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ON THE MOTION OF CHARGED PARTICLES IN NON-STANDARD TOKAMAK MAGNETIC FIELD

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Abstract. *In this paper we analyze the motion of charged particles in a non-standard magnetic field that is of a Clebsch type representation where a magnetic perturbation is considered. The safety factor of ASDEX-Upgrade is used. Some aspects of the solutions of the guiding center system of equations are featured.*

Key words: *magnetic field, plasma physics, tokamak*

1. INTRODUCTION

The analysis of the particle motion in a deterministic or perturbed toroidal magnetic field is of great importance in tokamak plasma physics.

In the drift approximation the equations of motion for the guiding center are written as:

$$\dot{\mathbf{Y}} = U\mathbf{b} + \frac{\varepsilon}{\Omega}\left[\frac{W^2}{2B}\mathbf{b}\times\nabla B + U^2\mathbf{b}\times(\mathbf{b}\cdot\nabla)\mathbf{b} + \frac{e}{m}(\mathbf{E}\times\mathbf{b})\right] \tag{1}$$

In our paper, we integrate the equations through a code to make relatively long-time simulations. We will use the classical approach based on the Runge-Kutta method. The expression (1) was already obtained in Balescu (1988) through the highest significant order, i.e. the order ε for $\dot{\mathbf{Y}}, \dot{U}, \dot{W}$ and ε^0 for $\dot{\Phi}$. The following notations are considered: $\mathbf{b} = \frac{\mathbf{B}}{B}$ is the unit vector parallel to the magnetic field, $U = \sigma\sqrt{2/m}\sqrt{\mathcal{E} - e\Phi - MB}$ is the parallel velocity with $\sigma = \pm 1$, $\mathcal{E}$ is the total energy, Φ is the electrostatic potential and M is the magnetic moment. $W = \sqrt{2MB/m}$ is the perpendicular velocity and m is the mass

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of the particle (ion or electron), $\mathbf{E}$ is the electric field, and $\Omega = eB/mc$ is the Larmor frequency.

We use in our paper the standard model expression for the toroidal magnetic field that in the Clebsch-type representation satisfies the divergence-free condition ($\nabla \cdot \mathbf{B} = 0$) exactly and also satisfies the condition of zero radial component for the electric current density ($j_r = 0$) (Balescu, 1988; Vanden Eijnden, 1998). The aforementioned expression is:

$$\mathbf{B} = \left(1 + \frac{r}{R}\cos\theta\right)^{-1} \frac{B_0 r}{q(r)R}\mathbf{e}_\theta + B_0\mathbf{e}_\zeta \tag{2}$$

or in a compact form

$$\mathbf{B} = \frac{B_0}{h}\left[g\mathbf{e}_\theta + \mathbf{e}_\zeta\right] \tag{3}$$

where

$$g \equiv \frac{r}{q(r)R} \quad and \quad h \equiv 1 + \frac{r}{R}\cos\theta \tag{4}$$

This definition for the toroidal magnetic field in the standard model of the Clebsch-type was introduced for the first time in Vanden Eijnden (1998). In Eq.(2), r is the radial coordinate, $q(r)$ is the safety factor (the inverse of the winding function), R is the major radius and θ is the poloidal angle. Here we use the safety factor corresponding to the ASDEX-Upgrade tokamak:

$$q(r) = 0.8 + r^2$$

The case for a perturbation of the safety factor will be considered in a future paper.

2. THE SYSTEM OF EQUATIONS FOR A PERTURBED MAGNETIC FIELD

In order to study the transport of fast fusion particles, we investigated analytically the relation between chaotic sheared magnetic field lines and the characteristics of the fast particle trajectories. We also analyzed the effect of the magnetic field on the assumption of the constant magnetic moment. The degree of the stochasticity and the magnetic shear are important for the assumption of the adiabaticity of the magnetic moment. The existence of the magnetic ripple breaks the invariants of the motion; this effect is important when considering fast alpha particles.

The main goal of our paper is to solve the set of equations of motion for the guiding center in a perturbed toroidal standard model magnetic.

We used the perturbed standard model toroidal magnetic of the form:

$$\mathbf{B} = \frac{B_0}{h}\left[g\mathbf{e}_\theta + \mathbf{e}_\zeta + h\delta\mathbf{b}\right] \tag{5}$$

where the perturbation of the magnetic field is $\delta\mathbf{b} = \delta\mathbf{b}(r, \theta, \zeta)$, with $|\delta\mathbf{b}| \ll 1$ and:

$$\delta\mathbf{b} = \delta b_r\mathbf{e}_r + \delta b_\theta\mathbf{e}_\theta + \delta b_\zeta\mathbf{e}_\zeta \tag{6}$$

In this paper we use the following approximations:

$$\eta \equiv \frac{r}{R} \ll 1; \quad h\delta b_i \simeq \delta b_i; \;\; gh \simeq \eta; \;\; g^2 \simeq 0; \tag{7}$$

$$h^2|\delta \mathbf{b}|^2 \simeq 0; \;\; gh\delta b_i \simeq \eta\delta b_i \simeq 0; \;\; h\frac{\partial \delta b_i}{\partial j} \simeq \frac{\partial \delta b_i}{\partial j}; \;\; (i,j = r,\theta,\zeta)$$

The approximate expressions for the amplitude of **B** and for the perturbed unit vector along the magnetic field are respectively:

$$B \simeq B_0\left(1 - \frac{r}{R}\cos\theta + \delta b_\zeta\right) \tag{8}$$

and

$$\mathbf{b} \simeq (g + \delta b_\theta)\mathbf{e}_\theta + \delta b_r \mathbf{e}_r + \mathbf{e}_\zeta \tag{9}$$

After relatively long but not difficult calculations, we obtained the following equations:

$$\dot{r} = U_0\delta b_r - \frac{U_0(U_0+U_1)}{\Omega_0 R}\sin\theta - \frac{2U_0U_1}{\Omega_0 R}\sin\theta\delta b_\zeta + \frac{U_0U_1}{\Omega_0 r}\frac{\partial \delta b_\zeta}{\partial \theta} \tag{10}$$

$$r\dot{\theta} = g\left(U_0 + U_1\frac{r}{R}\cos\theta\right) - \frac{U_0(U_0+U_1)}{\Omega_0 R} + U_0\delta b_\theta + \frac{U_0U_1}{\Omega_0}\frac{\partial \delta b_\zeta}{\partial r} - \tag{11}$$

$$-\frac{U_0U_1}{\Omega_0 R}\cos\theta\delta b_\zeta + \frac{U_0^2}{\Omega_0}[-\frac{1}{qR}\delta b_\theta + \frac{1}{qR}\frac{\partial \delta b_r}{\partial \theta} - \frac{1}{qR}\frac{\partial \delta b_\theta}{\partial r} + \frac{1}{R}\frac{\partial \delta b_r}{\partial \zeta} +$$

$$+\frac{2U_1}{U_0 R}\cos\theta\delta b_\zeta + \frac{1}{R}\cos\theta\delta b_\zeta]$$

$$R\dot{\zeta} = U_0\left(1 - \frac{r}{R}\cos\theta\right) + U_1\frac{r}{R}\cos\theta - U_1\delta b_\zeta + \tag{12}$$

$$+\frac{U_0}{\Omega_0 R}[U_1\sin\theta + U_0\cos\theta]\delta b_r + \frac{U_0}{\Omega_0 R}[U_0\cos\theta - U_1\sin\theta]\delta b_\theta$$

In the Eqs. (10, 11, 12) the following definitions were used:

$$U_0 = \sigma\sqrt{\frac{2}{m}}\sqrt{\mathcal{E} - MB_0} \tag{13}$$

and

$$U_1 = \frac{MB_0}{mU_0} \tag{14}$$

3. Comments and Results

1. If we compare, for example, the terms $U_0\delta b_\theta$ and $\frac{U_0^2}{\Omega_0}[-\frac{1}{qR}\delta b_\theta]$ from Eq.(11), we observe that the latter is not negligible anytime compared with the former, so the following relation can hold:

$$|U_0\delta b_\theta| \approx \left|\frac{U_0^2}{\Omega_0}[-\frac{1}{qR}\delta b_\theta]\right| \tag{15}$$

If we consider: $q \simeq 2$, $R \simeq 2m$, $\Omega_{0i} \simeq 10^8 s^{-1}$ and $U_0 \simeq v_{\parallel i} \in 10^2, 10^{10}]m \cdot s^{-1}$ for big islands and $U_0 \simeq v_{\parallel i} \in 10^2, 10^9]m \cdot s^{-1}$ for the small ones (Weyssow, 1990), the two terms satisfy Eq. (15). In this case the term $\frac{U_0^2}{\Omega_0}[-\frac{1}{qR}\delta b_\theta]$ can become important in the analysis of the equations of motion.

2. The radial magnetic field perturbation δb_r is also present in Eqs. (11, 12). If we take into account these perturbations, we can modify the equations of motion already obtained in Weyssow and Misguich (1997), even in the case when only the radial magnetic field perturbation is considered.

3. The divergence-free- condition must be satisfied for the total magnetic field. For the unperturbed magnetic field, this condition is satisfied because we used the standard model (2). In order for this condition to be satisfied for the total magnetic field, the following condition must be true: $\nabla \cdot \delta\mathbf{b}(r,\theta,\zeta) = 0$. In toroidal coordinates this condition is fulfilled, for example, by the following choice for the components of the perturbation:

$$\delta b_r = \frac{1}{r} f_r(\theta,\zeta), \quad \delta b_\theta = f_\theta(r,\zeta), \quad \delta b_\zeta = f_\zeta(r,\theta,\zeta) \tag{16}$$

The functions f_i with $i = (r,\theta,\zeta)$ are periodic in θ and ζ. The last expression in (16) can be correct only if the approximations (7) are considered.

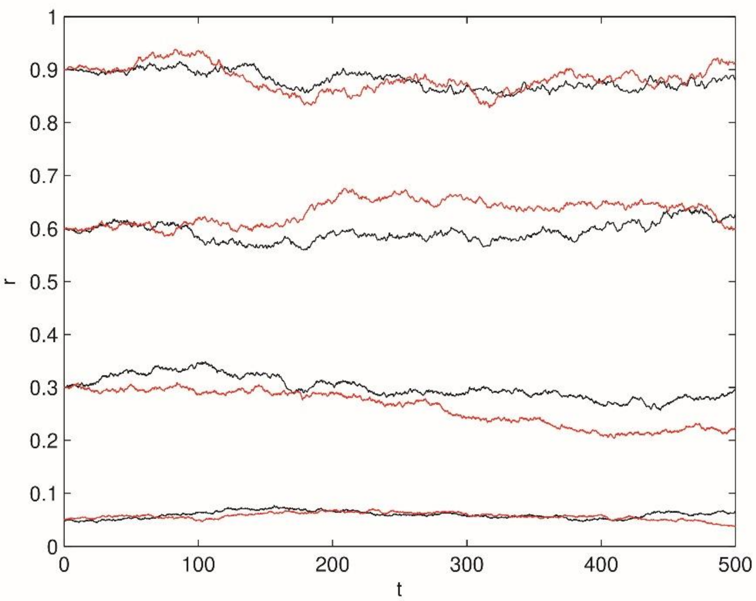

Fig. 1 The radial coordinate represented as a function of time.

Figure 1 corresponds to our system where the magnetic perturbation is considered to be of the form $\delta b_r = 0,\ \delta b_\theta = 0,\ \delta b_\zeta = \beta\cos(\alpha\theta + \gamma\zeta)$. The red orbits correspond to $\beta = 0.2,\ \alpha = 3$ and $\gamma = 2$ and the dark ones to $\beta = 0$. The orbits are deviated from the unperturbed $\beta = 0$ case when β is varying (which is anomal behaviour) but are not influenced essentially by the small or high values of α and γ. High values for α and γ generate an increased number of oscillations in the solution.

4. APPENDIX

In this appendix, we give the expressions necessary to derive the equations of motion (10-12).

1. Using the expression (9) and the definition of the electric field:

$$\mathbf{E} = E_r\mathbf{e}_r + E_\theta\mathbf{e}_\theta + E_\zeta\mathbf{e}_\zeta$$

we obtain :

$$\mathbf{E}\times\mathbf{b} = [E_\theta - E_\zeta(g+\delta b_\theta)]\mathbf{e}_r - [E_r - E_\zeta\delta b_r]\mathbf{e}_\theta + [E_r(g+\delta b_\theta) - E_\theta\delta b_r]\mathbf{e}_\zeta \quad (17)$$

2. The complete form of the expression: $\mathbf{b}\times\nabla B$

Using the expressions (8) and (9), as well as the gradient expressed in toroidal coordinates (for the standard model), we obtain:

$$\mathbf{b}\times\nabla B = -\left(\frac{B_0}{R}\sin\theta + \frac{B_0}{r}\frac{\partial\delta b_\zeta}{\partial\theta}\right)\mathbf{e}_r + \left(-\frac{B_0}{R}\cos\theta + B_0\frac{\partial\delta b_\zeta}{\partial r}\right)\mathbf{e}_\theta +$$
$$+\left(\frac{B_0}{R}\sin\theta\delta b_r + \frac{B_0}{R}\cos\theta\delta b_\theta + g\frac{B_0}{R}\cos\theta\right)\mathbf{e}_\zeta \quad (18)$$

3. The complete form of the expression: $\mathbf{b}\times(\mathbf{b}\cdot\nabla)\mathbf{b}$

Using also the expression (9), we obtain:

$$\mathbf{b}\times(\mathbf{b}\cdot\nabla)\mathbf{b} = [-\frac{\sin\theta}{R+r\cos\theta}\left(1+\frac{r^2}{q^2R^2}\right) - \frac{1}{R}\frac{\partial\delta b_\theta}{\partial\zeta} + \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r^2}{qR}\right)\delta b_r]\mathbf{e}_r +$$
$$+[\frac{r\sin\theta}{Rq(R+r\cos\theta)}\delta b_r - \frac{1}{qR}\frac{\partial}{\partial r}\left(\frac{r^2}{qR}\right) - \frac{1}{Rq}\delta b_\theta + \frac{1}{qR}\frac{\partial\delta b_r}{\partial\theta}$$
$$-\frac{1}{qR}\frac{\partial\delta b_\theta}{\partial r} - \frac{\cos\theta}{R+r\cos\theta} + \frac{1}{R}\frac{\partial\delta b_r}{\partial\zeta}]\mathbf{e}_\theta +$$
$$+[-\frac{\sin\theta}{R+r\cos\theta}\delta b_r + \frac{r}{q^2R^2}\frac{\partial}{\partial r}\left(\frac{r^2}{qR}\right) +$$
$$+\frac{r\cos\theta}{Rq(R+r\cos\theta)} + \frac{\cos\theta}{R+r\cos\theta}\delta b_\theta]\mathbf{e}_\zeta \quad (19)$$

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O KRETANJU NAELEKTRISANIH ČESTICA U NESTANDARDNOM TOKAMAK MAGNETNOM POLJU

U ovom radu analizirali smo kretanje naelektrisanih čestica u nestandardnom magnetnom polju reprezentacije Klebšovog tipa, gde je razmatrana magnetna perturbacija. Korišćen je faktor sigurnosti ASDEKS-Upgrade. Neki aspekti rešenja sistema jednačina vodećeg centra su istaknuti.

Ključne reči: *magnetno polje, fizika plazme, tokamak*