

**THE HIGGS DI-PHOTON DECAY IN THE STANDARD MODEL
EFFECTIVE FIELD THEORY[†]**

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Abstract. *Starting from the Standard Model (SM) of elementary particle physics, we assume that new physics effects can be encoded in higher-dimensional operators added in the SM Lagrangian. The resulting theory, the SM Effective Field Theory (SMEFT), is then used for high-accuracy phenomenological studies. Through this paper, the di-photon decay of the Higgs boson is used as a sample of a concrete calculation in the SMEFT framework.*

Key words: *beyond the Standard Model, effective field theories*

1. INTRODUCTION

The Standard Model (SM) is a quantum field theory, with the Lagrangian to consist of all possible renormalisable operators (those with dimension ≤ 4).¹ Here we are interested in the SM as an Effective Field Theory (Weinberg, 1979), or SMEFT for short.² If we assume that new physics lies not too far from the electroweak scale, to be capable of affecting the lower-energy physics, we could write an effective Lagrangian of the SM as

$$L_{SMEFT} = L_{SM} + \sum_{p=1}^{\infty} \sum_i \frac{c_{p,i}}{\Lambda^p} Q_i^{(4+p)}, \quad (1)$$

where L_{SM} is the usual renormalisable SM Lagrangian, Λ is the energy scale of the high-energy theory, $c_{p,i}$'s are the Wilson coefficients of the non-renormalisable operators and the sum over i runs over all possible operators of dimension $4 + p$, Q_i^{4+p} . In the following sections, for simplicity, we will absorb the energy scale in the Wilson coefficients. A list of

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¹ By “dimension” here we imply “dimension-of-mass”, in units where $\hbar=c=1$.

² See Ref. (Manohar, 2018) for recent pedagogical notes about EFTs.

all *independent* non-renormalisable operators of dimension up to 6 was given in Ref. (Grzadkowski et al., 2010). The SMEFT with up to dimension-6 operators in that basis was quantised in Ref. (Dedes et al., 2017) in linear R_ξ -gauges, and the full set of Feynman rules was presented there for general ξ 's.

It is convenient to use this explicit set of Feynman rules in order to calculate various amplitudes that are related to physical observables. Then, using recent experimental data we can set bounds on the Wilson coefficients, and finally gain insight into what the characteristics of a beyond the SM physical theory should be. An example of such an analysis has been accomplished in Ref. (Dedes et al., 2018), where the Higgs di-photon decay, $h \rightarrow \gamma\gamma$, was analysed in depth. A simple renormalisation scheme was presented, and the resulting amplitude was proved to be finite, independent of the renormalisation scale, gauge-choice invariant, and to respect the Ward identities of the theory. Furthermore, numerical results and bounds on the Wilson coefficients were provided, giving some first hints about possible high-energy models. Here we focus our attention on a subset of the operators appearing on the Higgs di-photon decay in SMEFT and present the calculational details in a pedagogical manner.

2. THE DECAY $h \rightarrow \gamma\gamma$ IN SM

Let us briefly review the SM calculation for the $h \rightarrow \gamma\gamma$ decay in linear R_ξ -gauges for arbitrary ξ . We will present the results and gain some insight for parts of the SMEFT calculation. The result for the $h \rightarrow \gamma\gamma$ amplitude in SM was first given in Ref. (Ellis et al., 1976) in the limit of small Higgs mass, and later for arbitrary Higgs mass in Refs. (Shifman et al., 1979; Bergstrom and Hulth, 1985). In all of the above references, the calculation was accomplished in linear (and non-linear for the later references) Feynman gauge. A calculation in linear R_ξ -gauges for arbitrary ξ was given in Ref. (Marciano et al., 2012). Thus far, Dimensional Regularisation (DR) has been used in all of the above works to handle divergent loop integrals. A detailed calculation of the amplitude in strictly four-dimensions was given in Ref. (Dedes and Suxho, 2013), proving the validity of the DR approach.

2.1. SM results

We assign to the momenta of the outgoing photons p_1 and p_2 and Lorentz indices μ and ν , respectively. As a consequence of gauge invariance, the amplitude should respect the Ward identity and, therefore, the Lorentz structure of the amplitude is expected to be $p_1 \cdot p_2 g^{\mu\nu} - p_1^\nu p_2^\mu$. It is convenient to introduce the following abbreviations:

$$\Omega^{\mu\nu} \equiv p_1 \cdot p_2 g^{\mu\nu} - p_1^\nu p_2^\mu, \quad \Omega \equiv \Omega^{\mu\nu} \varepsilon_\mu^*(p_1) \varepsilon_\nu^*(p_2), \quad (2)$$

where ε^* is the polarisation vector of an outgoing photon. Our results presented here are in agreement with the results found in the literature.

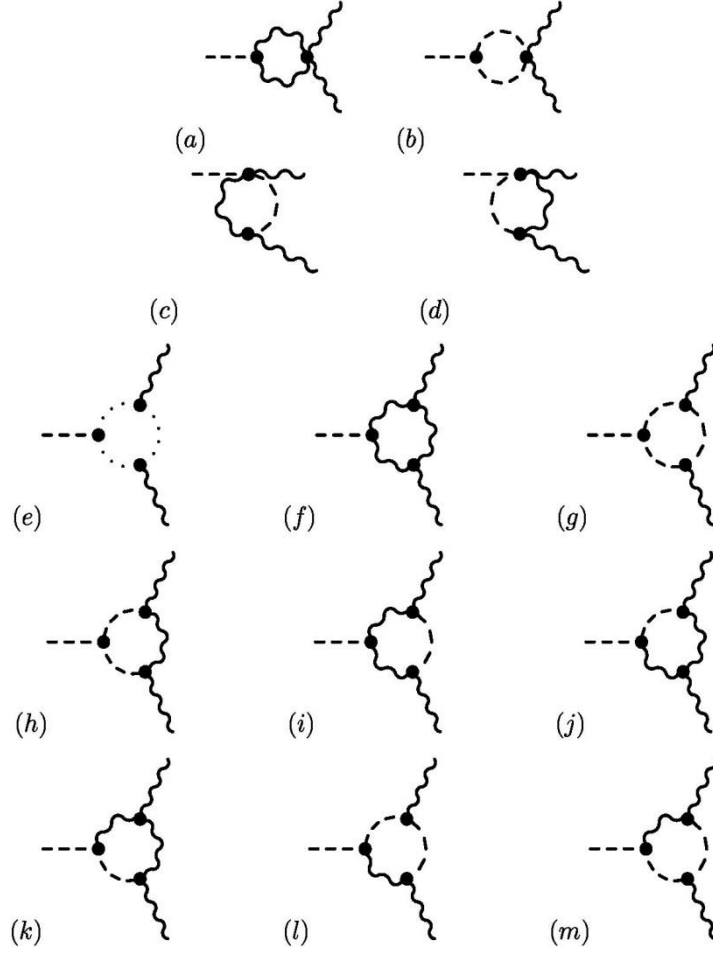


Fig. 1 Diagrams contributing to $h \rightarrow \gamma\gamma$ in the gauge sector in linear R_ξ -gauges in SM. Note that for diagrams (c) to (m) we have to multiply the result by 2 to account for the crossed diagrams. Also, an extra factor of 2 is needed in diagram (e) to account for the charge conjugated diagram.

The Feynman diagrams for the SM calculation are depicted in Fig. 1, where curly lines represent W^\pm -bosons, dashed lines represent G^\pm -bosons and dotted lines stand for ghosts (here we are mostly interested in the gauge-invariance of the result, so we are going to ignore the fermionic contributions altogether). The SM result can be found by adding the diagrams in unitary gauge, that is, diagrams (a) and (f) of Fig. 1 with the W^\pm -boson propagator evaluated in unitary gauge. The result is

$$iM_{un} = \frac{ig^4 g'^2 v \Omega}{2(4\pi)^2 (g^2 + g'^2) m_W^2} [2 + 3r + 3r(2 - r)f(r)], \quad (3)$$

where g and g' are the SM SU(2) and U(1) couplings, v is the vacuum expectation value of the theory, $r = 4m_W^2/m_h^2$ and $f(r)$ is given by

$$f(x) = \begin{cases} \arcsin^2\left(\frac{1}{\sqrt{x}}\right), & x \geq 1, \\ -\frac{1}{4} \left[\log\left(\frac{1+\sqrt{1-x}}{1-\sqrt{1-x}}\right) - i\pi \right]^2, & x \leq 1. \end{cases} \quad (4)$$

Let us also give here the scalar QED result (diagrams (b) and (g)),

$$iM_{sc} = \frac{4i\lambda v g^2 g'^2 \Omega}{(4\pi)^2 (g^2 + g'^2) m_h^2} [1 - \xi r f(\xi r)], \quad (5)$$

where ξ is the gauge-fixing parameter, λ is the Higgs quartic coupling, $r = 4m_W^2/m_h^2$ and the function f is given by eq. (4). Both results are seen to respect the Ward identities and be finite, as expected. In what follows we will call the rest of the result (i.e. full result minus the unitary gauge) the ξ -dependent result, for obvious reasons.

2.2. The ξ -dependence

After going through a straightforward calculation, one can easily see that the divergent parts of both the unitary gauge and the ξ -dependent diagrams integrate analytically to zero. To see that the finite ξ -dependent part integrates to zero is a by far more difficult task, since the results are long and the integrals complicated. After a laborious calculation, and by using the SM relations between coupling constants and particle masses,

$$m_h = \sqrt{\lambda} v, \quad m_W = \frac{1}{2} g v, \quad (6)$$

one can prove that the ξ -dependent part analytically cancels.

It is worth noting that we did *not* make use of the SM relations (6) to derive results (3) and (5). So, these infinite terms cancel also in SMEFT. In fact, the parameter λ appears only through the triple hG^+G^- interaction vertex, therefore being present only in scalar QED (diagrams (b) and (g)) and in the diagram (h). Now notice that scalar QED is automatically finite, and that (h) is finite by naive power counting. That is important since we immediately see that contributions from the operators C^φ , $C^{\varphi\Box}$ and $C^{\varphi D}$ that appear explicitly only in Higgs vertices are automatically finite, since λ is just rescaled with respect to the SM case.

3. THE DECAY $h \rightarrow \gamma\gamma$ IN R_ξ -GAUGES IN SMEFT

3.1. Introduction

Let us now move on to discuss the process $h \rightarrow \gamma\gamma$ in linear R_ξ -gauges for arbitrary ξ in the SMEFT. The calculation for the $h \rightarrow \gamma\gamma$ decay in the SMEFT was first accomplished in Refs. (Hartmann and Trott, 2015a; Hartmann and Trott, 2015b), using the background field method. Here we present only a small subset of the $h \rightarrow \gamma\gamma$ calculation; for the complete analysis see Ref. (Dedes et al., 2018) (see also Ref. (Dawson and Giardino, 2018) that recently appeared in the literature). We prove that, for each Wilson coefficient, all ξ -

dependent terms cancel among themselves, therefore proving that SMEFT respects gauge invariance, and we give the analytic results of our calculation.

3.2. Computational details for each operator

Each operator has its own special features when it comes to the calculational details. Therefore, we are going to briefly review everything needed for one to reproduce the result. We are not going to write down the intermediate results since these are quite lengthy and not that enlightening for one to understand the calculational steps. In what follows, we sometimes refer to the ‘‘SM result’’. By that, we mean the result one finds by calculating the amplitude having fixed all the *explicit* Wilson coefficients to zero but leaving everything else intact.

3.2.1. C^φ

The first operator we are about to discuss³ is also the simplest one. The operator C^φ appears *explicitly* only in the triple hG^+G^- vertex, therefore contributing only in diagrams (b) and (g) (the scalar QED diagrams) and diagram (h) of Fig. 1. That means that there are *no* diagrams in the unitary gauge, and, since we expect the result to be gauge invariant, the amplitude should vanish in every gauge. That is what we are about to verify.

One can perform this calculation easily starting from the SM result, as described in Section 2. The interacting vertex hG^+G^- goes from its SM value, $-i\lambda v$, to $+3iv^3C^\varphi$, so we have to modify scalar QED and diagram (h) by replacing $\lambda \rightarrow -3v^2C^\varphi$. After that change, we substitute the masses in the amplitude with their SM values making an error only in the second order in the Wilson coefficients, since everything is multiplied by C^φ . Now, that makes the explicit contribution, which is non-vanishing (but of course finite, as we explained in Subsection 2.2).

There are also *implicit* contributions, coming from the Higgs mass (Dedes et al., 2017)

$$m_h^2 = \lambda v^2 - \left(3C^\varphi - 2\lambda C^{\varphi\Box} + \frac{1}{2}\lambda C^{\varphi D}\right)v^4. \quad (7)$$

To get implicit contributions, we have to Taylor expand the full ξ -dependent SM result around $C^\varphi = 0$, keeping terms linear in C^φ . It is easy to see that this result is equal to minus the explicit one. Therefore, the result proportional to C^φ is gauge invariant (and in that case, identically zero).

3.2.2. $C^{\varphi\Box}$ and $C^{\varphi D}$

These operators appear always together in the linear combination

$$C^{\varphi\Box} - \frac{1}{4}C^{\varphi D}, \quad (8)$$

coming from the Higgs redefinition (Dedes et al., 2017)

³ There is a one-to-one correspondence between the non-renormalisable operators and the Wilson coefficients that multiply them. From now on we are simply going to call the operators by their Wilson coefficients, e.g. the phrase ‘‘the operator $C^{\varphi\Box}$ ’’ should be understood as ‘‘the operator that is accompanied by the Wilson coefficient $C^{\varphi\Box}$ ’’.

$$h = Z_h H, \quad (9)$$

where h is the physical Higgs field, and

$$Z_h = 1 - v^2 \left(C^{\varphi^{\boxplus}} - \frac{1}{4} C^{\varphi^D} \right). \quad (10)$$

Therefore, it suffices to discuss only one of them, say $C^{\varphi^{\boxplus}}$. It appears *explicitly* in all vertices involving the Higgs boson in Fig. 1. The hG^+G^- vertex goes from its SM value, $-i\lambda v$, to $-3i\lambda v^3 C^{\varphi^{\boxplus}}$, while all other vertices go from their SM value to $v^2 C^{\varphi^{\boxplus}}$ times their SM value. So, what we have to do is to rescale $\lambda \rightarrow 3\lambda$ in the SM result, and then multiply the new result by $v^2 C^{\varphi^{\boxplus}}$. Finally, we substitute masses with their SM values, which results in an error in the second order in the Wilson coefficients since everything is multiplied by $C^{\varphi^{\boxplus}}$. What we are left with constitutes the explicit contribution. Let us keep aside the unitary gauge result for the time being and focus our attention in the ξ -dependent part. We call this the explicit ξ -dependent result.

Next, we consider the *implicit* contribution which has to cancel the explicit ξ -dependent result. That is coming from the Higgs mass, defined in eq. (7). We also use the exact formula

$$m_W = \frac{1}{2} \bar{g} v. \quad (11)$$

A bar over the couplings g and g' denotes that the couplings are SMEFT couplings (Dedes et al., 2017). Finally, we Taylor expand around $C^{\varphi^{\boxplus}} = 0$, keeping the terms linear in $C^{\varphi^{\boxplus}}$. Once again, one can verify that the implicit result is equal to minus the explicit one, therefore the result proportional to $C^{\varphi^{\boxplus}}$ is independent of the ξ parameter, so this contribution is gauge invariant.

Now that we proved the gauge invariance for the result proportional to these two Wilson coefficients, we are ready to give the actual result. This is simply the unitary gauge SM result (where now the VEV and the couplings are defined in SMEFT), multiplied by $v^2 (C^{\varphi^{\boxplus}} - \frac{1}{4} C^{\varphi^D})$, i.e.

$$iM = \frac{i\bar{g}^4 \bar{g}'^2 v^3 \Omega}{2(4\pi)^2 (\bar{g}^2 + \bar{g}'^2) m_W^2} \left(C^{\varphi^{\boxplus}} - \frac{1}{4} C^{\varphi^D} \right) [2 + 3r + 3r(2-r)f(r)], \quad (12)$$

where $r = 4m_W^2/m_h^2$ and $f(r)$ is given by eq. (4).

3.2.3. C^W

Thus far we have considered only finite contributions. Let us now move on to an operator whose contribution is actually *infinite*. Of course, having infinite results in an amplitude representing a physical process means that we ought to use a renormalisation scheme, but that is beyond the scope of this paper. For a careful treatment of the renormalisation of $h \rightarrow \gamma\gamma$ in SMEFT see ref. (Dedes et al., 2018). Here we simply note that the decay $h \rightarrow \gamma\gamma$ in SMEFT

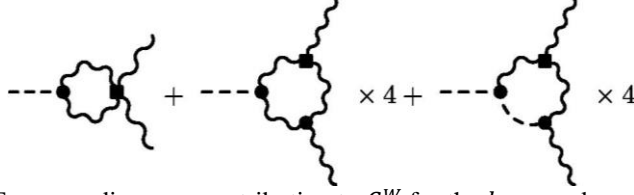


Fig. 2 Feynman diagrams contributing to C^W for the $h \rightarrow \gamma\gamma$ decay in SMEFT.

has a tree-level contribution, coming from a linear combination of three Wilson coefficients, namely

$$\cos^2\theta_w C^{\varphi B} + \sin^2\theta_w C^{\varphi W} - \sin\theta_w \cos\theta_w C^{\varphi WB}, \quad (13)$$

where θ_w is the SM weak mixing angle. Infinities, therefore, would be absorbed by the running of these coefficients. The beta functions of the Wilson coefficients at one-loop order in SMEFT are calculated in Refs. (Jenkins et al., 2013; Jenkins et al., 2014; Alonso et al., 2014).

The operator C^W has *no* implicit contribution. The Feynman rules are lengthy and complicated but, thankfully, many simplifications occur due to the on-shell conditions. In Fig. 2 we present the Feynman diagrams that contribute to the decay $h \rightarrow \gamma\gamma$ for the Wilson coefficient C^W . The particles in the loop are W^\pm -bosons (curly lines) and G^\pm -bosons (dashed lines). A square indicates that we consider *only* the part of the vertex proportional to C^W , whilst a dot denotes a SM vertex (zeroth order in the Wilson coefficients). We multiply by 4 the last two diagrams to account for the contribution of the charge-conjugated diagrams and of the crossed diagrams.

After performing the actual calculation, one can prove that C^W is gauge invariant, and the result is given by

$$iM = \frac{3i\bar{g}^3\bar{g}'^2 v\Omega C^W}{4\pi^2(\bar{g}^2 + \bar{g}'^2)} \left[-3E - 2 + rf(r) + 3\log\left(\frac{m_W^2}{\mu^2}\right) + 4\sqrt{r-1}\arctan\left(\frac{1}{\sqrt{r-1}}\right) \right], \quad (14)$$

where μ is the renormalisation scale, $\varepsilon = 4 - d$ with d being the dimensionality of space-time in dimensional regularisation, $E \equiv 2/\varepsilon - \gamma + \log(4\pi)$, $r = 4m_W^2/m_h^2$ and $f(r)$ is given in Eq. (4). As we claimed at the beginning of this section, the result is infinite, with the infinite part given by

$$iM|_{infinite} = -\frac{9i\bar{g}^3\bar{g}'^2 v\Omega C^W}{4\pi^2(\bar{g}^2 + \bar{g}'^2)} \frac{2}{\varepsilon}. \quad (15)$$

We verified that our result for the infinite part of the C^W contribution is in agreement with the beta functions from Ref. (Alonso et al., 2014).

4. CONCLUSIONS AND DISCUSSION

Due to its high-energy-model agnostic nature, the SMEFT is a very useful framework for the phenomenological study of beyond the SM physics. It is remarkable how the data provided by low-energy physics experiments can carry the remnants of physics at a (much) higher-energy scale — even if we have no direct findings of new physics, e.g. the discovery of new particles or forces — and SMEFT takes advantage of that feature.

Here we presented a sample of how detailed calculations can be performed in the SMEFT framework, by considering the $h \rightarrow \gamma\gamma$ decay. The full treatment of the aforementioned process can be found in Ref. (Dedes et al., 2018). There, we calculated the $h \rightarrow \gamma\gamma$ decay in general R_ξ -gauges using a convenient renormalisation scheme. Numerically important deviations from the SM were found in a subset of the effective operators, and the corresponding Wilson coefficients were meaningfully constrained using the current precision of the LHC measurements for the Higgs di-photon decay. Therefore, the SMEFT framework seems promising and suitable for the study of the effects of new physics, given the current status in the elementary particle physics field.

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DVOFOTONSKI RASPAD HIGSOVE ČESTICE U EFEKTIVNOJ TEORIJI POLJA STANDARDNOG MODELA

Polazeći od Standardnog modela (SM) fizike elementarnih čestica, možemo pretpostaviti da efekti nove fizike mogu biti kodirani u operatorima viših dimenzija koji su dodati u lagranžijan Standardnog modela. Dobijena teorija, efektivna teorija polja Standardnog modela (SMEFT), je iskorišćena za fenomenološka razmatranja velike preciznosti. U ovom radu uzet je dvofotonski raspad Higsogov bozona kao primer konkretnog računa u okviru SMEFT.

Ključne reči: *Iza Standardnog modela, efektivne teorije polja*