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INTRODUCING MATTER FIELDS IN $SO(2,3)_*$ MODEL OF NONCOMMUTATIVE GRAVITY†

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Marija Dimitrijević-Ćirić, Dragoljub Gočanin, Nikola Konjik, Voja Radovanović

University of Belgrade, Faculty of Physics, Studentski Trg 12, 11000 Belgrade, Serbia

Abstract. *This is a review of some of our recent work concerning Noncommutative Field Theory founded on $SO(2,3)_*$ gauge symmetry. One significant feature of this approach is that gravitational field, given by the vierbein, becomes manifest only after a suitable gauge fixing and it is formally united with other gauge fields. Starting from a model of pure noncommutative gravity, we extend it by introducing fermions and Yang-Mills gauge field. Using the enveloping algebra approach and the Seiberg-Witten map we construct corresponding actions and expand them perturbatively in powers of the canonical noncommutativity parameter $\theta^{\alpha\beta}$. Unlike in the case of pure noncommutative gravity, first non-vanishing noncommutative corrections are linear in the noncommutativity parameter and they describe the coupling of matter and gauge fields with gravity due to spacetime noncommutativity. This is augmented by the fact that some of these corrections pertain even in flat spacetime where they induce potentially observable noncommutative deformations. We discuss the effects of noncommutativity on electron's dispersion relation in the presence of constant background magnetic field – Landau levels. Our results could be useful for further investigation of phenothe menological consequences of spacetime noncommutativity.*

Key words: *NC gravity, Seiberg-Witten map, AdS gravity*

1. INTRODUCTION

Noncommutative (NC) Field Theory, i.e. the theory of relativistic fields on *noncommutative spacetime*, is a valuable effective theory of the underlying fundamental theory of quantum gravity. It is based on the method of *deformation quantization* via NC

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Corresponding author: Dragoljub Gočanin
University of Belgrade, Faculty of Physics, Studentski Trg 12, 11000 Belgrade, Serbia
E-mail: dgocanin@ipb.ac.rs
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$*$-products. One speaks of a deformation of an object/structures whenever there is a family of similar objects/structures for which we can parametrise their ''distortion'' from the original, "undeformed" one. In physics, this so-called *deformation parameter* appears as some fundamental constant of nature that measures the deviation from the classical (undeformed) theory. When it is zero, the classical theory is restored. To deform a continuous structure of spacetime, an abstract algebra of NC coordinates is introduced. These NC coordinates, denoted by $\hat{x}^\mu$, satisfy some non-trivial commutation relations, so it is no longer the case that $\hat{x}^\mu\hat{x}^\nu = \hat{x}^\nu\hat{x}^\mu$. Abandoning this basic property of spacetime leads to various new physical effects that were not present in the theory developed on "classical" spacetime. The simplest case of noncommutativity is the so called *canonical noncommutativity,* defined by

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu} \ , \tag{1}$$

where $\theta^{\mu\nu}$ are components of a constant antisymmetric matrix.

Instead of deforming abstract algebra of coordinates one can take an alternative, but equivalent, approach in which noncommutativity appears in the form of NC $*$-products of functions (fields) of ordinary commutative coordinates. Specifically, to establish canonical noncommutativity, we use the NC Moyal-Weyl $*$-product:

$$(\hat{f} * \hat{g})(x) = e^{\frac{i}{2}\theta^{\alpha\beta}\frac{\partial}{\partial x^\alpha}\frac{\partial}{\partial y^\beta}} f(x)g(y)|_{y\to x} \ . \tag{2}$$

The first term in the expansion is the ordinary point-wise multiplication of functions. The quantities $\theta^{\alpha\beta}$ are assumed to be small deformation parameters that have dimensions of $(length)^2$. They are fundamental constants, like the Planck length or the speed of light.

The subject of NC gravity has received a lot of attention and various approaches to this problem have been developed. In Refs. (Chamseeddine, 2001; Chamseeddine, 2004; Cardella and Zanon, 2003) a deformation of pure Einstein gravity via Seiberg-Witten (SW) map is proposed. The twist approach is explored in Aschieri et al., 2005; Aschieri et al., 2006; Ohl and Schenckel, 2009; Aschieri and Castellani, 2010. Lorentz symmetry in NC QFT is considered in Chaichian et al., 2004; Chaichian et al., 2005. The extension of NC gauge theories to orthogonal and symplectic algebras is treated in Bars et al., 2001; Bonora et al., 2000. Some other proposals can be found in Yang, 2009; Steinacker, 2010; Burić and Madore, 2008; Klammer and Steinacker, 2009; Harikumar and Rivelles, 2006; Dobrski, 2011; Burić et al., 2006; Burić et al., 2008. The connection to Supergravity (SUGRA) is made in Aschieri and Castellani, 2009a; Castellani, 2013. Finally, in Dimitrijević Ćirić et al., 2017a; Dimitrijević Ćirić et al., 2017b; Dimitrijević et al., 2012; Dimitrijević and Radovanović, 2014, an approach based on canonically deformed anti de Sitter (AdS) group $SO(2,3)$ is established. NC gravity is treated as a gauge theory and it becomes manifest only after a suitable symmetry breaking (gauge fixing). Action was constructed without the previous introduction of the metric tensor and the second order NC correction to the Einstein-Hilbert action was found explicitly. Special attention has been devoted to the meaning of the coordinates used. Namely, it was shown that coordinates in which we postulate canonical noncommutativity are the Fermi inertial coordinates, i.e. coordinates of an inertial observer along the geodesic. A commutator between arbitrary coordinates can be derived from the canonical ones (see Dimitrijević Ćirić et al., 2017a).

The success of the pure gravity model led us to consider matter and non-gravitational gauge fields in the $SO(2,3)_*$ framework. Dirac spinor field coupled to $U(1)$ gauge field on NC spacetime is introduced in Gočanin and Radovanović, 2018; Dimitrijević-Ćirić et al., 2018, and physical consequences such as NC deformation of free electron's dispersion relation and NC deformation of its Landau levels have been analysed. From a different perspective, the problem was also treated by Aschieri and Castellani (Aschieri and Castellani, 2009b; Aschieri and Castellani, 2012; Aschieri and Castellani, 2013; Aschieri, 2014). Here we will present the most important results concerning the $SO(2,3)_*$ framework.

2. Matter and Gauge Fields in $SO(2,3)$ Gauge Theory of Gravity

In the *first-order formalism* (gauge theories of gravity) fermions couple naturally to the gravitational field. On the other hand, to couple gauge fields to the gravitational field one normally requires the existence of *Hodge dual operation*. To define the Hodge dual, the metric tensor must be known explicitly, which means working in the *second-order formalism*. This difference becomes even more evident in the $SO(2,3)$ model of gravity. Namely, in this model t,he basic dynamical field is the $SO(2,3)$ gauge field, which splits into the $SO(1,3)$ spin-connection and vierbein (tetrade) only after the gauge fixing (symmetry breaking). In this section, we present classical (undeformed) actions involving the Yang-Mills gauge field and the Dirac spinor field in the $SO(2,3)$ gravity model.

2.1. Pure gravity

Before introducing fermions and the Yang-Mills gauge field, let us briefly review the basics of $SO(2,3)$ gauge theory of gravity and set the notation. Generators of $SO(2,3)$ gauge group are denoted by M_{AB} (group indices $A, B, \ldots$ take values 0,1,2,3,5) and they satisfy AdS algebra:

$$[M_{AB}, M_{CD}] = i(\eta_{AD}M_{BC} + \eta_{BC}M_{AD} - \eta_{AC}M_{BD} - \eta_{BD}M_{AC}) \ , \tag{3}$$

where η_{AB} is $5D$ flat metric with signature (+, –, –, –, +). By introducing momenta generators as $P_a = \frac{1}{l}M_{a5}$, where l is a constant length scale, we can recast the AdS algebra (3) in a more explicit form:

$$[M_{ab}, M_{cd}] = i(\eta_{ad}M_{bc} + \eta_{bc}M_{ad} - \eta_{ac}M_{bd} - \eta_{bd}M_{ac}) \ , \tag{4}$$

$$[M_{ab}, P_c] = i(\eta_{bc}P_a - \eta_{ac}P_b) \ , \qquad [P_a, P_b] = -\frac{i}{2l^2}M_{ab} \ . \tag{5}$$

In the limit $l \to \infty$ the AdS algebra reduces to the Poincaré algebra (Wigner-Inonu contraction). A realization of (3) can be obtained from $5D$ gamma-matrices Γ_A that satisfy Clifford algebra: $\{\Gamma_A, \Gamma_B\} = 2\eta_{AB}$; the generators are given by $M_{AB} = \frac{i}{4}[\Gamma_A, \Gamma_B]$. One choice of $5D$ gamma-matrices is $\Gamma_A = (i\gamma_a\gamma_5, \gamma_5)$, where γ_a are the usual $4D$ gamma-matrices. Indices $a, b, \ldots$ take values $0,1,2,3$. In this particular representation, $SO(2,3)$ generators are: $M_{ab} = \frac{i}{4}[\gamma_a, \gamma_b] = \frac{1}{2}\sigma_{ab}$ and $M_{5a} = \frac{1}{2}\gamma_a$. The total gauge field ω_μ takes values in the Lie algebra of $SO(2,3)$ and it decomposes into ω_μ^{ab} and $\omega_\mu^{a5} = \frac{1}{l}e_\mu^a$, that is

$$\omega_\mu = \frac{1}{2}\omega_\mu^{AB}M_{AB} = \frac{1}{4}\omega_\mu^{ab}\sigma_{ab} - \frac{1}{2l}e_\mu^a\gamma_a \ . \tag{6}$$

The field strength tensor is defined in the usual way:

$$F_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu - i[\omega_\mu, \omega_\nu]$$
$$= \frac{1}{2} F_{\mu\nu}^{AB} M_{AB} = \left(R_{\mu\nu}^{ab} - \frac{1}{l^2}(e_\mu^a e_\nu^b - e_\mu^b e_\nu^a)\right)\frac{\sigma_{ab}}{4} - F_{\mu\nu}^{a5}\frac{\gamma_a}{2}\,, \quad (7)$$

with

$$R_{\mu\nu}^{ab} = \partial_\mu \omega_\nu^{ab} - \partial_\nu \omega_\mu^{ab} + \omega_\mu^{ac}\omega_\nu^{cb} - \omega_\mu^{bc}\omega_\nu^{ca}\,, \quad lF_{\mu\nu}^{a5} = \nabla_\mu e_\nu^a - \nabla_\nu e_\mu^a = T_{\mu\nu}^a\,. \quad (8)$$

Equations (6), (7) and (8) suggest that one can identify ω_μ^{ab} with the spin connection of the Poincaré gauge theory, $l\omega_\mu^{a5}$ with the vierbein, $R_{\mu\nu}^{ab}$ with the curvature tensor and $lF_{\mu\nu}^{a5}$ with torsion. It was shown in the $70'$ that one can indeed make such an identification and relate AdS gauge theory with GR. Note that, in this framework, the vierbein field e_μ^a is treated as an additional gauge field, standing on equal footing with the spin-connection. This unification is an important feature of the theory with $SO(2,3)$ gauge symmetry. Vierbein is related to the metric tensor by $\eta_{ab} e_\mu^a e_\nu^b = g_{\mu\nu}$ and $e = \det(e_\mu^a) = \sqrt{-g}$.

A necessary step in obtaining GR from $SO(2,3)$ gauge theory of gravity is the gauge fixing, i.e. symmetry breaking from local $SO(2,3)$ down to local $SO(1,3)$. To do so, one introduces an *auxiliary field* $\phi = \phi^A \Gamma_A$. We break the symmetry by fixing the value of the auxiliary field, in particular, by setting $\phi^a = 0$ and $\phi^5 = l$. This field is a spacetime-scalar and a $SO(2,3)$-vector and it is constrained by: $\phi^A \phi_A = l^2$. It transforms in the adjoint representation of $SO(2,3)$ and its covariant derivative is

$$D_\mu \phi = \partial_\mu \phi - i[\omega_\mu, \phi]\,. \quad (9)$$

After the gauge fixing, the components of $D_\mu \phi$ reduce to $(D_\mu \phi)^a = e_\mu^a$ and $(D_\mu \phi)^5 = 0$. This is how we get the vierbein field e_μ^a from the auxiliary field ϕ. In Refs. (Stelle and West, 1980; MacDowell and Mansouri, 1977; Towsend, 1977; Wilczek, 1998), a commutative (undeformed) action for pure gravity with $SO(2,3)$ gauge symmetry was constructed. Also, in Chamseddine and Mukhanov, 2010; Chamseddine and Mukhanov, 2013, GR is formulated by gauging $SO(1,4)$ or, more suitably for SUGRA, $SO(2,3)$ group. Building on their work, the $SO(2,3)$ model of *pure gravity* action and its NC deformation were analyzed in Dimitrijević Ćirić et al., 2017a. We will not repeat that discussion here but merely present some of the main results. Before the gauge fixing, the action consists of three parts:

$$S_1 = \frac{ilc_1}{64\pi G_N} \mathrm{Tr} \int \mathrm{d}^4 x\ \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \phi\ , \quad (10)$$

$$S_2 = \frac{c_2}{128\pi G_N l} \mathrm{Tr} \int d^4 x\ \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} D_\rho \phi D_\sigma \phi \phi + h.c.\ , \quad (11)$$

$$S_3 = -\frac{ic_3}{128\pi G_N l} \mathrm{Tr} \int d^4 x\ \varepsilon^{\mu\nu\rho\sigma} D_\mu \phi D_\nu \phi D_\rho \phi D_\sigma \phi \phi\ , \quad (12)$$

After the gauge fixing, we finally obtain

$$S = \frac{-1}{16\pi G_N} \int d^4 x \left(\frac{c_1 l^2}{16} \varepsilon^{\mu\nu\rho\sigma} \varepsilon_{abcd} R_{\mu\nu}^{ab} R_{\rho\sigma}^{cd} + \sqrt{-g}\left((c_1 + c_2) R - \frac{6}{l^2}(c_1 + 2c_2 + 2c_3) \right) \right). \quad (13)$$

For the sake of generality, three *a priori* undetermined dimensionless constants are introduced. They can be fixed by some consistency conditions. The first part is the topological Gauss-Bonnet term which does not affect the equations of motion (note that we work in four-dimensional spacetime), and so, we can set $c_1 = 0$. The Einstein-Hilbert term requires $c_1 + c_2 = 1$, while the absence of the cosmological constant is provided with $c_1 + 2c_2 + 2c_3 = 0$.

Concerning the NC deformation of the model, here we just want to emphasize the most important conclusions (for details see Castellani, 2013; Dimitrijević Ćirić et al., 2017a). After deformation and perturbative expansion in powers of $\theta^{\alpha\beta}$, it was found that the first order NC correction to the commutative action equals zero. The first non-vanishing correction is quadratic in the NC parameter. The equations of motion for the vierbein and the spin-connection in the low energy limit indicate that noncommutativity is a source of both curvature and torsion. It was also found that in $SO(2,3)_*$ model, there are residual effects of noncommutativity in the limit of flat spacetime, namely, that there actually exists a canonical NC deformation of Minkowski space, and NC correction to the flat Minkowski metric suggests that the coordinates x^μ we started with, those which satisfy the NC-deformed commutation relations $[x^\mu \stackrel{*}{,} x^\nu] = i\theta^{\alpha\beta}$, are actually Fermi normal coordinates. These are the inertial coordinates of a local observer moving along a geodesic. The breaking of diffeomorphism symmetry due to canonical noncommutivity is understood as a consequence of working in a preferred reference frame given by the Fermi normal coordinates. A local observer moving along the geodesic measures $\theta^{\alpha\beta}$ to be constant. In any other reference frame this will not be the case.

2.2. Yang-Mills field

Introducing a non-Abelian $SU(N)$ gauge field, $A_\mu = A_\mu^I T_I$ requires an upgrade of the original gauge group $SO(2,3)$ to $SO(2,3) \times SU(N)$. Generators T_I of $SU(N)$ group are hermitian, traceless and they satisfy the (anti)commutation relations: $[T_I, T_J] = if_{IJK}T_K$ and $\{T_I, T_J\} = d_{IJK}T_K$, with antisymmetric structure constants f_{IJK} and totally symmetric symbols $d_{IJK} = Tr(\{T_I, T_J\}T_K)$. We use the normalization $Tr(T_I T_J) = \delta_{IJ}$. $SU(N)$ group indices I, J,... run from 1 to $N^2 - 1$. The total gauge potential of $SO(2,3) \times SU(N)$ group is given by

$$\Omega_\mu = \frac{1}{2}\omega_\mu^{AB} M_{AB} \otimes \mathbb{I} + \mathbb{I} \otimes A_\mu^I T_I \ , \tag{14}$$

and the corresponding total field strength $\mathbb{F}_{\mu\nu}$ is the sum of the gravitational part $F_{\mu\nu}$ and the Yang-Mills part $\mathcal{F}_{\mu\nu}$, that is

$$\mathbb{F}_{\mu\nu} = \frac{1}{2}F_{\mu\nu}^{AB} M_{AB} \otimes \mathbb{I} + \mathbb{I} \otimes \mathcal{F}_{\mu\nu}^I T_I \ , \tag{15}$$

with the usual $\mathcal{F}_{\mu\nu}^I = \partial_\mu A_\nu^I - \partial_\nu A_\mu^I + gf^{IJK}A^J A^K$, where g is the Yang-Mills coupling strength. We define action for Yang-Mills gauge field A_μ that is invariant under $SO(2,3) \times SU(N)$ transformations, as follows:

$$S_A = -\frac{1}{16l} T_r \int d^4x \ \varepsilon^{\mu\nu\rho\sigma} \left\{ f \mathbb{F}_{\mu\nu} D_\rho \phi D_\sigma \phi \phi + \frac{i}{6} f^2 D_\mu \phi D_\nu \phi D_\rho \phi D_\sigma \phi \phi \right\} + h.c. \tag{16}$$

It involves an additional auxiliary field f defined by

$$f = \frac{1}{2} f^{AB,I} M_{AB} \otimes T_I \ , \quad \delta_\varepsilon f = i[\varepsilon, f] \ , \tag{17}$$

where the gauge parameter

$$\varepsilon = \frac{1}{2}\varepsilon^{AB}M_{AB} \otimes \mathbb{I} + \mathbb{I} \otimes \varepsilon^I T_I \tag{18}$$

consists of the $SO(2,3)$ and the $SU(N)$ part. The field f transforms in the adjoint representation of $SO(2,3)$ and $SU(N)$ group. The role of this field is to produce the canonical kinetic term in curved spacetime for the $SU(N)$ gauge field in the absence of the Hodge dual. The auxiliary field $\phi = \phi^A\Gamma_A$ is invariant under $SU(N)$ gauge transformations and its full covariant derivative is given by

$$D_\sigma\phi = \partial_\sigma\phi - ig[\Omega_\sigma, \phi] = \partial_\sigma\phi - i[\omega_\sigma, \phi] \ . \tag{19}$$

By setting $\phi^a = 0$ and $\phi^5 = l$ in (16) we break $SO(2,3)$ gauge symmetry down to the local Lorentz $SO(1,3)$ symmetry and obtain

$$S_A = \frac{1}{2}\int d^4x\, e\, f^{ab,I}\mathcal{F}^I_{\mu\nu}e^\mu_a e^\nu_b + \frac{1}{4}\int d^4x\, e\,(f^{ab,I}f^I_{ab} + 2f^{a5,I}f^I_{a5}) \ . \tag{20}$$

Equations of motion (EoMs) for the auxiliary field are

$$f^I_{a5} = 0 \ , \quad f^I_{ab} = -e^\mu_a e^\nu_b \mathcal{F}^I_{\mu\nu} \ . \tag{21}$$

Evaluating (20) on these EoMs we eliminate from it the auxiliary field f and so we have

$$S_A[A_\mu] = -\frac{1}{4}\int d^4x\, e\, \eta^{ar}\eta^{bs}e^\mu_a e^\nu_b e^\rho_r e^\sigma_s \mathcal{F}^I_{\mu\nu}\mathcal{F}^I_{\rho\sigma} = -\frac{1}{4}\int d^4x \sqrt{-g}\ g^{\mu\rho}g^{\nu\sigma}\mathcal{F}^I_{\mu\nu}\mathcal{F}^I_{\rho\sigma}, \tag{22}$$

and this is exactly the canonical kinetic term for Yang-Mills gauge field in curved spacetime.

2.3. Dirac field

The problem of introducing fermions in the framework of $SO(2,3)$ gauge theory of gravity was solved in Gočanin and Radovanović (2018) and that procedure will not be repeated here; we will merely state the main results. In the context of $SU(N)$ Yang-Mills theory, we introduce a multiplet of Dirac spinors

$$\Psi = \begin{pmatrix}\psi_1 \\ \vdots \\ \psi_N\end{pmatrix} , \tag{23}$$

that transforms under infinitesimal gauge transformation as

$$(\delta_\varepsilon\Psi)_i = i(\varepsilon\Psi)_i = i\varepsilon^I(T_I)_{ij}\psi_j = \frac{i}{2}\varepsilon^I_{AB}(T_I)_{ij}M_{AB}\psi_i \ . \tag{24}$$

Its covariant derivative is given by

$$(D_\sigma\Psi)_i = \partial_\sigma\psi_i - ig\Omega_\sigma\psi_i = \nabla_\sigma\psi_i + \frac{i}{2l}e^a_\sigma\gamma_a\psi_i - igA^I_\sigma(T_I)_{ij}\psi_j \ . \tag{25}$$

The commutative fermionic action is defined by

$$S_\Psi = \frac{i}{12}\int \mathrm{d}^4x\, \varepsilon^{\mu\nu\rho\sigma}\{\bar{\Psi}D_\mu\phi D_\nu\phi D_\rho\phi D_\sigma\Psi - D_\sigma\bar{\Psi}D_\mu\phi D_\nu\phi D_\rho\phi\Psi\} , \tag{26}$$

and after the symmetry breaking, it reduces to

$$S_\Psi = i\int d^4x\, e\left\{\frac{i}{2}\bar\Psi e_s^\sigma\gamma^s\nabla_\sigma\Psi - \frac{i}{2}\nabla_\sigma\bar\Psi e_s^\sigma\gamma^s\Psi - \frac{2}{l}\bar\Psi\Psi + g\bar\Psi e_s^\sigma\gamma^s A_\sigma^I T_I\Psi\right\}. \quad (27)$$

This is exactly the action for the spinor field Ψ in curved spacetime we sought for, except for the unusual $2/l$ mass term which seems to be universal in the sense that every fermion in the theory, after quantisation, would have the same mass. But, we want to be able to have fermions with an arbitrary mass m. For that, we have to include additional $SO(2,3)\times SU(N)$ invariant terms in the action. We will call them "mass terms" $S_{m,i}$ $(i=1,2,3)$ and they are given by

$$S_{m,1} = \frac{i}{144}\left(\frac{m}{l}-\frac{2}{l^2}\right)\int d^4x\ \varepsilon^{\mu\nu\rho\sigma}[\bar\Psi D_\mu\phi D_\nu\phi D_\rho\phi D_\sigma\phi\phi\Psi + \bar\Psi\phi D_\mu\phi D_\nu\phi D_\rho\phi D_\sigma\phi\Psi]\ ,$$

$$S_{m,2} = \frac{-i}{144}\left(\frac{m}{l}-\frac{2}{l^2}\right)\int d^4x\ \varepsilon^{\mu\nu\rho\sigma}[\bar\Psi D_\mu\phi D_\nu\phi D_\rho\phi\phi D_\sigma\phi\Psi + \bar\Psi D_\mu\phi\phi D_\nu\phi D_\rho\phi D_\sigma\phi\Psi]\ ,$$

$$S_{m,3} = \frac{i}{72}\left(\frac{m}{l}-\frac{2}{l^2}\right)\int d^4x\ \varepsilon^{\mu\nu\rho\sigma}\ \bar\Psi D_\mu\phi D_\nu\phi\phi D_\rho\phi D_\sigma\phi\Psi\ . \quad (28)$$

After the symmetry breaking, the sum of the three mass terms in (28), denoted by S_m, reduces to

$$S_m = -\left(m-\frac{2}{l}\right)\int d^4x\ e\ \bar\Psi\Psi\ , \quad (29)$$

and when we add this to action (27), the universal $2/l$ term exactly cancels, and we are left with the fermions of mass m. In this way, we can get an arbitrary spectrum of fermion masses in the theory. Thus, we have a complete and consistent model of $SO(2,3)\times SU(N)$ invariant Yang-Mills theory. Now we want to deform it.

3. NC Deformation of Yang-Mills Theory

To canonically deform a gauge field theory, we take a model of commutative action endowed with some gauge symmetry, such as (16) or (26), and replace ordinary commutative multiplication by the noncommutative Moyal-Weyl $*$-product defined in (2). The fields of the deformed theory are denoted by a "hat" symbol and, by definition, their transformation laws under the deformed gauge transformations have the same structure as those for ordinary fields under ordinary gauge transformations. One introduces NC spinor field $\hat\Psi$, NC adjoint field $\hat\phi$ and NC gauge potential $\hat\Omega_\mu$; we use this gauge potential to construct NC field strength,

$$\hat{\mathbb{F}}_{\mu\nu} = \partial_\mu\hat\Omega_\nu - \partial_\nu\hat\Omega_\mu - i[\hat\Omega_\mu \stackrel{*}{,} \hat\Omega_\nu]\ . \quad (30)$$

The covariant derivatives of NC spinor and the adjoint field have the same structure as in the underlying commutative theory,

$$D_\mu\hat\Psi = \partial_\mu\hat\Psi - i\hat\Omega_\mu * \hat\Psi,$$
$$D_\mu\hat\phi = \partial_\mu\hat\phi - i[\hat\Omega_\mu \stackrel{*}{,} \hat\phi]\ . \quad (31)$$

Fields $\hat\Psi$ and $\hat\phi$, along with their covariant derivatives (31), transform in the fundamental and adjoint representation, respectively, under NC gauge transformations. Infinitesimally:

$$\delta_\varepsilon^* \hat{\Psi} = i\hat{\Lambda}_\varepsilon * \hat{\Psi}, \qquad \delta_\varepsilon^* D_\mu \hat{\Psi} = i\hat{\Lambda}_\varepsilon * D_\mu \hat{\Psi} \ ,$$
$$\delta_\varepsilon^* \hat{\phi} = i[\hat{\Lambda}_\varepsilon \stackrel{*}{,} \hat{\phi}], \qquad \delta_\varepsilon^* D_\mu \hat{\phi} = i[\hat{\Lambda}_\varepsilon \stackrel{*}{,} D_\mu \hat{\phi}] \ . \tag{32}$$

The NC field strength also transforms in the adjoint representation,

$$\delta_\varepsilon^* \hat{\mathbb{F}}_{\mu\nu} = i[\hat{\Lambda}_\varepsilon \stackrel{*}{,} \hat{\mathbb{F}}_{\mu\nu}] \ . \tag{33}$$

In all the above formulas, $\hat{\Lambda}_\varepsilon$ stands for a NC gauge parameter whereas ε is a commutative gauge parameter. The transformation law for NC gauge potential is:

$$\delta_\varepsilon^* \hat{\Omega}_\mu = \partial_\mu \hat{\Lambda}_\varepsilon - i[\hat{\Omega}_\mu \stackrel{*}{,} \hat{\Lambda}_\varepsilon] \ , \tag{34}$$

where Ω_α is the ordinary, commutative gauge potential. In general, however, there is a problem concerning the closure condition for NC gauge transformations. Generally, if the parameter $\hat{\Lambda}$ is supposed to be Lie algebra-valued, $\hat{\Lambda}(x) = \hat{\Lambda}^A(x) T_A$, it follows that

$$\begin{aligned}[\delta_{\Lambda_1}^* \stackrel{*}{,} \delta_{\Lambda_2}^*]\hat{\Psi} &= \left(\hat{\Lambda}_1 * \hat{\Lambda}_2 - \hat{\Lambda}_2 * \hat{\Lambda}_1\right) * \hat{\Psi} \\ &= \frac{1}{2}\left([\hat{\Lambda}_1^A \stackrel{*}{,} \hat{\Lambda}_2^B]\{T_A, T_B\} + \{\hat{\Lambda}_1^A \stackrel{*}{,} \hat{\Lambda}_2^B\}[T_A, T_B]\right) * \hat{\Psi},\end{aligned} \tag{35}$$

and so, the commutator of two infinitesimal NC gauge transformations does not generally close in the Lie algebra itself, since the anti-commutator $\{T_A, T_B\}$ does not in general belong to this algebra. To overcome this difficulty, we will apply the enveloping algebra approach. The enveloping algebra of a gauge group is "large enough" to ensure the closure property of NC gauge transformations, if we allow the NC gauge parameter $\hat{\Lambda}$ to take values in it. The NC gauge potential $\hat{\Omega}_\mu$ then also belongs to the enveloping algebra and can be represented in its basis. However, the enveloping algebra has an infinite basis, and so it seems that by invoking it we actually introduced an infinite number of new degrees of freedom (new fields) in the NC theory. The solution to the problem is the Seiberg-Witten map [38, 39]. It relies on the fact that NC fields can be represented as a perturbation series in powers of the deformation parameter $\theta^{\alpha\beta}$, with expansion coefficients built out of the commutative fields, e.g. NC field $\hat{\Psi}$ can be represented as:

$$\hat{\Psi} = \Psi - \frac{1}{4}\theta^{\alpha\beta}\Omega_\alpha(\partial_\beta + D_\beta)\Psi + O(\theta^2) \ , \tag{36}$$

It is clear that at the lowest perturbative order NC fields consistently reduce to their undeformed counterparts.

The complete NC-deformed Yang-Mills action (NC actions will be also denoted by a "hat" symbol) invariant under deformed $SO(2,3)_* \times SU(N)_*$ gauge transformations is obtained by applying the above-described procedure to the commutative actions S_Ψ, S_m, and S_A, given by (16), (26) and (28), respectively. For example, the NC deformation of the pure Yang-Mills action S_A is given by

$$\begin{aligned}\hat{S}_A = -\frac{1}{16Nl} T_r \int d^4x\, \varepsilon^{\mu\nu\rho\sigma}\{&\hat{f} * \hat{\mathbb{F}}_{\mu\nu} * D_\rho\hat{\phi} * D_\sigma\hat{\phi} * \hat{\phi} \\ &+ \frac{i}{6}\hat{f} * \hat{f} * D_\mu\hat{\phi} * D_\nu\hat{\phi} * D_\rho\hat{\phi} * D_\sigma\hat{\phi} * \hat{\phi}\} + h.c.\end{aligned} \tag{37}$$

Now we can take the SW-map and represent NC-deformed fields in terms of ordinary commutative ones. In general, the resulting action, as a perturbative series in $\theta^{\alpha\beta}$, possesses the gauge symmetry of the undeformed action, order-by-order. This important property is ensured by the SW-map. The whole procedure, including some methods for simplifying the calculation, can be found in Gočanin and Radovanović (2018) and Dimitrijević-Ćirić et al. (2018).

After the symmetry breaking and elimination of the f-field - as it turns out, when working up to the first order in $\theta^{\alpha\beta}$, to eliminate it, one only needs to insert the undeformed equations of motion (21) in the first order NC action, in particular, there is no need for calculating the first order NC correction of these equations, since it will, whatever form it takes, annihilate the undeformed action (20) because of its specific structure - it becomes:

$$\hat{S}_A^{(1)} = -\frac{\theta^{\alpha\beta}}{16}\int d^4x\, e\, d_{IJK}\, g^{\mu\rho} g^{\nu\sigma}\left(\mathcal{F}^I_{\alpha\beta}\mathcal{F}^J_{\mu\nu}\mathcal{F}^K_{\rho\sigma} - 4\mathcal{F}^I_{\alpha\mu}\mathcal{F}^J_{\beta\nu}\mathcal{F}^K_{\rho\sigma}\right)\ . \tag{38}$$

This is the first order NC correction to the pure Yang-Mills action in curved spacetime. Note that this result agrees with the one obtained in [40], which employs NC deformation by minimal substitution. Analogous procedure for spinors has been done in Gočanin and Radovanović (2018).

After the symmetry breaking, the spinorial part of the full NC action, involving the "kinematic" term $\hat{S}_\Psi$ and the "mass terms" $\hat{S}_m$, reduces to

$$\begin{aligned}\hat{S}_\Psi^{(1)} + \hat{S}_m^{(1)} = \theta^{\alpha\beta}\int d^4x\, e\, \bar{\Psi}[&-\frac{1}{8}R^{ab}_{\alpha\mu}e^\mu_a\gamma_b\mathcal{D}_\beta - \frac{i}{32}R^{ab}_{\alpha\beta}\varepsilon_{abc}{}^d e^\sigma_d\gamma^c\gamma^5\mathcal{D}_\sigma \\ &+\frac{1}{16}R^{ab}_{\alpha\beta}e^\sigma_b\gamma_a\mathcal{D}_\sigma\end{aligned}$$

$$\begin{aligned}&-\frac{i}{16}R^{bc}_{\alpha\mu}e^\mu_a\varepsilon^a{}_{bcm}\gamma^m\gamma^5\mathcal{D}_\beta - \frac{i}{24}R^{ab}_{\alpha\mu}\varepsilon_{abc}{}^d e^c_\beta\left(e^\mu_d e^\sigma_s - e^\mu_s e^\sigma_d\right)\gamma^s\gamma^5\mathcal{D}_\sigma - \frac{i}{8l}T^a_{\alpha\beta}e^\sigma_a + \\ &\frac{i}{8l}T^a_{\alpha\mu}e^\mu_a\mathcal{D}_\beta + \frac{1}{16l}T^a_{\alpha\beta}e^\mu_a\sigma_\mu{}^\sigma\mathcal{D}_\sigma + \frac{1}{8l}T^a_{\alpha\mu}e^\mu_b\sigma_a{}^b\mathcal{D}_\beta - \frac{1}{12l}T^a_{\alpha\mu}\varepsilon_{ab}{}^{cd}e^b_\beta e^\mu_c e^\sigma_d\gamma^5\mathcal{D}_\sigma + \\ &\frac{7i}{48l^2}\varepsilon_{abc}{}^d e^a_\alpha e^b_\beta e^\sigma_d\gamma^c\gamma^5\mathcal{D}_\sigma - \frac{1}{4l}\sigma_\alpha{}^\sigma\mathcal{D}_\beta\mathcal{D}_\sigma - \frac{1}{4}(\nabla_\alpha e^a_\mu)(e^\mu_a e^\sigma_b - e^\sigma_a e^\mu_b)\gamma^b\mathcal{D}_\beta\mathcal{D}_\sigma - \\ &\frac{i}{8}\eta_{ab}(\nabla_\alpha e^a_\mu)(\nabla_\beta e^b_\nu)\varepsilon^{cdrs}e^\mu_c e^\nu_d e^\sigma_s\gamma_r\gamma_5\mathcal{D}_\sigma + \frac{i}{12}(\nabla_\alpha e^a_\mu)(\nabla_\beta e^b_\nu)\varepsilon_b{}^{cds}e^\mu_c e^\nu_d e^\sigma_s\ \gamma_a\gamma_5\mathcal{D}_\sigma - \\ &\frac{1}{12l}e^c_\alpha(\nabla_\beta e^b_\nu)\varepsilon_{bc}{}^{ds}e^\nu_d e^\sigma_s\gamma_5\mathcal{D}_\sigma - \frac{1}{8l}(\nabla_\alpha e^a_\mu)(e^\mu_a e^\sigma_b - e^\sigma_a e^\mu_b)e^c_\beta\sigma^b{}_c\mathcal{D}_\sigma - \\ &\frac{1}{8l}(\nabla_\alpha e^a_\mu)e^\mu_b\sigma_a{}^b\mathcal{D}_\beta + \frac{3}{96l}R^{ab}_{\alpha\beta}\sigma_{ab} + \frac{1}{16l}R^{ab}_{\alpha\mu}e^\mu_a e^c_\beta\sigma_{bc} - \frac{1}{16l}R^{ab}_{\alpha\mu}e_{\beta a}e^\mu_c\sigma_b{}^c - \frac{19}{288l^2}T^a_{\alpha\beta}\gamma_a + \\ &\frac{19}{144l^2}T^a_{\alpha\mu}e^\mu_a\gamma_\beta + \frac{1}{16l^2}T^a_{\alpha\mu}e_{\beta a}\gamma^\mu - \frac{19}{144l^2}(\nabla_\alpha e^a_\mu)e^\mu_a\gamma_\beta + \frac{1}{16l^2}(\nabla_\alpha e^a_\mu)e_{\beta a}\gamma^\mu - \frac{1}{12l^3}\sigma_{\alpha\beta} + \\ &\frac{3i}{8}\mathcal{F}_{\alpha\beta}e^\sigma_s\gamma^s\mathcal{D}_\sigma - \frac{i}{4}\mathcal{F}_{\alpha\sigma}e^\sigma_s\gamma^s\mathcal{D}_\beta + \frac{1}{8l}\mathcal{F}_{\alpha\beta}]\Psi + h.c.\end{aligned} \tag{39}$$

where we introduced the $SO(1,3)\times SU(N)$ covariant derivative:

$$\mathcal{D}_\sigma\Psi = (\nabla_\sigma - igA^I_\sigma T_I)\Psi\ . \tag{40}$$

This result exhibits the type of couplings between fermions and gravity that emerge due to spacetime noncommutativity. Evidently, some of them pertain even in flat spacetime. From the curved spacetime NC actions (39) and (40) we can derive NC-deformed action for Yang-Mills theory in Minkowski space. It comes down to

$$\hat{S}_{flat} = \int d^4x\ \{i\bar{\Psi}\gamma^\sigma\mathcal{D}_\sigma\Psi - \frac{1}{4}g^{\mu\rho}g^{\nu\sigma}\mathcal{F}^I_{\mu\nu}\mathcal{F}^I_{\rho\sigma}$$

$$+\theta^{\alpha\beta}[-\frac{1}{2l}\bar{\Psi}\sigma_{\alpha}{}^{\sigma}\mathcal{D}_{\beta}\mathcal{D}_{\sigma}\Psi+\frac{7i}{24l^2}\varepsilon_{\alpha\beta}{}^{\rho\sigma}\bar{\Psi}\gamma_{\rho}\gamma_5\mathcal{D}_{\sigma}\Psi+\frac{3i}{4}\bar{\Psi}\mathcal{F}_{\alpha\beta}\gamma^{\sigma}\mathcal{D}_{\sigma}\Psi$$
$$-\frac{i}{2}\bar{\Psi}\mathcal{F}_{\alpha\sigma}\gamma^{\sigma}\mathcal{D}_{\beta}\Psi-\frac{1}{6l^3}\bar{\Psi}\sigma_{\alpha\beta}\Psi+\frac{1}{4l}\bar{\Psi}\mathcal{F}_{\alpha\beta}\Psi]$$
$$-\frac{\theta^{\alpha\beta}}{16}d_{IJK}\ g^{\mu\rho}g^{\nu\sigma}\left(\mathcal{F}^I_{\alpha\beta}\mathcal{F}^J_{\mu\nu}\mathcal{F}^K_{\rho\sigma}-4\mathcal{F}^I_{\alpha\mu}\mathcal{F}^J_{\beta\nu}\mathcal{F}^K_{\rho\sigma}\right)\}\,. \tag{41}$$

As we can see, the first non-vanishing NC correction is linear in $\theta^{\alpha\beta}$, and this leads to some potentially observable physical effects. We will examine them in the case of NC Electrodynamics.

4. Electron in Background Magnetic Field

From the NC-deformed action (41), in the case of $U(1)$ gauge symmetry, we can derive NC-deformed Dirac equation

$$\left(i\gamma^{\mu}\partial_{\mu}-m+\gamma^{\mu}A_{\mu}+\theta^{\alpha\beta}\mathcal{M}_{\alpha\beta}\right)\psi=0\ , \tag{42}$$

with the linear NC correction given by

$$\theta^{\alpha\beta}\mathcal{M}_{\alpha\beta}=\theta^{\alpha\beta}\{-\frac{1}{2l}\sigma_{\alpha}{}^{\sigma}\mathcal{D}_{\beta}\mathcal{D}_{\sigma}+\frac{7i}{24l^2}\varepsilon_{\alpha\beta}{}^{\rho\sigma}\gamma_{\rho}\gamma_5\mathcal{D}_{\sigma}-\left(\frac{m}{4l^2}+\frac{1}{6l^3}\right)\sigma_{\alpha\beta}$$
$$+\frac{3i}{4}\mathcal{F}_{\alpha\beta}\gamma^{\sigma}\mathcal{D}_{\sigma}\ -\frac{i}{2}\mathcal{F}_{\alpha\mu}\gamma^{\mu}\mathcal{D}_{\beta}-\left(\frac{3m}{4}-\frac{1}{4l}\right)\mathcal{F}_{\alpha\beta}\}\ , \tag{43}$$

and investigate a special case of constant magnetic field $B=B\boldsymbol{e}_z$. We choose $A_{\mu}=(0,By,0{,}0)$ accordingly. An appropriate ansatz for (42) is

$$\psi=\begin{pmatrix}\varphi(y)\\ \chi(y)\end{pmatrix}e^{-iEt+ip_xx+ip_zz}\ . \tag{44}$$

Undeformed relativistic Landau levels are

$$E^{(0)}_{n,s}=\sqrt{p_z^2+m^2+(2n+s+1)B}\ . \tag{45}$$

Working perturbatively up to the first order in the parameter of noncommutativity and assuming, for simplicity, that only $\theta^{12}=-\theta^{21}=\theta\neq 0$, we get the NC correction to the relativistic Landau energy levels,

$$E^{(1)}_{n,s}=-\frac{\theta s}{E^{(0)}_{n,s}}\left(m+\frac{B(2n+s+1)}{E^{(0)}_{n,s}+m}\right)\left[\frac{m}{12l^2}-\frac{1}{3l^3}\right]+\frac{\theta B^2}{2E^{(0)}_{n,s}}(2n+s+1). \tag{46}$$

In the absence of the magnetic field, they reduce to

$$E^{(1)}_{n,s}=-\frac{\theta s}{E^{(0)}_{n,s}}\left[\frac{m^2}{12l^2}-\frac{m}{3l^3}\right]. \tag{47}$$

Since $s=\pm 1$ we see that constant noncommutative background causes Zeeman-like splitting of undeformed energy levels. The non-relativistic limit of NC energy levels is obtained by expending (46). If we also take $p_z=0$ (which correspondes to an electron constrained to a NC plane) the expansion reads:

$$
\begin{aligned}
E_{n,s} &= E^{(0)}_{n,s} + E^{(1)}_{n,s} + \mathcal{O}(\theta^2)\\
&= m_{eff} + \frac{2n+s+1}{2m} - \frac{(2n+s+1)^2}{8m^3}B^2_{eff} + \frac{(2n+s+1)^3}{16m^5}B^3_{eff} + \mathcal{O}(\theta^2),
\end{aligned} \tag{48}
$$

where we introduced $m_{eff} = m + \frac{\theta s}{3l^3} - \frac{\theta sm}{12l^2}$ as an *effective mass* and $B_{eff} = (B + \theta B^2)$ as an *effective magnetic field*. If we compare this expression with the one for undeformed energy levels $E^{(0)}_{n,s}$, we can conclude that the only effect of noncommutaivity is to modify (renormalise) the mass of an electron and the value of the background magnetic field. This interpretation of constant noncommutativity is in accord with string theory. In the famous paper by Seiberg and Witten [39], it is argued, in the context of string theory, that coordinate functions of the endpoints of an open string constrained to a D-brane in the presence of a constant Neveu-Schwarz B-field (equivalent to a constant magnetic field on the brane) satisfy constant noncommutativity algebra. The implication is that a relativistic field theory on noncommutative spacetime can be interpreted as a low energy limit, i.e. an effective theory, of the theory of open strings.

5. CONCLUSION

We discussed the coupling of matter fields with gravity in the framework of NC $SO(2,3)_*$ gauge theory of gravity. Using the Seiberg-Witten map and the enveloping algebra approach we constructed gauge invariant NC actions that can be represented as a perturbative series in powers of $\theta^{\alpha\beta}$. In this way, we formulated NC Electrodynamics and NC Yang-Mills theory in curved space-time induced by NC $SO(2,3)_*$ gravity. The flat spacetime limit of this model enables one to study the behaviour of an electron in a background electromagnetic field. Especially, corrections to the relativistic Landau levels of an electron in a constant magnetic field are derived along with their non-relativistic limit. It can be seen both from (46) and (48) that the NC correction to (non)-relativistic Landau levels depends on the mass m, the principal quantum number n and the spin s. In particular, the NC correction to energy levels will be different for different levels.

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UVOĐENJE POLJA MATERIJE U $SO(2,3)_*$ MODEL NEKOMUTATIVNE GRAVITACIJE

Ovo je pregled nekih skorašnjih rezultata koji se tiču nekomutativne teorije polja zasnovane na lokalnoj $SO(2,3)_*$ simetriji. Jedan od bitnih aspekata ove teorije je to da se gravitaciono polje, opisano tetradom, ispoljava tek nakon odgovarajuće kalibracije kao i to da je gravitaciono polje formalno ujedinjeno sa ostalim kalibracionim poljima.. Polazeći od modela čiste nekomutativne gravitacije, proširićemo ga uvođenjem fermiona i Jang-Milsovog kalibracionog polja. Koristeći metod obavijajuće algebre i Sajberg-Vitenovo preslikavanje konstruisana su odgovarajuća dejstva koja su potom razvijena perturbativno po kanonskom parametru nekomutativnosti $\theta^{\alpha\beta}$. Za razliku od čiste nekomutativne gravitacije, prve nenulte korekcije u razvoju dejstva su linearne po parametru nekomutativnosti. One opisuju interakciju materije i kalibracionih polja sa gravitacijom usled nekomutativnosti prostor-vremena. Na ovo se nadovezuje i to da ove nekomutativne korekcije opstaju čak i u ravnom prostor-vremenu i gde uzrokuju potencijalno opservabilne efekte. Razmotrićemo uticaj nekomutativnosti na disperzionu relaciju elektrona u pozadinskom magnetnom polju, tj. nekomutativne Landauove nivoe. Naš rezultat bi mogao imati uticaj na dalje ispitivanje fenomenoloških posledica nekomutativnosti prostor-vremena.

Ključne reči: *nekomutativna gravitacija, Sajberg-Vitenovo preslikavanje, AdS gravitacija*