficiently large*" t_0, which is assumed to be unique and universal in the standard theory of Eq.(8).

Physically, Eq.(9) establishes that a *single* system can take some instant of time t' only from the interval $[t_0 - \Delta t, t_0 + \Delta t]$ with the probability density $\rho(t)$. While this answers the above question (e) (Eq.(8) follows from Eq.(9) in the limit of $\Delta t \to 0$), it also emphasizes a need for $\Delta t \ll t_0$ as an answer to the question (b) that suggests a rather sharp probability density $\rho(t)$. In a more transparent discrete version, Eq.(9) introduces an ensemble $\{\hat{\sigma}_\mu, p_\mu, \mu = 0,1,2,3, \dots\}$ of states $\hat{\sigma}_\mu = \hat{U}(t_\mu)\hat{\sigma}(0)\hat{U}^+(t_\mu)$, $t_\mu \in [t_0 - \Delta t, t_0 + \Delta t]$, with the probability p_μ, such that $\sum_\mu p_\mu = 1$.

Therefore, Eq.(9) introduces a "*proper mixture*" (D'Espagnat, 1999) described by the mixed state $\hat{\sigma}(t_0)$ for a *statistical ensemble* consisting of single systems (single elements of the ensemble), each of which being in one of the offered states (some state $\hat{\sigma}_\mu$) and with the related local time instant ($t_\mu \in [t_0 - \Delta t, t_0 + \Delta t]$). (Of course, the ensemble is supposed to regard the same initial state). Bearing in mind that t_0 must be sufficiently large, we may say, in a picturesque way, that *local time dynamically "matures"* for every *single* element of the ensemble.

6. Initial "Instant of Time"

Placing $t_0 = 0$ in Eq.(9) gives rise to uncertainty of the "initial instant of time", which labels the initial state $\hat{\sigma}(0)$. For an ensemble of systems, this uncertainty does not produce any problem as the initial state $\hat{\sigma}(0)$ is allowed in Eq.(9) to be a mixed state, whatever the origin/reason for the mixed-ness might be. Bearing in mind that Eq.(9) describes a proper mixture, formally the non-unique initial state emphasizes *ignorance* about the single system's state and the related instant of time, this provides an answer to the first part of the above question (d); a complete answer to the (d) question requires a deeper insight that we will provide below.

In order to comply with the apparent absence of experimental evidence for the nonzero Δt, the Δt-uncertainty cannot be arbitrary. To this end, it seems unavoidable to require non-orthogonality (i.e. indistinguishability) of the states determined by the interval $[t_0 - \Delta t, t_0 + \Delta t]$—a precise answer to the above question (b). Formally, this requires that the unitary time evolution for an arbitrary initial, e.g., pure state $|\psi\rangle$, cannot even produce approximate orthogonality (operational distinguishability) of the states, i.e. $\langle \psi(t)|\psi(t')\rangle \neq 0$, $\forall t, t' \in [t_0 - \Delta t, t_0 + \Delta t]$ should be fulfilled. While the quantitative criteria can be found elsewhere (Jeknić-Dugić et al., 2014), below we complete the answer to the question (d) while also providing an answer to the question (a).

Consider a bipartite quantum system, $C = 1 + 2$. Then Eq. (3) equally applies to *both isolated* subsystems 1 and 2, thus providing local times for both of them. Now suppose that down the dynamical chain for a single C system, the interaction between the subsystems becomes non-negligible. According to the above algorithm, the sufficiently strongly interacting subsystems 1 and 2 *start to constitute an isolated whole* with a unique local time for that single whole as well as for both subsystems. Therefore, a *sufficiently strong interaction* between certain systems *may imply a constitution of a new isolated whole*, which is now subject of the algorithm that *introduces a new local time* for the whole as well as for all of its constituent, mutually interacting, subsystems. This immediately answers the question (a): the interaction-induced dynamical *change of local time* (or dynamical redistribution of local times for a many-particle total system) makes LT incompatible with the lt. On the other hand, it implies a *new start of the dynamics for the newly formed composite system*, as well as for all of its subsystems. Bearing in mind the above given answer to this question, i.e. partly to the question (d), we strongly emphasize that the effect of the dynamical transition from the negligible interaction (approximately isolated subsystems 1 and 2) to the "strongly" interacting subsystems 1 and 2 makes the uncertainty of the initial "instant of time" practically negligible and virtually unobservable in realistic situations (Jeknić-Dugić et al., 2016). This completes our answer to the above question (d): neither may there be a problem with the initial mixed state nor does the uncertainty of the initial "time instant" introduce arbitrariness or vagueness for the fundamental dynamical law Eq.(9).

7. Some Open Questions

The following related questions are in order. First, why not use the mixed states in the dynamical chain equation (6)? Second, what is the system's dynamics in between the "weak" and the "(sufficiently) strong" interaction?

As for the first question, we regard mixed states poorly defined for single systems. That is, we are not aware of a consistent meaning of "probability" when already a *single* system should carry/be-responsible-for all the ignorance. Some arguments in contrast to our standpoint can be found in Zurek (2018) and a more general discussion in Khrennikov (2014). Accepting the point of view by Zurek (2018), *we already have a solution to* the long-standing (almost an aporia) *quantum measurement problem* (Jeknić-Dugić et al., 2014).

Independently of the kind of states in Eq. (6), *the second question still remains open*. That is, if for certain states in the chain (6) the interaction is neither "weak" nor "strong", it is not clear if unitary dynamics may be applied to these states. Then at least for certain sub-chains in the system's dynamical chain (transition from the "weak" to the "strong" interaction), the fundamental dynamical law may not be known yet. This line of reasoning—which may result in unitary dynamics as a special case—will be presented elsewhere.

8. SOME COSMOLOGICAL ASPECTS

Starting from Eq.(9), a number of interesting findings, including some desired results in the foundations of quantum theory and the open quantum systems theory, can be obtained (Jeknić-Dugić et al., 2014; Jeknić-Dugić et al., 2016). As an example, we emphasize the following: for small systems (a small number of degrees of freedom), the state Eq.(9) is virtually indistinguishable from the state Eq.(8); the quantitative criterion used is the so-called *fidelity of states* (Nielsen and Chuang 2000). Therefore, the nontrivial implications of LT can be expected practically only for many-particle systems, e.g. for the coarse-grained energy spectrum for the system. Without further ado, here, as an answer to the above question (f), we distinguish only a few cosmological aspects of the concept of Local Time.

Within the Local Time scheme, every single (at least approximately) isolated system *dynamically changes* and, possibly, *dynamically acquires* its own local time, which remains undeterminable for the system itself as well as for an external observer. As for the latter, determining the local-time-instant for a system would contradict the stipulated (see above) indistinguishability, $\langle \psi(t)|\psi(t')\rangle \neq 0, \forall t, t' \in [t_0 - \Delta t, t_0 + \Delta t]$, i.e. it would contradict the no-cloning theorem (Nielsen and Chuang, 2000; Jeknić-Dugić et al., 2014). While every proper system may be used to serve as a clock by an observer, *non-measurability* of the local time instant and *unobserved* uncertainty $\Delta t \neq 0$ may make an impression that the clock measures the universal Newtonian time for the unique state Eq.(8).

Just like the composite system C consisting of the mutually independent (sufficiently-weakly interacting) isolated subsystems 1 and 2 (Section 6), the Universe as a whole cannot be joined by unique time. That is, just like in any mixture, even if every single "local" (approximately isolated) system has its own local time, *the Universe as a whole cannot be described by unique local time*. The most that can be done is to provide a (dynamically changing) local-subsystems' time-distribution for the Universe, very much like the spatial-position distribution of the gas molecules. Some kind of averaging the local times does not seem to fit with the Enss' theorem and its interpretation presented by Eq.(9). The absence of the unique (local) time for the Universe may be regarded as an interpretation of the Wheeler-DeWitt equation (Jeknić-Dugić et al., 2014):

$$\hat{H}|\psi\rangle = 0. \tag{10}$$

Finally, the algorithm presented in Section 4 provides a novel picture of the Universe cosmological dynamics. Assuming that in the early stages of its dynamics the Universe can be regarded as a single isolated system (i.e. consisting of strongly interacting subsystems), the algorithm (Section 4) directly implies the existence of the unique local time for the early Universe as a whole; *that* Time may be universal in the sense of the (A) statement (cf. Introduction). However, for some subsequent states in the Universe dynamical chain, a possible dynamical structuring of the Universe into a set of approximately isolated systems leads to the lack of the universal time for the reasons discussed above. This kind of transition from the existence to non-existence of the Universe local time may possibly be regarded as a kind of "phase transition", i.e. *a change in the rules for describing the Universe as a dynamical physical system*. The importance of the existence of "local" subsystems of the Universe in the cosmological context can be found e.g. in Halliwell (2010).

Therefore, we may recognize at least two novel points of relevance for (quantum) cosmology that, to the best of our knowledge, have not yet been fully appreciated. Investigation along these lines may constitute one of the prominent routes in the further development of the Local Time scheme.

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O POJMU LOKALNOG VREMENA

Dat je kratak pregled osnova ne davno uvedenog, originalnog, pojma Lokalnog Vremena u kvantnoj teoriji. Poseban naglasak je stavljen na odredjene kosmološke aspekte.

Ključne reči: *zasnivanje kvantne teorije, kvantno rasejanje, otvoreni kvantni sistemi, kosmologija*