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THE CONNECTION BETWEEN MATHEMATICS AND PHYSICS FROM THE ASPECT OF REASONING BASED ON PROPORTIONS AND ERRORS IN THE CONCLUSION †

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Branka Radulović[1], Marina Dorocki[2]

[1]University of Novi Sad, Faculty of Sciences, Department of Physics, Novi Sad, Republic of Serbia
[2]High school "Isidora Sekulić", Novi Sad, Republic of Serbia

Abstract. *Numerous researchers emphasized the connection between mathematics and physics, as well as the pupil's lack of linking knowledge gained within these subjects. In this paper, examples of the connection between mathematics and physics will be examined by the scientific reasoning test and the initial test created for this research. In the research related to the initial knowledge and scientific research, 235 students of the first and second grade of gymnasiums in Novi Sad and Bačka Palanka participated. The results of the research have shown that most of the students did not correctly answer questions related to reasoning based on proportions. The obtained result points to the pupil's misunderstanding of the notion of the smallest interval of an instrument, that is, the value of an instrumental error, which is the basis of the correct reading of the experimental results in the teaching of physics. From the point of view of physics, it is important for pupils to point out the mistakes they have made in concluding, in order to correct their claims and the exact information associated with existing ones in long-term memory.*

Key words: *connection between physics and mathematics, proportionality based reasoning, errors in conclusions, students*

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Corresponding author: Branka Radulović
University of Novi Sad, Faculty of Science, Department of Physics, Trg Dositeja Obradovića 4, 21000 Novi Sad
E-mail: branka.radulovic@df.uns.ac.rs
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1. INTRODUCTION

Numerous researchers dealt with the question of linking knowledge within the teaching process. Đorđević, in his work (Đorđević, 2004) emphasizes the importance of the relation between teaching and cognitive development of students and represents it as one of the central and essential problems of education and contemporary teaching. In this way, Đorđević put questions and problems related to the zone of proximal development to the forefront, which surely deserves to be in the focus of methodical-didactic research. Antonijević more closely determines this problem by linking it to the question of the connection of knowledge and concepts within the contents of the curriculum. He points out that the knowledge that the students adopt in teaching represent knowledge that is appropriately didactically and methodically shaped, transformed and adapted to the needs of carrying out the learning process in teaching (Antonijević, 2006). However, although didactic and methodically shaped information received by pupils during the teaching process, students often do not notice the correlation between information presented in a variety of different subjects, and even often consider them as separate and non-interacting facts. As this is contrary to the accepted theories of the cognitive development of students, the aim of this paper is to examine the development of student reasoning based on proportions, as well as to look at the connection with socio-demographic variables in order to find directions or patterns that will accelerate student reasoning.

2. THEORETICAL FRAMEWORK OF RESEARCH

The basis of the connection of knowledge within the teaching process is logical and critical thinking. Critical thinking undoubtedly implies that logical rules of the valid conclusion are followed in the performance of the courts (Pešić, 2007). These rules serve to identify errors in the conclusion and have been separated correctly from the incorrect conclusion. For the development of critical thinking, Pešić, as the key competence, states - the ability to interpret and analyze information, to perform logically correct and acceptable conclusions and to review the principles and acceptability of courts, arguments, beliefs, and interpretations (Pešić, 2007). If we look at the separated abilities, we can notice their clear application in the teaching of physics within the computational and experimental assignments. At higher levels of education, the need to describe the physical phenomena or solve problems requires both the understanding of theory and its mathematical formulation (Greca, de Ataíde, 2017). Greca and Moreira in their work cite the scheme of the connection between the mathematical, physical and mental models necessary for understanding physical phenomena (Greca, Moreira, 2002). If the student understands how physics is related to mathematics in the construction of concepts, she/he will have a more appropriate image of the use of the mathematical models for physics (Greca, de Ataíde, 2017). Uhden and Pospiech underline the importance of the connection between mathematics and physics because of the significant impact on the cognitive development of students and knowledge transfer (Uhden, Pospiech, 2010). Dirac emphasizes that the physicist, in his study of natural phenomena, has two methods of making progress: (1) the method of experiment and observation, and (2) the method of mathematical reasoning (Dirac, 1940). According to Greca, de Ataíde (2017), Paty highlights three forms of mathematics to be used in the construction of physical knowledge. The first form is the analog vision, which is related to kinematics. The second form is the vision of mathematics as a language that reflects the real which is invoked with the Galileo's book of nature and the third form is mathematics as a language that is intrinsically linked to the construction of physical thinking and a theoretical physics problem. According to Greca, de Ataíde (2017), Feynman say that mathematics is not just language, she is a language plus reasoning, i.e. mathematics is a tool for reasoning. Bitner (1991) and Valanides (1996) emphasize that five formal operational reasoning modes consisting of controlling variables, proportional, probabilistic, correlational, and combinatorial reasoning and critical thinking skills have been identified as essential abilities for success in advanced secondary school science and mathematics courses. In this paper, special attention will be paid to proportionality based reasoning. The propositional reasoning is the ability to draw conclusions on the basis of sentence connectives such as and, if, or, and not (Rips, 1983). The theory of positional reasoning proposes that reasoning is a semantic process based on mental models. It assumes that people are able to maintain models of only a limited number of alternative states of affairs, and they accordingly use models representing as much information as possible in an implicit way (Johnson-Laird, Byrne, & Schaeken, 1992). According to Lamon (1993), Lesh, Post, and Behr emphasize that proportional reasoning plays such a critical role in a student's mathematical development that it has been called a watershed concept, a cornerstone of higher mathematics, and the capstone of elementary concepts. In mathematics and physics, proportionality is a mathematical relation between two quantities. Thus, it has been implicitly understood that proportional reasoning consists of being able to construct and algebraically solve proportions (Lamon, 1993). Because of the influence of Piaget's theory in which proportional reasoning was the hallmark of the formal operations stage of development (Han, 2013; Lamon, 1993). As is known, during the stage of formal-logical operations abstract thinking develops and students become able to think in abstract categories, they can make conclusions based on abstract assumptions and formulate general laws and principles (Stepanović, 2004). This is especially important for teaching physics.

There are a number of researchers that were found in the work of Lamon (1993), which put the questions at the center of all the research questions in the light of context variables, remediating areas of misconception or difficulties of students that have with proportional reasoning and to identify some of the variables that affect the problem of difficulty. Inspired by this, a research was conducted aimed at examining the development of student-based reasoning based on proportions, as well as looking at the connection with socio-demographic variables in order to find directions or patterns that would accelerate student reasoning. Likewise, additional initial research was conducted aimed at more clearly determining students' mathematical knowledge needed in physics classes, as well as the initial knowledge of students about the basic physical settings they studied during primary education.

3. METHODOLOGY

The subject of this research was the discernible learning of knowledge and skills in mathematics on concrete physical problems. Therefore, two surveys were conducted. Within the first survey, which included 137 students of the first (67 pupils) and the second grade (70 pupils) of the high school "Svetozar Marković" from Novi Sad and "20. October" from Bačka Palanka, the degree of student-based reasoning based on proportions was examined.

This survey was conducted in January-February 2017. The questions for this part of the research were taken from the Lawson Classroom Test of Scientific Reasoning (CTSR). In order to better define whether the lower result on the test of the reasoning is the result of their lack of understanding of the teaching material of the first grade of the gymnasium or of the lesser pre-knowledge of elementary education, another research involving 98 students of the first grade of the high school "Isidora Sekulić" from Novi Sad has started. This research examined the students' knowledge of mathematical functions and operations that they needed when teaching physics in gymnasiums, the students' knowledge of the basic physical settings that they studied during elementary education and the attitude of students towards teaching subjects and physics in particular. Another survey was conducted in September 2017.

3.1. Examples of tasks

Examples of tasks on the scientific reasoning test

Figure 1 (a) shows two censuses. One is narrower than the other, but the sections are marked at the same heights. In the wider measuring cylinder, the water flows down to the fourth interval, then the water flows into the narrow censor where it comes to the sixth interval. Now the two censuses are emptying and in the wider measuring cylinder the water flows to the sixth interval. If this water flows into the hot measuring cylinder, to what interval will the water be in it?

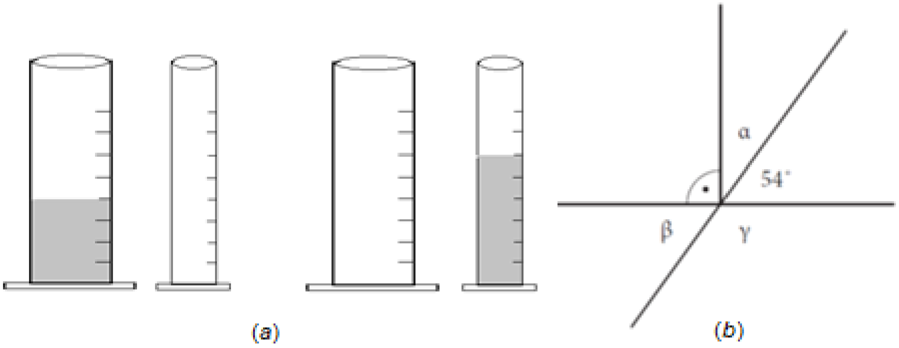

Fig. 1 Display a case question

Examples of questions at the initial test

Tied to mathematics:

1. In one school of 120 students, 40% are excellent, 20% are good, 15% good and everyone else is sufficient. How many students do you have?

2. Calculate the angles α, β and γ shown in Figure 1 (b).

Tied to physics:

1. The term for pressure is $p = F / S$, where F is the force acting on the surface of the cross-section of the body S. If the surface of the body is increased three times for how much and how will the pressure change?

2. If a body has an acceleration whose numerical value is $a = 2\ m/s^2$, does that mean that it exceeds every second by two meters?

3.2. Data analysis

By using the SPSS 20.0 software program for statistical analysis, the average score and standard deviation for knowledge tests were determined. The statistical significance of the obtained difference in student achievements was tested by descriptive analysis, ANOVA, and t-test. An eta-square indicator is used to determine the size of the effect of an independent variable on the dependent.

4. Results and Discussion

4.1. Achievement of students based on proportionality test

The single-factor analysis of variance showed that there is a statistically significant difference in pupil achievement, depending on the age of the student, F (df = 1, N = 136) = 39.156, p = 0.000, $\eta^2 = 0.225$. The value of the eta-square of the parameter shows the great influence of the age on the degree of reasoning based on proportions. The T-test showed that the median value of the achievement of the students of the second grade (M = 3.14, SD = 1.77) statistically significantly differs from the median achievement of the students of the first grade (M = 1.45, SD = 1.36), t (df = 135) = 6.257, p = 0.000.

Figure 2 shows the proportion of correct answers depending on age.

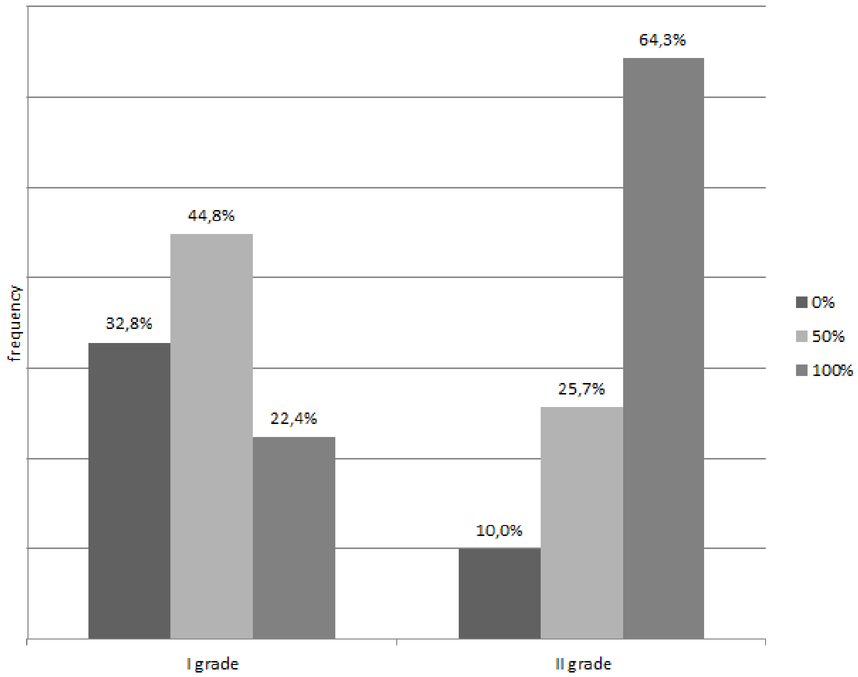

Fig. 2 Frequency of accurate responses depending on the age of the student

As can be seen from Histogram 1, significantly more students of the second grade than first-grade students correctly answered all questions related to proportionality-based reasoning. It has also been found that even a third of first-grade students did not give any correct answer. This data points to pupil misunderstanding of basic terms related to proportions, as well as a misunderstanding of the notion of the smallest interval, which is the prerequisite for correct experimentation in the teaching of physics.

If a specific example is given in the Methodology section, student responses are divided. 43.8% of students think that the level of water in the narrower censor will reach the 8th interval, while almost the same percentage (43.1%) think that the water level will reach the 9th interval. The correct answer is that the water level will reach the 9th interval. Explanation of the correct answer: If it starts from the fact that the volume of water is a constant in both cases, which could be mathematically represented as a product of the base and pendulum of the column of liquid. So the first case would get a relation $4 \cdot S_1 = 6 \cdot S_2$. From this relation, the ratio of the surfaces of the transverse sections of the cuff can be determined. For the second case, the relationship is $6 \cdot S_1 = x \cdot S_2$. If the obtained ratio of surfaces from the first relation is used, then it can be calculated that the new water level will be on the 9th interval. As stated in Han (2013), proportional reasoning can be conceptualized as the identification of two extensive variables that are applicable to a problem. This application of given data and relationships can find an additional value (problems with a missing value) or allow comparison of two values (comparison problem). 38% of students said as an explanation that if the height for the first time increased for the 2 intervals, then it will now increase at two intervals as well. The obtained result points to the wrong conclusion and the need to use experimental exercises in the teaching of physics much more with pupils in the elementary school, in order to make pupils clearer the least interval of the instrument and, consequently, the notion of instrumental error. If the naïve errors were to be broken, the students would be trained for proper handling of the apparatus, proper performance of the experiment, and correct conclusions based on the exact results obtained. According to Han (2013), McLaughlin emphasizes that in learning, proportional reasoning is recognized as fundamental reasoning construct necessary for mathematics and science achievement. It is therefore important to encourage scientific reasoning through all teaching subjects and especially through the teaching subject of physics.

The influence of the sex was not statistically significant, F (df = 1, N = 136) = 2.799, p = 0.097. Namely, the mean value of boys achievement (M = 2.63, SD = 1.75) is slightly higher than the achievement of girls (M = 2.11, SD = 1.80), but that difference is not statistically significant. Similar results were obtained in the research Valanides (1996).

4.2. Achievement of students on the initial test

The mean value of pupil achievement at initial testing is M = 10.43, SD = 2.74. As the initial test consisted of two parts, one part of the test concerned the understanding of mathematical operations, while the other part concerned questions related to the understanding of physical concepts, therefore the student achievements for individual parts of the test were calculated. The study found that the median achievement of the students for the first part of the test was $M_m = 5.77$, $SD_m = 1.20$, while the median achievement for the second part of the test was $M_f = 4. 69$, $SD_f = 2.14$. The Pearson coefficient of correlation between these two achievements showed a positive correlation of moderate strength, $r = 0.286$, $p = 0.005$. The results of the research Greca & de Ataíde (2017) also showed a positive correlation between the Understanding of physics concepts and Mathematical understanding of basic differential calculus ($r = 0.46$, $p \leq 0.05$) and the understanding of physics concepts and the epistemic view of the role of mathematics in physics ($r = 0.57$, $p \leq 0.05$).

If we observe the above examples related to the understanding of mathematical operations in the Methodology section, the answer to the first question is exactly 77%, which indicates that the students understand the percentage account. A similar percentage of students (76%) correctly answered the first question about understanding physical concepts. If we look at the principle of solving this question, it can be seen that the same method of reasoning can be applied as explained in the above example. 37% of students answered correctly the second question. This data indicates a student's misunderstanding of the physical interpretation of fractions.

The results of this part of the research showed that the students did better the math test than the physics test, which was shown by the inexplicable application of acquired knowledge to specific problems. The pupils’ poor results in the knowledge tests can be explained by their attitude towards teaching subjects. Therefore, within the initial research, pupils' attitudes about teaching subjects, as well as attitudes toward physics as a teaching subject, were examined in concrete terms.

4.3. The attitude of pupils towards teaching subjects in elementary school

The research found that about a third of students stated that their favorite subjects were foreign languages or Serbian. About a quarter of pupils sad that the most favorite subject for them is mathematics, and almost the same percentage sad that subjects in the group of natural sciences are the most favorite, with the notion that no student has listed physics as the most favorite subject. The primacy of this group has been taken over by biology and geography. On the other hand, the following results were obtained on the question that was their most overworked subject (Figure 3).

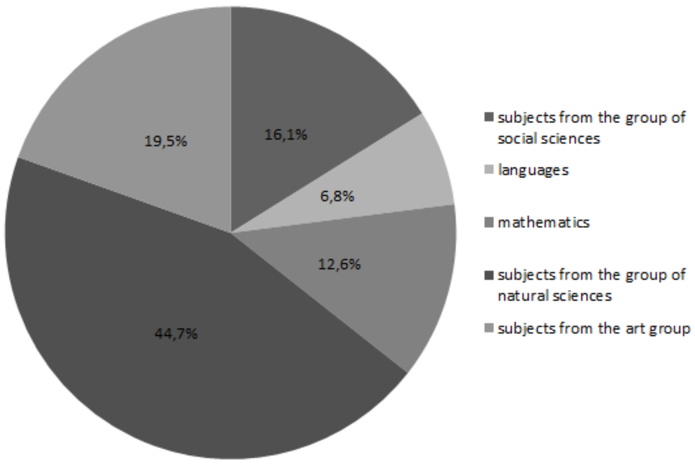

Fig. 3 The attitude of students about the most overworked subject in elementary school

As can be seen from Figure 3, almost half of the respondents listed one of the groups of natural sciences as the most overwhelming object. In particular, chemistry was singled out by about 30% of students and physics perceived by about 10% of students. The main epithets that have been singled out in the student's description of physics as a teaching element in elementary school are that they are difficult, incomprehensible and complicated. The existence of a negative attitude about an object negatively influences student engagement during the course of a specific subject matter. The involvement of students is defined as learners' motivation to learn specific content. Motivational effects affect the learners' decision to learn and do exercises, as well as the mental effort, which is invested in mastering the material (Žuapnec et al., 2018). If the student is not interested in the content or the task, he/she will not invest enough mental effort to learn the teaching material, which will result in his / her low performance (Paas et al., 2005).

5. Conclusions

Scientific literacy as an instructional goal typically includes students' understanding of the nature of science and scientific reasoning. Proportional reasoning, as one of the five formal operational reasoning modes, plays such a critical role in the student's mathematical development that it was called a watershed concept, a cornerstone of higher mathematics, and the capstone of elementary concepts. The aim of this study was to examine student based reasoning based on proportions, as well as to determine students' knowledge of mathematics and physics, and students' attitudes toward teaching subjects and physics in particular. The results of the research have shown that it is necessary to pay more attention to correlations between teaching subjects. The ability to perform logically correct and justified conclusions based on the mathematical interpretation of physical problems is particularly important in the teaching of physics. From the point of view of physics, it is important for pupils to point out the mistakes they have made in concluding, in order to correct their claims and the exact information associated with existing ones in long-term memory.

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POVEZANOST MATEMATIKE I FIZIKE SA ASPEKTA REZONOVANJA ZASNOVANOG NA PROPORCIJAMA I POGREŠKAMA U ZAKLJUČIVANJU

Brojni istraživači su u svojim istraživanjima isticali povezanost matematike i fizike, ali i učeničko nepovezivanje znanja stečenih u okviru ovih predmeta. U okviru ovog rada razmotriće se primeri povezanosti matematike i fizike zadati testom naučnog rezonovanja i inicijalnim testom kreiranim za ovo istraživanje. U istraživanju vezanom za ispitivanje inicijalnog znanja i stepena naučnog rezonovanja učestvovalo je 235 učenika prvog i drugog razreda gimnazija u Novom Sadu i Bačkoj Palanci. Rezultati istraživanja su pokazali da većina učenika nije tačno odgovorila na pitanja vezana za rezonovanje zasnovano na proporcijama. Dobijeni rezultat ukazuje na učeničko nerazumevanje pojma najmanjeg podeoka nekog instrumenta, odnosno vrednost instrumentalne greške, što predstavlja osnovu pravilnog očitavanja eksperimentalnih rezultata u nastavi fizike. Sa aspekta metodike fizike, važno je učenicima ukazati na greške koje su učinili prilikom zaključivanja, kako bi korigovali svoje tvrdnje i tačne informacije vezali za postojeće u dugotrajnoj memoriji.

Ključne reči: *povezanost fizike i matematike, rezonovanje zasnovano na proporcijama, pogreške u zaključivanju, učenici*