

SCALAR QUASINORMAL MODES OF THE NEAR EXTREMAL REISSNER-NORDSTRÖM BLACK HOLE IN NC SPACE-TIME[†]

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Marija Dimitrijević Ćirić¹, Nikola Konjik¹, Andelo Samsarov²¹Faculty of Physics, University of Belgrade Studentski trg 12, 11000 Beograd, Serbia²Ruder Bošković Institute, Theoretical Physics Division, Bijenička 54, 10002 Zagreb, Croatia

Abstract. *A way to search for a signal of space-time noncommutativity is to study a quasinormal mode spectrum of the Reissner-Nordström black hole in the presence of noncommutativity. In this paper, we chose a particular NC deformation defined by the angular twist. We investigate a noncommutative (NC) deformation of a complex scalar field, minimally coupled with a classical (commutative) near extremal Reissner-Nordström background. The theory is manifestly invariant under the deformed $U(1)_*$ gauge symmetry group. Using the EOM for the NC complex scalar field in RN background, the QNM spectrum is calculated for a particular range of parameters corresponding to the near extremal limit.*

Key words: *quasinormal modes, near extremal RN black hole, NC scalar field*

1. INTRODUCTION

The ringdown phase of a black hole (the phase when a black hole emits gravitational waves) may broadly be divided into three stages: a short period of strong initial outburst of radiation, which is then followed by a long period of damped oscillations, dominated by quasi-normal modes (QNMs). The third stage is the so-called late time tails.

The feature of QNMs that they do not depend on the details of a perturbation but only on the parameters of a black hole makes them a convenient carrier of information on the properties of black holes. Black hole QNMs (Regge and Wheeler, 1957) also provide key signatures of gravitational waves. Moreover, the recent experimental discovery of gravitational waves including the ringdown phase arising from black hole mergers (Abbott et al., 2016) has opened up new possibilities for the observation of QNMs.

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Corresponding author: Nikola Konjik

Faculty of Physics, University of Belgrade, Studentski trg 12, 11000 Beograd, Serbia

E-mail: konjik@ipb.ac.rs

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Besides carrying the intrinsic information about black holes, it is reasonable to assume that QNMs may also carry information about the properties of the underlying space-time structure. In particular, if the underlying space-time structure is deformed in such a way that it departs from the usual notion of the smooth space-time manifold, then this deviation should also in some way be imprinted in the QNMs' spectrum.

QNMs spectra have been analyzed in the presence of the noncommutative (NC) structure of space-time (Giri, 2007; Gupta et al., 2015).

Our goal in this paper is to study the quasi-normal modes of an NC scalar field in a fixed background of the Reissner-Nordström (RN) black hole. More precisely, we fix a particular NC deformation of space-time by "turning on" the twist (3). After fixing the NC deformation in our model, we study the propagation of scalar and $U(1)$ gauge fields in the non-propagating geometry of a RN black hole. In particular, we study the QNMs spectrum of the charged massive scalar field. The RN geometry is non-dynamical and, unlike the scalar and gauge field, it does not feel the deformation.

In the next sections, we introduce the NC space-time and we construct the NC $U(1)_*$ gauge theory coupled with a charged scalar field. Then we use the Seiberg-Witten map (Seiberg and Witten, 1999) to expand the NC fields in terms of the corresponding commutative fields and obtain an expanded action. The action and the corresponding equations of motion are expanded up to first order in the NC parameter a . We calculate the scalar field equation of motion in the RN background. This equation is our starting point for discussing QNMs solutions in the extremal RN case in Section 4. Both numerical and analytic solutions show that there is a Zeeman-like splitting of the QNMs frequencies for a fixed angular momentum number l . Solutions depend on the magnetic number m in such a way that m always couples with the NC parameter a .

2. NONCOMMUTATIVE SPACE-TIME FROM THE ANGULAR TWIST

The Reissner-Nordström (RN) black hole is a well-known solution to Einstein equations. It represents a charged non-rotating black hole and it is given by

$$ds^2 = \left(1 - \frac{2MG}{r} + \frac{Q^2G}{r^2}\right) dt^2 - \frac{dr^2}{1 - \frac{2MG}{r} + \frac{Q^2G}{r^2}} - r^2(d\theta^2 + \sin^2\theta d\varphi), \quad (1)$$

where M is the mass of the RN black hole and Q is the charge of the RN black hole. The RN space-time is static and spherically symmetric, and it has four Killing vectors. In particular, $X_1 = \partial_t$ and $X_2 = \partial_\varphi = x\partial_y - y\partial_x$ are the Killing vectors that will be of importance in the paper.

To define NC deforming of space-time we choose a Killing twist

$$\mathcal{F} = e^{-\frac{i}{2}\theta^{AB}X_A \otimes X_B}, \quad (2)$$

where θ^{AB} is a constant skewsymmetric matrix $\theta^{AB} = \begin{pmatrix} 0 & a \\ -a & 0 \end{pmatrix}$, with a small NC parameter a . Indices $A, B = 1, 2$, while $X_1 = \partial_t$, $X_2 = x\partial_y - y\partial_x$ are commuting vector fields, $[X_1, X_2] = 0$. We call (3) "angular twist" because the vector field $X_2 = x\partial_y - y\partial_x$ is nothing else but a generator of rotations around z -axis, that is $X_2 = \partial_\varphi$. In particular, the twist (5) does not act on the RN metric and it does not act on the functions

of the RN metric because X_1 and X_2 are Killing vectors for the RN metric. In this way, we ensure that the geometry remains undeformed. The $*$ -product of functions follows from (2) and is given by

$$\begin{aligned} f * g &= \mu_* \mathcal{F}^{-1} \{f \otimes g\} = \\ &= fg + \frac{ia}{2} (\partial_t f (x \partial_y g - y \partial_x g) - \partial_t g (x \partial_y f - y \partial_x f)) + \mathcal{O}(a^2). \end{aligned} \quad (3)$$

This $*$ -product is noncommutative, associative, and in the limit $a \rightarrow 0$, it reduces to the usual point-wise multiplication. In this way, we obtain the noncommutative algebra of functions, i.e. the noncommutative space-time. In the special case of coordinates, the $*$ -commutation relations are

$$[t^*, x] = -ia y, \quad [t^*, y] = iax, \quad (4)$$

while all other coordinates commute.

We have already mentioned that the vector field X_2 is nothing else but the generator of rotations around the z-axis. Let us rewrite the twist (3) in the spherical coordinate system

$$\mathcal{F} = e^{-\frac{i}{2}\theta^{\alpha\beta} \partial_\alpha \otimes \partial_\beta} = e^{-\frac{ia}{2}(\partial_t \otimes \partial_\varphi - \partial_\varphi \otimes \partial_t)}, \quad (5)$$

with $\alpha, \beta = t, \varphi$. Note that the twist has the same (5) form in the cylindrical coordinate system.

Now we rewrite all the formulas for the $*$ -product of functions and the differential calculus in the spherical coordinate system. Here we present the most important results, as the rest can be calculated easily:

$$f * g = fg + \frac{ia}{2} (\partial_t f (\partial_\varphi g) - \partial_t g (\partial_\varphi f)) + \mathcal{O}(a^2), \quad (6)$$

$$dx^\mu * f = f * dx^\mu = f dx^\mu. \quad (7)$$

Note that now $x^\mu = (t, r, \theta, \varphi)$.

In the case of the Hodge dual, we have to use the definition of the Hodge dual in curved space-time. This leads to

$$\begin{aligned} *_H F &= *_H \left(\frac{1}{2} F_{\mu\nu} * dx^\mu \wedge_* dx^\nu \right) = \frac{1}{2} F_{\mu\nu} * \left(\frac{1}{\sqrt{-g}} \varepsilon^{\mu\nu\rho\sigma} g_{\rho\alpha} g_{\sigma\beta} dx^\alpha \wedge_* dx^\beta \right) \\ &= \frac{1}{2\sqrt{-g}} \varepsilon^{\mu\nu\rho\sigma} g_{\rho\alpha} g_{\sigma\beta} (F_{\mu\nu} * dx^\alpha \wedge_* dx^\beta) \\ &= \frac{1}{2\sqrt{-g}} \varepsilon^{\mu\nu\rho\sigma} g_{\rho\alpha} g_{\sigma\beta} F_{\mu\nu} dx^\alpha \wedge dx^\beta, \end{aligned} \quad (8)$$

that is, we obtained the commutative (undeformed) Hodge dual. In this calculation, we used (7). In addition, we used the fact that the metric tensor $g_{\mu\nu}$ does not depend on t, φ coordinates and therefore $g_{\mu\nu} * f = g_{\mu\nu} \cdot f$ for an arbitrary function f . In a more general case, when the twist F is not a Killing twist for the space-time metric $g_{\mu\nu}$, we cannot use this definition of the Hodge dual, since in general $g_{\mu\nu} * f \neq f * g_{\mu\nu}$ and $g_{\rho\alpha} * g_{\sigma\beta} \neq g_{\sigma\beta} * g_{\rho\alpha}$. These will spoil the covariance of the $*_H F$ under the NC gauge transformations and make the

construction of NC gauge invariant action complicated (see Aschieri and Castellani 2013 for a detailed discussion).

In what follows, we will work with the twist (5) and we will develop the NC scalar $U(1)_*$ gauge theory on the RN background.

3. SCALAR $U(1)_*$ GAUGE THEORY

The twist (5) allows us to study the behavior of an NC scalar field in the gravitational field of a Reissner-Nordström black hole.

Let us start from a more general action, describing the NC $U(1)_*$ gauge theory of a complex charged scalar field on an arbitrary background. The only requirement on the background is that ∂_t and ∂_φ are Killing vectors.

If a one-form gauge field $\hat{A} = \hat{A}_\mu * dx^\mu$ is introduced into the model through a minimal coupling, the relevant action becomes

$$S[\hat{\phi}, \hat{A}] = \int (d\hat{\phi} - i\hat{A} * \hat{\phi})^+ \wedge_{**H} (d\hat{\phi} - i\hat{A} * \hat{\phi}) - \int \frac{\mu^2}{4!} \hat{\phi}^+ * \hat{\phi} \varepsilon_{abcd} e^a \wedge * e^b \wedge * e^c \wedge * e^d. \quad (9)$$

Here μ is the mass of the scalar field $\hat{\phi}$, while its charge is q .

In order to write the mass term for the scalar field $\hat{\phi}$ geometrically, we introduced vierbein one-forms $e^a = e_\mu^a * dx^\mu$ and $g_{\mu\nu} = \eta_{ab} e_\mu^a * e_\nu^b$. In index notation, the action is of the form

$$S_\phi = \int d^4x \sqrt{-g} * (g^{\mu\nu} * D_\mu \hat{\phi}^+ * D_\nu \hat{\phi} - \mu^2 \hat{\phi}^+ * \hat{\phi}). \quad (10)$$

The scalar field $\hat{\phi}$ is a complex charged scalar field transforming in the fundamental representation of NC $U(1)_*$. Its covariant derivative is defined as

$$D_\mu \hat{\phi} = \partial_\mu \hat{\phi} - i\hat{A}_\mu * \hat{\phi}.$$

The background gravitational field $g_{\mu\nu}$ is not specified at this time. However, it is important that its Killing vectors are ∂_t and ∂_φ since only in that case the action (10) has this simple form. Note that $*$ -products in $\sqrt{-g} * g^{\alpha\beta} * g^{\mu\nu}$ can all be removed since the twist (5) does not act on the metric tensor.

One can check that the action (10) is invariant under the infinitesimal $U(1)_*$ gauge transformations defined in the following way:

$$\begin{aligned} \delta_* \hat{\phi} &= i\hat{\Lambda} * \hat{\phi}, \\ \delta_* \hat{A}_\mu &= \partial_\mu \hat{\Lambda} + i[\hat{\Lambda}, \hat{A}_\mu], \\ \delta_* g_{\mu\nu} &= 0, \end{aligned} \quad (11)$$

with the NC gauge parameter $\hat{\Lambda}$.

3.1. Seiberg-Witten map

There are different approaches to the construction of NC gauge theories. In this paper, we use the enveloping algebra approach (Jurčo et al., 2001) and the Seiberg-Witten (SW) map (Seiberg and Witten, 1999). The SW map allows us to express NC variables as functions of the corresponding commutative variables. In this way, the problem of charge quantization in $U(1)_*$ gauge theory does not exist. In the case of NC Yang-Mills gauge theories, the SW map guarantees that the number of degrees of freedom in the NC theory is the same as in the corresponding commutative theory. In other words, no new degrees of freedom are introduced.

Using the SW-map, NC fields can be expressed as functions of the corresponding commutative fields and can be expanded in orders of the deformation parameter a . Expansions for an arbitrary Abelian twist deformation are known to all orders (Aschieri and Castellani, 2012). Applying these results to the twist (5), expansions of fields up to first order in the deformation parameter a follow. They are given by:

$$\hat{\phi} = \phi - \frac{1}{4}\theta^{\rho\sigma}A_\rho(\partial_\sigma\phi + D_\sigma\phi), \quad (12)$$

$$\hat{A}_\mu = A_\mu - \frac{1}{2}\theta^{\rho\sigma}A_\rho(\partial_\sigma A_\mu + F_{\sigma\mu}). \quad (13)$$

The $U(1)$ covariant derivative of ϕ is defined as $D_\mu\phi = (\partial_\mu - iA_\mu)\phi$ in the case of $U(1)$ gauge theory. It is important to note that the coupling constant q between fields ϕ and A_μ , the charge of ϕ , is included into A_μ , namely $A_\mu = qA_\mu$.

3.2. Expanded actions and equations of motion

Using the SW-map solutions and expanding the $*$ -products in (10) we find the action up to first order in the deformation parameter a . It is given by

$$S = \int d^4x \sqrt{-g} (g^{\mu\nu} D_\mu \phi^+ D_\nu \phi - \mu^2 \phi^+ \phi + \frac{\mu^2}{2} \theta^{\alpha\beta} F_{\alpha\beta} \phi^+ \phi + \frac{\theta^{\alpha\beta}}{2} g^{\mu\nu} (-\frac{1}{2} D_\mu \phi^+ F_{\alpha\beta} D_\nu \phi + (D_\mu \phi^+) F_{\alpha\nu} D_\beta \phi + (D_\beta \phi^+) F_{\alpha\mu} D_\nu \phi)). \quad (14)$$

Varying the action with respect to ϕ^+ , we obtain the equation of motion for the field ϕ

$$g^{\mu\nu} ((\partial_\mu - iA_\mu) D_\nu \phi - \Gamma_{\mu\nu}^\lambda D_\lambda \phi) - \mu^2 \phi + \frac{\mu^2}{2} \theta^{\alpha\beta} F_{\alpha\beta} \phi - \frac{1}{4} \theta^{\alpha\beta} g^{\mu\nu} ((\partial_\mu - iA_\mu) (F_{\alpha\beta} D_\nu \phi) - \Gamma_{\mu\nu}^\lambda F_{\alpha\beta} D_\lambda \phi - 2(\partial_\mu - iA_\mu) (F_{\alpha\nu} D_\beta \phi) + 2\Gamma_{\mu\nu}^\lambda F_{\alpha\lambda} D_\beta \phi - 2(\partial_\beta - iA_\beta) (F_{\alpha\mu} D_\nu \phi)) = 0. \quad (15)$$

3.3. Scalar field in the Reissner-Nordström background

Finally, let us specify the gravitational background to be that of a charged non-rotating black hole, the Reissner-Nordström (RN) black hole. Since we are interested in the QNMs of the scalar field, we will consider the equation (15) and assume that the gravitational field $g_{\mu\nu}$ and the $U(1)$ gauge field A_μ are fixed to be the gravitational field and the electromagnetic field of the RN black hole.

Let us introduce $f = 1 - \frac{2MG}{r} + \frac{Q^2G}{r^2}$, where M is the mass of the RN black hole, while Q is the charge of the RN black hole.

The RN black hole is non-rotating, therefore the only non-zero component of the gauge field is the scalar potential

$$A_0 = -\frac{qQ}{r}. \quad (16)$$

The corresponding electric field is given by

$$F_{r0} = \frac{qQ}{r^2}. \quad (17)$$

Remembering that the only non-zero components of the NC deformation parameter $\theta^{\alpha\beta}$ are $\theta^{t\varphi} = -\theta^{\varphi t} = a$, we obtain the following equation

$$\begin{aligned} & \left(\frac{1}{f}\partial_t^2 - \Delta + (1-f)\partial_r^2 + \frac{2MG}{r^2}\partial_r + 2iqQ\frac{1}{rf}\partial_t - \frac{q^2Q^2}{r^2f} - \mu^2\right)\phi \\ & + \frac{aqQ}{r^3}\left(\left(\frac{MG}{r} - \frac{GQ^2}{r^2}\right)\partial_\varphi + rf\partial_r\partial_\varphi\right)\phi = 0. \end{aligned} \quad (18)$$

Note that Δ is the usual Laplace operator.

In order to solve this equation, we assume an ansatz

$$\phi_{lm}(t, r, \theta, \varphi) = R_{lm}(r)e^{-i\omega t}Y_l^m(\theta, \varphi), \quad (19)$$

with the spherical harmonics $Y_l^m(\theta, \varphi)$. Inserting (19) into (18) leads to an equation for the radial function $R_{lm}(r)$

$$\begin{aligned} & fR_{lm}'' + \frac{2}{r}\left(1 - \frac{MG}{r}\right)R_{lm}' - \left(\frac{l(l+1)}{r^2} - \frac{1}{f}\left(\omega - \frac{qQ}{r}\right)^2 + \mu^2\right)R_{lm} \\ & - ima\frac{qQ}{r^3}\left(\left(\frac{MG}{r} - \frac{GQ^2}{r^2}\right)R_{lm} + rfR_{lm}'\right) = 0. \end{aligned} \quad (20)$$

The zeroth order of this equation corresponds to the equation for the radial function R_{lm} in Hod (2010) and Richartz and Giugno (2014).

4. SOLUTIONS FOR QNMS

We are interested in a special solution to the equation (20), the quasi-normal mode solution with proper boundary conditions mentioned in Introduction. In order to find the spectrum of QNMs in our model, let us firstly rewrite the equation (21) in a more convenient way.

Taking into account that the outer and inner horizon of the RN black hole are given by $r_{\pm} = GM \pm \sqrt{G^2M^2 - GQ^2}$, we introduce the following variables and abbreviations

$$x = \frac{r-r_+}{r_+}, \quad \tau = \frac{r_+-r_-}{r_+}, \quad k = 2\omega r_+ - qQ, \quad (21)$$

$$\kappa = \frac{\omega k - \mu^2 r_+}{\sqrt{\omega^2 - \mu^2}}, \quad \Omega = \frac{\omega r_+ - qQ}{\sqrt{G^2M^2 - GQ^2}} r_+ = 2\frac{\omega r_+ - qQ}{\tau}. \quad (22)$$

Then the radial equation of motion (21) reduces to

$$\begin{aligned}
 x(x + \tau) \frac{d^2 R}{dx^2} + (2x + \tau - ima \frac{qQ}{r_+} \frac{x(x + \tau)}{(x + 1)^2}) \frac{dR}{dx} - [l(l + 1) + \mu^2 r_+^2 (x + 1)^2 \\
 - \frac{(\omega r_+^2 x^2 + kr_+ x + r_+ \frac{\Omega \tau}{2})^2}{r_+^2 x(x + \tau)} - ima \frac{GMqQ}{r_+^2 (x + 1)^2} + ima \frac{GqQ^3}{r_+^3 (x + 1)^3}] R = 0.
 \end{aligned} \quad (23)$$

The strategy that we adopt here is similar to that in Hod (2010). It allows us to analyze the equation (23) in two different regions, one being relatively far from the horizon, $x \gg \tau$, and the other being relatively close to the horizon, $x \ll 1$. It is important to emphasize that these two regions have to be chosen in such a way as to ensure that their region of a common overlap exists (or is likely close to exist). The restriction to a near extremal limit that we make, as well as an appropriate choice for a range of the system parameters makes this possible.

The described procedure leads to a condition for the QNM frequency ω :

$$\begin{aligned}
 & \frac{\Gamma(1 - 2i\sigma)\Gamma(-2i\sigma - 2\tilde{\rho})}{\Gamma(\frac{1}{2} - i\sigma - ik - \tilde{\rho})\Gamma(\frac{1}{2} - i\sigma + ik - i\Omega - \tilde{\rho})\Gamma(\frac{1}{2} - i\sigma - ik)} \\
 & = - \frac{\Gamma(1 + 2i\sigma)\Gamma(2i\sigma + 2\tilde{\rho})\tau^{-2\tilde{\rho}}}{\Gamma(\frac{1}{2} + i\sigma - ik + \tilde{\rho})\Gamma(\frac{1}{2} + i\sigma + ik - i\Omega + \tilde{\rho})\Gamma(\frac{1}{2} + i\sigma - ik)} \times (-2i\sqrt{\omega^2 - \mu^2 r_+ \tau})^{-2i\sigma}.
 \end{aligned} \quad (24)$$

In general, this condition cannot be solved analytically. In what follows, we present some numerical results for the QNMs frequencies, obtained by Wolfram Mathematica and resulting from the analytic condition (24). In particular, we shall be concerned with the fundamental quasi-normal mode (the lowest absolute value of the imaginary part). Using some more additional approximations, it is possible to find an analytic solution (Dimitrijević Ćirić et al. 2018)

We give separately the dependence of the fundamental QNMs frequency ω (i.e. its real and imaginary part) on the charge of the scalar field q . We plot the results for $Q/M = 0.999999$, corresponding to $\tau = 0.0028244321$. For simplicity, we set $G = 1$. When deriving (61) we used the approximation that the NC deformation parameter a is of the same order as the extremality τ . Therefore, $a = 0.01$ corresponds to the case $Q/M = 0.999999$. We assumed that the mass of the scalar field is fixed at $\mu = 0.05$.

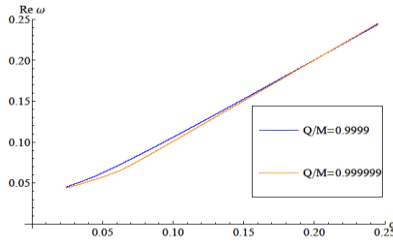


Fig. 1 Dependence of $\text{Re } \omega$ on the charge q of the scalar field with the mass $\mu = 0.05$, $l = 1$.

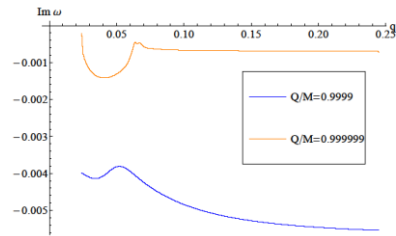


Fig. 2 Dependence of $\text{Im } \omega$ on the charge q of the scalar field with the mass $\mu = 0.05$, $l = 1$.

It needs to be said that the calculations leading to the results depicted in Figures 1-2 were carried out¹ for $l = 1$, as well as for the three values of m , namely $m = 0, \pm 1$. However, as readily seen from the figures, the curves corresponding to three different values of m cannot be distinguished, actually. Nevertheless, this does not mean that these three curves coincide identically. On the contrary, they are not identical, as can be easily verified by simply improving the resolution and by letting the graphs show a higher level of details. For that purpose, we also plot the differences of frequencies $\omega^\pm = \omega_{m=\pm 1} - \omega_{m=0}$, which indeed appear to be nonvanishing. In this regard, we notice that the deformation a and the azimuthal quantum number m always come in pair, implying that the mode with $m = 0$ actually corresponds to the limit $a \rightarrow 0$, that is, the absence of deformation. It is therefore clear that the differences ω^\pm encode the effect of a space-time deformation.

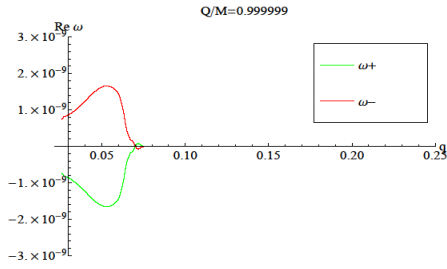


Fig 3 Dependence of frequency splitting of $\text{Re } \omega$ on the charge q of the scalar field with the mass $\mu = 0.05$, $l = 1$.

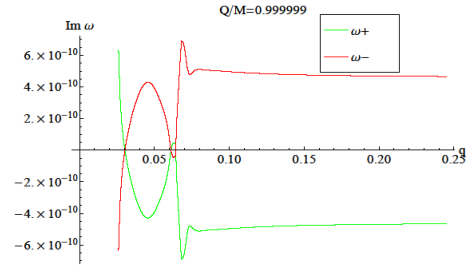


Fig 4 Dependence of frequency splitting of $\text{Im } \omega$ on the charge q of the scalar field with the mass $\mu = 0.05$, $l = 1$.

The NC effect can be described as a Zeeman-like splitting in the spectrum, manifested by the coupling between the deformation parameter a and the azimuthal (magnetic) quantum number m . In Figures 3 and 4, we plot the dependence of frequency splitting of the frequency ω showed in Fig. 1 and Fig. 2 with $\omega^\pm = \omega_{m=\pm 1} - \omega_{m=0}$, of the real/imaginary part of ω . The green line represents $\omega^+ = \omega_{m=1} - \omega_{m=0}$, while the red line represents $\omega^- = \omega_{m=-1} - \omega_{m=0}$.

The frequency splitting is small, as expected. To have an idea of how small it is, one can estimate $\delta\omega/\omega$ for the imaginary part of ω in the case of q dependence from Figures 3 and 4 and obtain $\delta\omega/\omega \approx 10^{-6}$. However, the effect is very important qualitatively, since it predicts a Zeeman-like splitting of the QNMs spectrum in the presence of noncommutativity.

The frequency splitting manifests itself as a coupling between the deformation parameter a and the azimuthal (magnetic) quantum number m . At first glance, a similar behaviour can be found in the QNMs spectrum of the Kerr black hole evaluated in the limit of slow rotation (Konoplya and Zhidenko, 2014), where the magnetic quantum number m couples with the black hole angular momentum J . The described feature would suggest the existence of a specific kind of duality between noncommutative and non-rotating systems on one side, and standard commutative and rotating systems on the other

¹ Note that $l = 0$ corresponds to a trivial situation where NC effects disappear.

side. This duality has already been observed in some lower dimensional systems (Gupta et al. 2015) However, a closer inspection shows that these two spectra are not equivalent or dual to each other, since in the case of a rotating black hole in linear approximation, in addition to the term proportional to mJ , there is another contribution, proportional to J alone, which is nonzero for $m = 0$. This is different from the dependence on the noncommutative scale a encountered in our analysis. This shows that a true relationship between these two settings (commutative and noncommutative) still needs to be found and we plan to address this problem in future work.

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SKALARNE KVAZINORMALNE MODE PRIBLIŽNO EKSTREMALNE RAJSNER-NORDSTROMOVE CRNE RUPE U NK PROSTORVREMENU

Put kojim tragamo za signalom da je prostorvreme nekomutativno je da proučavamo spektar kvazinormalnih moda Rajsner-Nordstromove (RN) crne rupe u prisustvu nekomutativnosti. U ovom radu biramo specifičnu NK deformaciju definisanu sa angularnim tvistom. Proučavamo nekomutativnu deformaciju kompleksnog skalarnog polja koje je minimalno spregnuto sa komutativnom približno ekstremalnoj RN pozadini. Teorija je manifestno invarijantna na $U(1)_$ gradijentne transformacije. Koristeći jednačine kretanja za NK kompleksno skalarno polje u pozadini RN crne rupe, spektar kvazinormalnih moda je izračunat za određen opseg parametara koji odgovaraju približno ekstremalnoj limitu.*

Ključne reči: kvazinormalne mode, približno ekstremalna RN crna rupa, NK skalarno polje