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# PECULIAR FIVE-DIMENSIONAL BLACK HOLES†

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**Abstract**. *In this article, we review two black hole solutions to the five-dimensional Lovelock gravity. These solutions are characterized by the non-vanishing torsion and the peculiar property that all their conserved charges vanish. The first solution is a spherically symmetric black hole with torsion, which also has zero entropy in the semi-classical approximation. The second solution is a black ring, which is the five-dimensional uplift of the BTZ black hole with torsion in three dimensions.*

**Key words**: *black hole, torsion, alternative theories of gravity*

## 1. Introduction

Gravity is the interaction which we are aware of for the longest period of time of all the known interactions but, paradoxically, it is also the one we know the least about. Quantum gravity is the goal which drives the most of modern research in high-energy physics. Unfortunately, the realm of quantum gravity is beyond our current experimental abilities and researchers have to come up with ingenious ideas how to go around this. Fortunately, the effects of quantum gravity are visible in black hole physics already in the semi-classical level. This makes black holes the most important objects in gravity and this is the very reason why they were extensively studied in the past century. Now, it is well known that general relativity cannot be the whole story and for this reason, for different purposes, research went in the direction of alternative theories of gravity. Some hope to construct a good theory of quantum gravity in this way, others, less ambitious, hope to gain a small insight into the quantum effect of gravity.

Lovelock gravity is an interesting generalization of general relativity, which is a unique ghost-free higher derivative theory of gravity with second-order field equations. In three and four dimensions, Lovelock gravity reduces to general relativity. Originally,

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Lovelock gravity is formulated in the second order, i.e. the metric tensor is a dynamical field. More interesting is the first order formulation in which we treat the vielbein and spin connection as independent dynamical variables of our theory. A theory formulated in this way is no longer torsion-less but may and does contain solutions with non-trivial torsion. The first order formulation is convenient because it contains torsion-less theory as a limit and is conceptually necessary for coupling with fermionic matter fields and supersymmetric extensions of the theory. The Einstein-Cartan theory, the first-order formulation of general relativity, has the property that all vacuum solutions are without torsion, the vacuum solutions of the Lovelock gravity are more complicated since there exist solutions with non-zero torsion. Lovelock gravity possesses a large number of black hole solutions (see Aros et al., 2001; Boulware and Deser, 1985; Camanho and Edelstein, 2013; Canfora et al., 2007; Canfora et al., 2008; Cai Rong-Gen and Ohta, 2006; Cai Rong-Gen et al., 2010; Cvetković and Simić, 2016; Cvetković and Simić, 2018; Dotti et al., 2007; Garraffo and Giribet, 2008; Kastor and Mann, 2006; Maeda et al., 2011, and references therein). Many of them possess exotic properties, for example, zero mass (Cai Rong-Gen et al., 2010; Cvetković and Simić, 2016; Cvetković and Simić, 2018), peculiar topology of the event horizon (Cvetković and Simić, 2016; Kastor and Mann, 2006; Maeda et al., 2011; Ray, 2015), etc. This brings us to the problem of black hole uniqueness. Solutions of general relativity are highly constrained, but things change when we go into higher dimensions. In higher dimensions, black hole solutions appear which have the non-spherical topology of the event horizon, more precisely, black string, black ring and black brane (Emparan and Reall, 2002; Emparan and Reall, 2008; Horowitz and Strominger, 1991; Kastor and Mann, 2006). It is not uncommon that these black holes suffer from various instabilities, for example, black strings and branes suffer from Gregory-Laflamme instability (Gregory and Lafflamme, 1993), and will decay into a black hole with a spherical horizon. We see that gravity in higher dimensions is a very interesting area of research, full of surprising discoveries, whose importance is in its applications in many different areas.

In the end, a few words on notation. We use the following conventions: Lorentz signature is mostly minus, local Lorentz indices are denoted by the middle letters of the Latin alphabet, while space-time indices are denoted by the letters of the Greek alphabet. For notational simplicity, we mostly use differential forms instead of coordinate notation, in all formulas the wedge product is not written explicitly.

## 2. LOVELOCK GRAVITY

Lovelock gravity (Lovelock, 1971; Lovelock, 1972) is a minimalistic generalization of general relativity and is one of many alternative theories of gravity which is under constant investigation.

The first-order formulation of gravity as dynamical variables has the vielbein $e^i$ 1-form and the spin connection $\omega^{ij} = -\omega^{ji}$ 1-form. In local coordinates $x^\mu$, we can expand the vielbein and the connection 1-forms as $e^i = e^i_\mu dx^\mu$, $\omega^{ij} = \omega^{ij}_\mu dx^\mu$. Gauge symmetries of the theory are local translations (diffeomorphisms) and local Lorentz rotations, parameterized by $\xi^\mu$ and $\varepsilon^{ij}$, respectively.

From the dynamical variables, we can construct field strengths torsion $T^i$ and curvature $R^{ij}$ (2-forms), which are given as

$$T^i = \nabla e^i \equiv de^i + \omega^{ik} e_k \,,$$

$$R^{ij} = d\omega^{ij} + \omega^i_k \omega^{kj} \,,$$

where $\nabla = dx^\mu \nabla_\mu$ is the exterior covariant derivative.

Metric tensor $g_{\mu\nu}$ can be constructed from the vielbein $e^i_\mu$ and flat metric $\eta_{ij}$

$$g_{\mu\nu} = \eta_{ij} e^i_\mu e^j_\nu \,, \quad \eta_{ij} = (+,-,-,\ldots) \,.$$

The anti-symmetry of $\omega^{ij}$ is equivalent to the metric condition $\nabla g = 0$. The geometry whose connection is restricted by the metric condition (metric-compatible connection) is known as Riemann-Cartan geometry.

The connection $\omega^{ij}$ determines the parallel transport in the local Lorentz basis. Because parallel transport is a geometric operation it is independent of the basis. This property is encoded into PGT via the so-called *vielbein postulate*, which implies

$$\omega_{ijk} = \Delta_{ijk} + K_{ijk} \,,$$

where $\Delta$ is the Levi-Civita connection, and $K_{ijk} = -\frac{1}{2}(T_{ijk} - T_{kij} + T_{jki})$ is the contortion.

The Lagrangian of Lovelock gravity in D dimensions is given by

$$L = \sum_{p=0}^{[D/2]} \frac{\alpha_p}{D-2p} L_p \,,$$

where $\alpha_p$ are real parameters and $L_p$ is dimensionally continued Euler density defined in the following manner

$$\varepsilon_{i_1 i_2 \ldots i_D} R^{i_1 i_2} \ldots R^{i_{2p-1} j_{2p}} e^{D-2p} \ldots e^D \,.$$

Because we will be concerned with Lovelock gravity in five dimensions, we will only give equations of motion for this case. A variation of the action with respect to the vielbein $e^i$ and spin-connection $\omega^{ij}$ leads to the following field equation

$$\varepsilon_{ijkln}(\alpha_0 e^j e^k e^l e^n + \alpha_1 R^{jk} e^l e^n + \alpha_2 R^{jk} R^{ln}) = 0 \tag{1}$$

and

$$\varepsilon_{ijkln}(e^k e^l + 2\alpha_2 R^{kl}) T^n = 0 \,. \tag{2}$$

Note that from the equation (2) it does not follow that torsion is zero in the vacuum, the explicit examples of this are given in the following sections where vacuum solutions with non-vanishing torsion are constructed.

Finding the solution to the equations (1) and (2) is greatly simplified in the torsion-less sector because the equations (2) are automatically solved in this case. For this very reason finding solutions with non-trivial torsion that exist for arbitrary values of parameters is extremely hard and still out of reach.

## 3. SPHERICALLY SYMMETRIC BLACK HOLE

This section is based on the results in the reference (Cvetković and Simić, 2018).

### 3.1. Killing vectors

We search for the static solution of equations that possesses SO(4) symmetry. Killing vectors that correspond to this symmetry are.

$$\begin{aligned}
&\xi_0 = \partial_t ,\\
&\xi_1 = cos\theta\,\partial_\psi - cot\psi\, sin\theta\,\partial_\theta ,\\
&\xi_2 = cos\varphi\,\partial_\theta - cot\theta\, sin\varphi\,\partial_\varphi ,\\
&\xi_3 = \partial_\varphi ,\\
&\xi_4 = sin\theta\, cos\,\varphi\partial_\psi + cot\psi\, cos\theta\, cos\varphi\,\partial\theta - \frac{cot\psi}{sin\theta} sin\varphi\partial_\varphi ,\\
&\xi_5 = sin\theta\, sin\varphi\,\partial_\psi + cot\psi\, cos\theta\, sin\varphi\partial_\theta + \frac{cot\psi}{sin\theta} cos\varphi\,\partial_\varphi ,\\
&\xi_6 = sin\varphi\,\partial_\theta + cot\theta\, cos\varphi\,\partial_\varphi .
\end{aligned} \tag{3}$$

Independent Killing vectors are $\xi_0$, $\xi_1$, $\xi_2$ and $\xi_3$. Others are obtained as the commutator of the later. Besides the invariance under Killing vectors, we have invariance under local Lorentz transformations which form is fixed and the only non-zero are given by

$$\varepsilon^{23} = -\frac{\sin\theta}{\sin\psi}, \quad \varepsilon^{34} = -\frac{\sin\varphi}{\sin\theta} .$$

### 3.2. Form of the vielbein and spin connection

Invariance under Killing vectors greatly restricts the most general form of the vielbein and spin connection.

The most general metric which is invariant under Killing vectors in coordinates $x^\mu = (t, r, \psi, \theta, \phi)$ is of the form

$$ds^2 = N^2 dt^2 - B^{-2} dr^2 - r^2 (d\psi^2 + \sin\psi^2 d\theta^2 + \sin\psi^2 \sin\theta^2 d\varphi^2) , \tag{4}$$

where functions $N$ and $B$ depend only on $r$.

The vielbeins $e^i$ are chosen to be of the form

$$\begin{aligned}
&e^0 = Ndt, \quad e^1 = B^{-1}dr, \quad e^2 = r\,d\psi,\\
&e^3 = r\sin\psi\, d\theta, \quad e^4 = r\sin\psi \sin\theta\, d\varphi.
\end{aligned} \tag{5}$$

The most general form of the spin connection is

$$\begin{aligned}
&\omega^{01} = A_0 dt + A_1 dr, \quad \omega^{02} = A_2 d\psi, \\
&\omega^{03} = A_2 \sin\psi d\theta, \quad \omega^{04} = A_2 \sin\psi \sin\theta, \\
&\omega^{12} = A_3 d\psi, \quad \omega^{13} = A_3 \sin\psi d\theta \\
&\omega^{14} = A_3 \sin\psi \sin\theta d\varphi, \quad \omega^{23} = \cos\psi d\theta + A_4 \sin\psi \sin\theta d\varphi, \\
&\omega^{24} = -A_4 \sin\psi d\theta + \cos\psi \sin\theta d\varphi, \quad \omega^{34} = A_4 d\psi + \cos\theta d\varphi,
\end{aligned} \tag{6}$$

where $A_i$ are arbitrary functions of the radial coordinate.

### 3.3. Solution

Solution to the equations of motion (1) and (2) with the most general form of the vielbein (5) and spin connection (6) is found by straightforward computation with the use of computer assistance.

A solution exists only if functions $N$ and $B$ are equal and there are two solutions one of which is a well-know Boulware-Deser black hole (Boulware and Deser, 1985) and the other is

$$N^2 = B^2 = -\frac{\alpha_1}{8\alpha_2}\left(r^2 - \frac{16\alpha_2 C}{7\alpha_1} r - \frac{r_+^2}{r^2}\right) . \tag{7}$$

The solution for the functions that determine the spin connection is as follows

$$\begin{aligned}
&A_1 = A_2 = A_3 = 0, \\
&A_0 = \frac{\alpha_1}{\alpha_2} r + C, \\
&A_4 = \sqrt{1 - 2\frac{\alpha_0}{\alpha_1} r^2}.
\end{aligned} \tag{8}$$

Constants $C$and $r_+$ characterize the solution. For simplicity, we take $C = 0$ in the following analysis of the properties. This solution exists only if the constraint between parameters holds

$$\alpha_1^2 - 12\alpha_0\alpha_2 = 0 , \tag{9}$$

and if the ratio $\alpha_1/\alpha_2$ is negative. From the formula (7) we see that the metric of the black hole is asymptotically Anti de Sitter.

### 3.4. Properties of the solution

For the definitions and properties of curvature and torsion invariants see reference (Obukhov, 2006). Now we will give the results for the most important invariants of the black hole.

The scalar Cartan curvature is of the form

$$R = -\frac{2\alpha_1}{\alpha_2} . \tag{10}$$

The scalar Riemann curvature is given by

$$\breve{R} = \frac{\alpha_1}{2\alpha_2}(-5 - \frac{12\alpha_2}{\alpha_1 r^2} + \frac{3r_+^8}{r^8}) . \tag{11}$$

The quadratic torsion invariant is

$$*(T^i * T_i) = \frac{3\alpha_1}{2\alpha_2}(1 - \frac{r_+^8}{r^8}) . \tag{12}$$

From these invariants, we conclude that there is a singularity in the center of the black hole. An interesting point is that singularity is not visible in the Cartan curvature invariant but in the torsion invariant, which is an unusual property.

Next, we turn to the thermodynamics of the black hole and give the results for its temperature and entropy.

The temperature of the black hole is given by

$$T = \frac{(N^2)'}{4\pi} = -\frac{\alpha_1}{4\alpha_2} r_+ . \tag{13}$$

The proportionality of the temperature to the radius of the event horizon is not common for the black holes with a spherical topology of the event horizon, except in the case of three dimensions, and it is a nice illustration that interesting things can happen in alternative theories of gravity.

The entropy is calculated in the semi-classical level, by calculating the Euclidean partition function which has an interpretation of free energy, and it is concluded that it vanishes

$$S = 0 . \tag{14}$$

This result is very interesting because it is drastically different from the one in general relativity. As such it is a good check for any entropy formula. Also, because it is expected that the explanation of black hole microstates is universal, it is puzzling why this solution has such a low number of states compared to black holes in general relativity.

To every Killing vector $\xi_i$, we can associate conserved charge $Q(\xi_i)$, the charges are calculated in the original reference using the Nester formula (Nester, 1991) and it is obtained that all charges are zero

$$Q(\xi_i) = 0 . \tag{15}$$

## 4. BLACK RING

This section is based on the results obtained in the reference (Cvetković and Simić, 2016).

### 4.1. Ansatz

The search for a new class of solutions is inspired by Canfora et al. (Canfora et al., 2007), who found a solution of the type $AdS_2 \times S^3$ when the coefficients in the Lagrangian satisfy the relation

$$\alpha_1^2 - 12\alpha_0\alpha_2 = 0\,. \tag{16}$$

We shall now construct another class of solutions of the "complementary" type $\Sigma^3 \times S^2$, where $\Sigma^3$ is a three-dimensional space-time and $S^2$ is a two-dimensional sphere. We start from the following ansatz for curvature

$$\begin{aligned} R^{ab} &= qe^a e^b, \\ R^{3a} &= R^{4a} = 0, \\ R^{34} &= -\frac{1}{r_0^2} e^3 e^4, \end{aligned} \tag{17}$$

and torsion

$$\begin{aligned} T^a &= p\varepsilon^{abc} e_b e_c, \\ T^3 &= T^4 = 0. \end{aligned} \tag{18}$$

In the ansatz we have three real number $q$, $r_0$ and $p$ which are a priory arbitrary before substituting the ansatz in the equations of motion (1) and (2), which will lead to a relation among them. We decomposed the indices $a,b,c... = 0,1,2$ and 3 and 4 which are written explicitly, and we also defined $\varepsilon^{abc} = \varepsilon^{abc34}$.

### 4.2. Solution

The three-dimensional space-time remains arbitrary after substituting the ansatz in the equations of motion, but there is only one reasonable black hole solution in this number of dimensions which is a BTZ black hole with(-out) torsion (Garcia et al., 2003; Obukhov, 2003). Because of this, the vielbein takes the following form.

$$\begin{aligned} &e^0 = Ndt, \quad e^1 = N^{-1}dr, \quad e^2 = r(d\varphi + N_\varphi dt), \\ &e^3 = r_0 d\theta, \quad e^4 = r_0 sin\theta\, d\chi, \end{aligned} \tag{19}$$

where the functions $N$ and $N_\varphi$ are given by

$$N^2 = -2m + \frac{r^2}{l^2} + \frac{j^2}{r^2}, \quad N_\varphi = \frac{j}{r^2}\,. \tag{20}$$

We introduced the AdS radius in the following manner

$$\frac{1}{l^2} = \mathrm{q} + \frac{p^2}{4}, \tag{21}$$

and $m$ and $j$ are parameters of the solution which are related to mass and angular momentum, respectively.

The spin connection is of the form

$$\begin{aligned} \omega^{ab} &= \breve{\omega}^{ab} - \frac{p}{2}\varepsilon^{abc} e_c, \\ \omega^{34} &= -\cos\theta\, d\chi, \end{aligned} \tag{22}$$

where $\omega^{ab}$ is the Riemann spin connection given by the following expressions

$$\breve{\omega}^{01} = -\frac{r}{l^2}dt - \frac{j}{r}d\varphi,$$
$$\breve{\omega}^{02} = -\frac{j}{Nr^2}dr, \tag{23}$$
$$\breve{\omega}^{12} = Nd\varphi.$$

As previously stated, the equations of motion introduce a relation among the parameters in the ansatz, which reads

$$q = -\frac{1}{2r_0^2}\,. \tag{24}$$

Also, as usual for the solutions with torsion of Lovelock gravity, a solution does not exist generally but in the sector of the theory in which a constraint between the coefficients in the Lagrangian holds

$$\alpha_1^2 - 8\alpha_0\alpha_2 = 0\,. \tag{25}$$

### 4.3. Properties of the black ring

The black ring as the product manifold of a BTZ black hole and a two-dimensional sphere inherits their Killing vectors. The complete set of Killing vectors consists of those originating from the BTZ black hole

$$\xi_0 = \partial_t, \quad \xi_1 = \partial_\varphi\,, \tag{26}$$

and those inherited from the sphere

$$\xi_2 = \partial_\chi, \quad \xi_3 = \sin\chi\partial_\theta + \cot\theta\cos\chi\partial_\chi, \quad \xi_4 = \cos\chi\partial_\theta - \cot\theta\sin\chi\partial_\chi\,. \tag{27}$$

For every Killing vector, we have conserved charge $Q(\xi_i)$, the charges are calculated in reference (Cvetković and Simić, 2016) using the Nester formula and, as in the previous solution, it is concluded that all the charges are zero

$$Q(\xi_i) = 0\,. \tag{28}$$

This is even more striking than in the case of a spherically symmetric black hole for which, because it does not rotate, only zero mass was an unexpected result. Namely, the black ring is a five-dimensional generalization of a rotating BTZ black hole which has non-zero angular momentum in three dimensions but, as we see, the black ring has a vanishing angular momentum nonetheless.

## 5. CONCLUSION

In this paper, we gave a short review of two black hole solutions that exist in five-dimensional Lovelock gravity.

First, we reviewed a spherically symmetric black hole. We explained what its Killing vectors are and what is the most general form of the metric and spin connection compatible with them. Afterward, we presented the solution itself and gave its properties. An interesting property is that all conserved charges vanish, which means that the mass of this solution is zero, too. This is a peculiar property which is in conflict with our intuition that black holes are made by the collapse of ordinary matter. Another peculiar property of this black hole is zero entropy. The vanishing entropy in the semi-classical approximation does not imply that the entropy calculated in full quantum theory is zero. It tells us that the entropy is much smaller than expected by the factor $1/G$, which immediately leads to the conclusion that this black hole has far fewer microstates than the usual black hole with non-vanishing entropy in the semi-classical approximation. For this reason, the solution is very interesting as a consistency check of every proposal for the black hole micro-states.

Second, we constructed a black ring with torsion which is a black hole which horizon of events does not have a spherical topology but the topology $S^1 \times S^2$. This is the reason for its name. The black ring also has all charges equal to zero, including its mass and angular momentum. This is, again, counter-intuitive, even more, if we take into account that this solution is nothing more than a rotating BTZ black hole times a two-dimensional sphere. Because a rotating BTZ black hole possesses mass and angular momentum, it is not clear what makes black rings so different from it.

## REFERENCES


Aros, R., Troncoso, R. and Zanelli, J., 2001. Phys. Rev. **D63**, [hep-th/0011097], DOI: 10.1103/PhysRevD.63.084015.

Boulware, D. G. and Deser, S., 1985. Phys. Rev. Lett. **55**, 2656.

Camanho, X. O. and Edelstein, J. D., 2013. Class. Quant. Grav. **30**, arXiv:1103.3669 [hep-th], DOI: 10.1088/0264-9381/30/3/035009.

Canfora, F., Giacomini, A. and Willinson, S., 2007. Phys. Rev. **D76**, 044021, arXiv:0706.2891 [gr-qc], DOI: 10.1103/PhysRevD.76.044021.

Canfora, F., Giacomini, A. and Troncoso, R., 2008. Phys. Rev. **D77**, 024002, arXiv:0707.1056 [hep-th], DOI: 10.1103/PhysRevD.77.024002.

Cai Rong-Gen and Ohta, N., 2006. Phys. Rev. **D74**, 064001, [hep-th/0604088], DOI: 10.1103/PhysRevD.74.064001.

Cai Rong-Gen, Caob, Li-Ming and Ohta, N., 2010. Phys. Rev. **D81**, 024018, [hep-th/0911.0245], DOI: 10.1103/PhysRevD.81.024018.

Cvetković, B. and Simić, D., 2016. Phys. Rev. **D94**, no.8, 084037, arXiv:1608.07976 [gr-qc], DOI: 10.1103/PhysRevD.94.084037.

Cvetković, B. and Simić, D., 2018. Class. Quant. Grav. **35**, no.5, 055005, arXiv:1708.08766[gr-qc], DOI: 10.1103/PhysRevD.96.064031.

Dotti, G., Oliva, J. and Troncoso, R., 2007. Phys. Rev. **D76**, 064038, arXiv:0706.1830 [hep-th], DOI: 10.1103/PhysRevD.76.064038.

Emparan, R. and Reall, H. S., 2002. Phys. Rev. Lett. **88**, [hep-th/0110260], DOI: 10.1103/PhysRevLett.88.101101.

Emparan, R. and Reall, H. S., 2008. Living Rev. Rel. **11**, arXiv:0801.3471 [hep-th], DOI: 10.12942/lrr-2008-6.

Garcia, A., Hehl, F. W., Heinecke, C. and Macias, A., 2003. Phys. Rev. **D67**, 124016, DOI: 10.1103/PhysRevD.67.124016.

Garraffo, C. and Giribet, G., 2008. Mod. Phys. Lett. **A23**, 1801, arXiv:0805.3575 [gr-qc], DOI: 10.1142/S0217732308027497.

Gregory, R. and Laflamme, R., 1993. Phys. Rev. Lett. **70**, 2837, [hep-th/9301052], DOI: 10.1103/PhysRevLett. 70.2837.
Horowitz, G. T. and Strominger, A., 1991. Nucl. Phys. **B360**, 197, DOI: 10.1016/0550-3213(91)90440-9.
Kastor, D. and Mann, R. B., 2006. JHEP **0604**, 048, [hep-th/0603168], DOI: 10.1088/1126-6708/2006/04/048.
Lovelock, D., 1971. J. Math. Phys. **12**, 498, DOI: 10.1063/1.1665613.
Lovelock, D., 1972. J. Math. Phys. **13**, 874, DOI: 10.1063/1.1666069.
Maeda, H., Willison, S. and Ray, S., 2011. Class. Quant. Grav. **28**, 165005, arXiv:1103.4184 [gr-qc], DOI: 10.1088/0264-9381/28/16/165005.
Nester, J. M., 1991. Mod. Phys. Lett. **A6**, 2655, DOI: 10.1142/S0217732391003092.
Obukhov, Y. N., 2003. Phys. Rev. **D68**, 124015, DOI: 10.1103/PhysRevD.68.124015.
Obukhov, Yu. N., 2006. Int. J. Geom. Meth. Mod. Phys. **3**, 95, DOI: 10.1142/S021988780600103X.
Ray, S., 2015. Class. Quant. Grav. **32**, no.19, 195022, arXiv:1505.03830 [gr-qc], DOI: 10.1088/0264-9381/32/19/195022.

# ZANIMLJIVE PETODIMENZIONALNE CRNE RUPE

*U ovom radu ćemo proučiti dve crne rupe koje su rešenja petodimenzionalne Lavlokove gravitacije. Ova rešenja su karakterisana nenultom torzijom i interesantnom osobinom da su svi njihovi očuvani naboji nula. Prvo rešenje je sfernosimetrična crna rupa sa torzijom koja takođe ima nultu entropiju u semiklasičnoj aproksimaciji. Drugo rešenje je crni prsten, koji je petodimenzionalna generalizacija BTZ crne rupe sa torzijom u tri dimenzije.*

Ključne reči: *crna rupa, torzija, alternativne teorije gravitacije*