ATTRACTOR BEHAVIOUR OF HOLOGRAPHIC INFLATION MODEL FOR INVERSE COSINE HYPERBOLIC POTENTIAL[†]

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Abstract. *We study a model of tachyon inflation and its attractor solution in the framework of holographic cosmology. The model is based on a holographic braneworld scenario with a D3-brane located at the holographic boundary of an asymptotic ADS₅ bulk. The tachyon field that drives inflation is represented by a Dirac-Born-Infeld (DBI) action on the brane. We examine the attractor trajectory in the phase space of the tachyon field for the case of inverse cosine hyperbolic tachyon potential.*

Key words: *DBI Lagrangians, holographic inflation, attractor solution*

1. INTRODUCTION

The cosmological inflation scenario successfully solves the horizon and other related problems of the standard Big Bang cosmology. Tachyon models belong to a class of inflationary models where inflation is driven by the tachyon scalar field originating in M or string theory (Fairbairn and Tytgat, 2002; Frolov et al., 2002; Gibbons, 2003; Li and Liddle, 2014; Shiu et al., 2003; Steer and Vernizzi, 2004). The effective field theory Lagrangian is of the DBI form (Sen, 1999; 2002)

$$\mathcal{L} = -\ell^{-4}V(\theta/\ell)\sqrt{1 - g^{\mu\nu}\theta_{,\mu}\theta_{,\nu}}, \tag{1}$$

where ℓ is an appropriate length scale, V is dimensionless tachyon potential, $\theta_{,\mu} = \partial\theta / \partial x^{\mu}$ and the tachyon field θ is of the dimension of length (we use natural units, i.e. the speed of light c is equal to 1).

We study a braneworld inflation model in the framework of a holographic cosmology (Apostolopoulos et al., 2009; Bilić, 2016). This cosmological model is based on the effective four-dimensional Einstein equations on the holographic boundary in the framework of anti-de Sitter/conformal field theory (AdS/CFT) correspondence. A D3-brane is located at the holographic boundary of an asymptotic ADS$_5$ bulk (Bilić et al., 2019).

An interesting property of the model is that the universe evolution starts from a point at which the energy density and cosmological scale are both finite, bypassing the Big Bang singularity of the standard cosmology. In this way, the inflation phase proceeds immediately after the initial moment ($t = 0$) (Bilić, 2018; Bilić et al., 2017a; 2017b; 2017c).

The paper is organized as follow. Section 2 contains a short recapitulation of tachyon dynamics for the model in the slow-roll regime. Section 3 is reserved for attractor dynamics, while in Section 4 we study attractor solution for an inverse cosine hyperbolic tachyon potential. After the conclusion in Section 5, we end up with a short Appendix.

2. TACHYON DYNAMICS

We consider tachyon inflation in the framework of holographic cosmology, where the line element of the holographic braneworld corresponds to a spatially flat FRW universe:

$$ds^2 = g_{\mu\nu}dx^{\mu}dx^{\nu} = dt^2 - a^2(t)(dr^2 + r^2 d\Omega^2). \tag{2}$$

The Friedmann equations are modified for the holographic case and take the forms (Bilić et al., 2019):

$$h^2 - \frac{1}{4}h^4 = \frac{\kappa^2}{3}\ell^4 \rho, \tag{3}$$

$$\dot{h}\left(1 - \frac{1}{2}h^2\right) = -\frac{\kappa^2}{2}\ell^3(p + \rho), \tag{4}$$

where, $h \equiv \ell H$ is a dimensionless expansion rate and κ is fundamental dimensionless coupling (Bilić et al., 2017b):

$$\kappa^2 = \frac{8\pi G_{\mathrm{N}}}{\ell^2}. \tag{5}$$

Here, the scale ℓ can be identified with the AdS curvature radius.

The (covariant) Hamiltonian is given by (Bilić et al., 2017b):

$$H = \ell^{-4}V\sqrt{1 + \eta^2}, \tag{6}$$

where:

$$\eta = \ell^4 V^{-1} \sqrt{g_{\mu\nu} \pi^\mu \pi^\nu}, \quad \pi^\mu = \frac{\partial L}{\partial \theta_{,\mu}}, \tag{7}$$

and Hamilton equations (in comoving frame) are:

$$\dot{\theta} = \frac{\eta}{\sqrt{1+\eta^2}}, \tag{8}$$

$$\dot{\eta} = -\frac{3h\eta}{\ell} - \frac{V_{,\theta}}{V} \sqrt{1+\eta^2}, \tag{9}$$

where $V_{,\theta} = dV/d\theta$. The pressure and the energy density are, respectively:

$$p = \mathcal{L}, \quad \rho = \mathcal{H}, \tag{10}$$

and for the spatially homogenous tachyon field $\theta(t)$ we get:

$$p = -\ell^{-4} V \sqrt{1-\dot{\theta}^2}, \quad \rho = \frac{\ell^{-4} V}{\sqrt{1-\dot{\theta}^2}}, \tag{11}$$

while the equation of motion for the tachyon field and the modified Friedmann equation read, respectively:

$$\frac{\ddot{\theta}}{1-\dot{\theta}^2} + 3\frac{h}{l}\dot{\theta} + \frac{V_{,\theta}}{V} = 0, \tag{12}$$

$$h^2 = 2\left[1 - \sqrt{1 - \frac{\kappa^2}{3}\ell^4 \rho}\right]. \tag{13}$$

To study the attractor behaviour of the model we work in the slow-roll regime. The horizon-flow parameters ε_j (slow-roll parameters) are defined as:

$$\varepsilon_0 = \frac{h_*}{h}, \quad \varepsilon_{i+1} = \frac{d\ln|\varepsilon_i|}{dN}, \quad i \geq 0, \tag{14}$$

where h_* is the Hubble rate at some chosen time and N is the number of e-folds (Steer and Vernizzi, 2004). From these two defining expressions for the slow-roll parameters one can derive very useful expression:

$$\dot{\varepsilon}_i = h\varepsilon_i \varepsilon_{i+1}, \quad i \geq 0. \tag{15}$$

In the slow-role regime, the tachyon field changes slowly with respect to time and following conditions hold (Bilić et al., 2019):

$$\dot{\theta}^2 \ll 1, \quad |\ddot{\theta}| \ll 3\frac{h}{\ell}\dot{\theta}. \tag{16}$$

Employing these conditions to the equation of motion for the tachyon field (12) one gets approximate expression for $\dot{\theta}$ in the slow-roll regime:

$$\dot{\theta} \simeq -\frac{\ell V_{,\theta}}{3hV}.$$

(17)

As a consequence, the modified Friedmann equation (13) in the slow-roll regime takes the form:

$$h^2 \simeq 2\left[1 - \sqrt{1 - \frac{\kappa^2}{3}V}\,\right].$$

(18)

In the following, we will use these expressions to study attractor behaviour of the model.

3. ATTRACTOR BEHAVIOUR

For inflation to be predictive, one needs to ensure that inflation is independent of initial conditions. That is, one should ensure that there is an attractor solution to the dynamics, such that differences between solutions corresponding to different initial conditions rapidly vanish.

Starting from the classical background and spatially homogenous tachyon field $\theta(t)$, the equation of motion in the phase plane $(\theta, \dot{\theta})$ has the general form:

$$\frac{d\dot{\theta}}{d\theta} = g(\theta, \dot{\theta}),$$

(19)

where the function $g(\theta, \dot{\theta})$ is expressed through the Lagrangian (pressure) as (see Appendix):

$$g(\theta, \dot{\theta}) = -\frac{1}{\dot{\theta} p_{,\dot{\theta}\dot{\theta}}}\left[3\frac{h}{\ell}p_{,\dot{\theta}} + \dot{\theta}p_{,\dot{\theta}\theta} - p_{,\theta}\right].$$

(20)

The expression (19) is the defining equation for the phase space trajectory of the tachyon field $\theta(t)$. The attractor trajectory is defined as:

$$\frac{d\dot{\theta}}{d\theta} \simeq 0,$$

(21)

or, equivalently:

$$g(\theta, \dot{\theta}) \simeq 0,$$

(22)

and for the tachyon case, in the slow-roll regime, one gets:

$$-\frac{1}{\dot{\theta}}\left[3\frac{h}{l}\dot{\theta} + \frac{V_{,\theta}}{V}\right] \simeq 0.$$

(23)

We have demonstrated in (Stojanović et al., 2020) that the attractor behaviour of the system for general potential $V(\theta)$ leads to the expression for deviation of the expansion rate h in the form:

$$\delta h(\theta) = \delta h(\theta_i)e^{-3N}, \tag{24}$$

where the number of e-folds N is defined as (Bilić et al., 2019):

$$N \equiv \frac{1}{\ell}\int_{t_i}^{t_f} h\, dt \simeq -3\int_{\theta_i}^{\theta_f} \frac{h^2 V}{\ell^2 V_{,\theta}}d\theta. \tag{25}$$

The subscripts i and f are regarded to the beginning and the end of inflation, respectively. Therefore, regardless of the initial condition, the attractor behaviour implies that late-time solutions are the same up to a constant time shift, which cannot be measured.

4. INVERSE COSINE HYPERBOLIC TACHYON POTENTIAL

If the tachyon potential has the inverse cosine hyperbolic form:

$$V = \frac{1}{\cosh(\omega\,|\,\theta\,|\,/\ell)}, \tag{26}$$

where ω is dimensionless constant, then we have:

$$\frac{V_{,\theta}}{V} = -\frac{\omega}{\ell}\tanh(\omega\,|\,\theta\,|\,/\ell), \tag{27}$$

and the equation (23) reduces to:

$$3\frac{h}{\ell}\dot{\theta} - \frac{\omega}{\ell}\tanh(\omega\,|\,\theta\,|\,/\ell) \simeq 0. \tag{28}$$

The value of θ_i as a function of h_i, according to (18) and (26), is:

$$\theta_i \simeq \frac{\ell}{\omega}\operatorname{arccosh}\left[\frac{\kappa^2}{3\left(1-(1-\frac{h_i^2}{2})^2\right)}\right]. \tag{29}$$

For a given θ, the function $\dot{\theta}$ is obtained from the expression:

$$\dot{\theta} \simeq -\frac{\omega\tanh(\omega\,|\,\theta\,|\,/\ell)}{\sqrt{18(1-\sqrt{1-\frac{\kappa^2}{3}\frac{1}{\cosh(\omega\,|\,\theta\,|\,/\ell)}})}}, \tag{30}$$

which was derived from (17) and (18), with the help of (26). In addition, the connection between η_i and $\dot{\theta}_i$ is given by the equation (9):

$$\eta_i = \sqrt{\frac{\dot{\theta}_i^2}{1-\dot{\theta}_i^2}}. \tag{31}$$

To solve the system of Hamiltonian equations, the initial values θ_i and η_i can be determined from equations (29) and (31), for fixed h_i. Contrary to the exponential potential case (Stojanović et al., 2020), in this case, there does not exist analytical expression for the number of e-folds N as a function of h_i and ω. The expression $N(h_i,\omega)$ is obtained by (numerically) solving the integral:

$$N \simeq -3\int_{\theta_i}^{\theta_f} \frac{h^2 V}{\ell^2 V_{,\theta}} d\theta \simeq -\frac{3}{\omega\ell} \int_{\theta_i}^{\theta_f} \frac{h^2}{\tanh(\omega|\theta|/\ell)} d\theta, \tag{32}$$

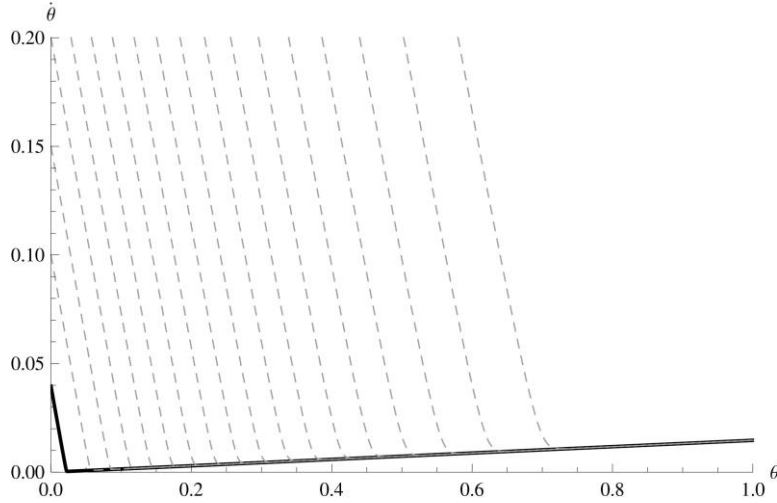where h and θ are connected through expression (18).

Fig. 1 $\dot{\theta}$ versus θ diagram, calculated numerically for several initial values of $\dot{\theta}$ and fixed parameters $\omega = 0.164$, $\kappa = 1$. The solid line represents the solution of the equation (19) obtained in respect to the initial values constrained by the equation (30). Dashed lines represent solutions of the same equation with arbitrary initial conditions.

The obtained diagram (for $\kappa = 1$), in the phase space plane $(\theta, \dot{\theta})$, is presented in Fig.1. Trajectories in the phase space for the tachyon field $\theta(t)$ rapidly approach an attractor solution, which drives inflation. The presence of the attractor trajectory makes a model largely insensitive to initial conditions since the attractor solution is unique and is approached exponentially quickly

5. CONCLUSIONS

We studied a model of tachyon inflation and its attractor solution in the framework of holographic cosmology, i.e. holographic braneworld scenario with a D3-brane located at the holographic boundary of an asymptotic ADS_5 bulk. The inflation tachyon field was represented by a DBI action on the brane. We examined the attractor trajectory in the phase space of the tachyon field for the case of inverse cosine hyperbolic tachyon potential. The numerical results confirm the existence of the attractor behaviour for the chosen potential.

APPENDIX

We start from energy density, equation (10), i.e Hamiltonian, as Legendre transformation of Lagrangian:

$$\rho = \mathcal{H} = \dot{\theta} p_{,\dot{\theta}} - p. \tag{33}$$

Taking time derivative of (33) yields:

$$\dot{\rho} = \dot{\theta}\frac{dp_{,\dot{\theta}}}{dt} - p_{,\dot{\theta}}\dot{\theta}. \tag{34}$$

Let us remind that the equation of motion for spatially homogenous tachyon field (12) is the continuity equation of the form:

$$\dot{\rho} + 3\frac{h}{\ell}(\rho + p) = 0, \tag{35}$$

which can be brought, with the help of (33), in the form:

$$\dot{\rho} = -3\frac{h}{\ell}\dot{\theta}p_{,\dot{\theta}}. \tag{36}$$

Combining equations (34) and (36) leads to (Armendariz-Picon et al., 1999):

$$\frac{d}{dt}p_{,\dot{\theta}} + 3\frac{h}{\ell}p_{,\dot{\theta}} = p_{,\theta}. \tag{37}$$

The first term is:

$$\frac{d}{dt}p_{,\dot{\theta}} = p_{,\dot{\theta}\dot{\theta}}\ddot{\theta} + p_{,\dot{\theta}\theta}\dot{\theta}, \tag{38}$$

and equation (37) can be put in the form:

$$\frac{\ddot{\theta}}{\dot{\theta}} = -\frac{1}{\dot{\theta}p_{,\dot{\theta}\dot{\theta}}}\left[3\frac{h}{\ell}p_{,\dot{\theta}} + \dot{\theta}p_{,\dot{\theta}\theta} - p_{,\theta}\right]. \tag{39}$$

Having in mind that:

$$\frac{d\dot{\theta}}{d\theta} = \frac{d\dot{\theta}}{dt}\frac{dt}{d\theta} = \frac{\ddot{\theta}}{\dot{\theta}},$$ (40)

equation (39) becomes:

$$\frac{d\dot{\theta}}{d\theta} = g(\theta,\dot{\theta}),$$ (41)

where:

$$g(\theta,\dot{\theta}) = -\frac{1}{\dot{\theta}p_{,\dot{\theta}\dot{\theta}}}\left[3\frac{h}{\ell}p_{,\dot{\theta}} + \dot{\theta}p_{,\dot{\theta}\theta} - p_{,\theta}\right],$$ (42)

which coincides with the expression (20).

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ATRAKTORSKO PONAŠANJE MODELA HOLOGRAFSKE INFLACIJE ZA INVERZNI KOSINUS HIPERBOLIČKI POTENCIJAL

Razmatramo, u okviru holografske kosmologije, model tahionske inflacije i njegovo atraktorsko rešenje. Model je zasnovan na holografskom scenariju za brane, sa D3-branom smeštenom na holografskom obodu asimptotskog ADS5 prostora. Tahionsko polje na brani, koje vodi inflaciju, predstavljeno je dejstvom DBI tipa. Ispitana je atraktorska trajektorija u faznom prostoru tahionskog polja za slučaj inverznog kosinus hiperboličkog potencijala.

Ključne reči: *Lagranžijani DBI tipa, holografska inflacija, atraktorsko rešenje*