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PROSPECTS IN STRING FIELD THEORY *

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Abstract. This is a review of the motivations for and recent progress in open string field theory

1. Introduction

The most authoritative candidate to represent the UV completion of our low energy field theory models, in particular the standard model and Einstein-Hilbert gravity, is (super)string theory. Superstring theories seems to have all the ingredients needed to describe the fundamental physical interactions. In particular it provides a consistent quantization of gravity. However how the various physical phenomena are accommodated in the string theory framework, starting from the alimentary particle physics and ending with the physics of extreme large distances and times, is still an open problem, to say the least, notwithstanding the strenuous endeavor of string theorists and undeniable, but partial, progress made in many separate applications of string theory. Although the picture is still not precise it is plausible that the latter can accommodate the physics of the standard model of elementary particles as well as a description of the evolution of the universe; it does also shed light on the black hole physics. However many questions remain unanswered, the identification of the vacuum to start with.

The panorama of superstring theory, on one side, is the same as about twenty years ago. There are five consistent superstring theories in 10D, one open-closed (type I) and four closed (IIA, IIB and two heterotic ones). In addition we have another consistent theory in 11D (M theory), whose low energy limit is 11D supergravity. There exist also other consistent non-supersymmetric theories, but the attention is mostly focused on the supersymmetric ones. The latter are connected by dualities and appear as limiting cases of a unique theory, characterized by a large moduli space, when the relevant moduli take on specific limiting values. It is this unique theory that people understand when generically referring to superstring

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theory. The ordinary way to extract low energy information is to compactify the extra dimensions or else to consider configurations of branes. In a way or another it is possible, for instance, to reproduce the spectrum and various qualitative features of the standard model and to produce effective models for the evolution of the universe, describing for instance inflation.

On the other hand in this panorama a different point of view was introduced by Maldacena with his idea of the AdS/CFT correspondence. A stack of D3-branes in type IIB superstring theory generate an AdS geometry that splits physics into two separated systems, a supersymmetric gauge theory and a supergravity theory. However since the theory is unique, the two systems must be related in a one-to-one way. This argument is the basis of the correspondence. The latter is a duality of the strong-weak coupling type, so that it can be directly verified only in the presence of supersymmetry: the original case refers to N=4 conformal gauge theory in 4D and a supergravity theory in 10D; such an amount of supersymmetry guarantees the persistence of certain properties while going from weak to strong coupling. AdS/CFT has been nevertheless hypothesized also in the case of reduced or no supersymmetry, or for non-conformal theories. The basic idea is the holographic correspondence between a gauge theory on the boundary of an AdS space and a (super)gravity theory that lives on the bulk of the latter. This brings into the game a new concept: gauge theories and gravity theories seem to be *complementary* rather than *supplementary*, they complete each other rather than being two separate entities, they seem to describe in different ways the same basic underlying physics.

The AdS/CFT correspondence, as it is commonly used, relates two field theories, but it should not be forgotten that, in the original case, it is formulated in the framework of superstring theory and it requires at least type IIB theory on $AdS_5 \times S^5$ for the full duality to work. So we are led to conclude that the natural framework for this kind of correspondence is string theory. And since the low energy effective theory of the open strings on a stack of D-branes is a gauge theory, while gravity is generated by closed strings, one is naturally led to think that the basic duality is the one between open and closed strings. Even more, since closed strings source D-branes, it seems unlikely to be able to treat open and closed string as separate entities (except for closed strings at the perturbative level).

A logical conclusion of the previous argument is that a full understanding of holographic dualities can be acquired only in the framework of string theory, and that the underlying duality to be considered is the open-closed string duality. Now the question is: what is the best context in which these problems can be analyzed? The abovementioned (super)string theories are first quantized theories, and although one can go a long way even without a second quantized string theory, there seems to be insurmountable difficulties if one tries to draw a completely satisfactory picture of the theory. For instance, while there are no obstructions in constructing on-shell perturbative amplitudes of a given string theory, there is no unambiguous guide in constructing off-shell amplitudes.

Thus it is extremely desirable, if not compulsory, to have a second quantized string theory available. In this regard, the present situation is a as follows. We have a covariant formulation (à la Witten) of second quantized bosonic open string theory (OSFT) with a cubic interaction term, which is well defined and consistent (see below). Witten formulated also a boundary SFT, a theory of 2D theories so to speak, defined on a unit disk with perturbations on the boundary, which, however, has serious normalization problems. As for bosonic closed string theories, their second quantized version can be formulated in analogy to the OSFT, but the cubic interaction term is not enough to cover the moduli space (see below), so one is obliged to introduce infinite many interaction terms, ending up with a nonpolynomial theory; as a consequence perhaps this theory cannot be properly called a field theory. Coming to the second quantized superstring theories, there is the analog of the bosonic OSFT, also proposed by Witten; this theory however has contact singularities. The modifications that have been proposed avoid these singularities but have other problems. A successful alternative is Berkovits' open superstring field theory, modelled on the WZW model; this passes many tests, its basic drawback being that it is formulated only for the NS sector while the R sector (the fermionic one) is at present missing.

Faced with the above situation one is tempted to wonder whether a second quantized superstring field theory is at all possible. On the other hand a bosonic closed SFT does not seem to be (at least) technically viable (see above). If this is so one is obliged to conclude that the only consistent SFT at our disposal is Witten's open SFT. Does it make sense to focus on this theory and take it seriously? It should be pointed out that we have no a priori reason to believe that a unique and complete SFT theory exists. On the other hand the objections against Witten's OSFT (the tachyon, the tadpoles contributions) seem by now to be overcome: the recent successes of this theory indicate that these problems are not intrinsic to the theory but rather to the way we solve it. The traditional motivation: OSFT is an extremely useful playground while waiting for the its consistent full supersymmetric version is by now too diminishing and perhaps misleading. We will return to this point at the end of the paper. In the following we will focus on some aspects of OSFT à la Witten.

2. The bosonic open SFT

This is a short summary of first quantized open bosonic theory.

First quantized open string theory in the critical dimension D=26 is formulated in terms of quantum oscillators α_n^μ, $-\infty < n < \infty$, $\mu = 0, 1, \ldots, 25$, which come from the mode expansion of the string scalar field

$$X^\mu(z) = \frac{1}{2}x^\mu - \frac{i}{2}p^\mu \ln z + \frac{i}{\sqrt{2}}\sum_{n\neq 0}\frac{\alpha_n^\mu}{n}z^{-n}$$

having set the characteristic square length of the string $\alpha' = 1$. They satisfy the algebra $[\alpha_m^\mu, \alpha_n^\nu] = m\eta^{\mu\nu}\delta_{n+m,0}$, η being the space–time Minkowski metric. The vacuum is defined by $\alpha_n^\mu|0\rangle = 0$ for $n > 0$ and $p^\mu|0\rangle = 0$. The states of the theory are constructed by applying to the vacuum the remaining quantum oscillators $\alpha_n^{\mu\dagger} = \alpha_{-n}^\mu$, with $n > 0$. Any such state $|\phi\rangle$ is given momentum k^μ by multiplying it by the eigenstate e^{ikx}. This state with momentum will be denoted by $|\phi, k\rangle$. In order for such states to be physical they must satisfy the conditions

$$L_n^{(X)}|\phi, k\rangle = 0, \quad n > 0, \qquad (L_0^{(X)} - 1)|\phi, k\rangle = 0 \tag{2.1}$$

where $L_n^{(X)}$ are the matter Virasoro generators

$$L_n^{(X)} = \frac{1}{2}\sum_{k=-\infty}^{\infty} : \alpha_{n-k}^\mu \alpha_k^\nu : \eta_{\mu\nu} \tag{2.2}$$

Here $\alpha_0 = p$ and :: denotes normal ordering. The conditions (2.1) are the quantum translation of the classical vanishing of the energy–momentum tensor.

The conditions (2.1) define the physical spectrum of the theory (in D=26). All the states are ordered according to the level, the level being a natural number specified by the eigenvalue of $L_0^{(X)} + L_0^{(gh)} - \frac{p^2}{2}$. The lowest lying state (level 0) is the tachyon represented by the vacuum with momentum k and square mass $M^2 = -1$. The next (level 1) is the massless vector state $\zeta_\mu\, \alpha_{-1}^\mu|0\rangle e^{ikx}$ with $k^2 = 0$ and $\zeta \cdot k = 0$, which is interpreted as a gauge field. The other states are all massive, with increasing masses proportional to the Planck mass.

To each of these states is associated a vertex operator. For instance, to the tachyon we associate $V_t(k) =: e^{ik\cdot X}$:; to the vector state $V_A(k, \zeta) =: \zeta \cdot \dot{X} e^{ik\cdot X}$:, where the dot on top of X denotes the tangent derivative with respect to the world–sheet boundary (the real axis in the z UHP); and so on. In this way one can formulate rules to calculate any kind of amplitude of these operators $\langle V_1(k_1) \dots V_N(k_N)\rangle$, as far as these amplitudes are *on shell*. At low energy, $\alpha' \to 0$, such amplitudes reproduce those of the corresponding field theory (for instance, the amplitudes of V_A reproduce the amplitudes of a Maxwell field theory). If we want to compute *off–shell* amplitudes the above rules are insufficient and in general we have to resort to a field theory of strings. This was a major motivation for introducing string field theories.

So far we have ignored ghosts. Indeed the b, c ghosts, which come from the gauge fixing of reparametrization invariance via the Faddeev–Popov recipe, play a minor role in perturbative string theory. They play a much more important role in SFT. They are also expanded in modes c_n and b_n and one can construct the corresponding Virasoro generators

$$L_n^{(gh)} =: \sum_k (2n + k)\, b_{-k} c_{k+n} : \tag{2.3}$$

Both (2.2) and (2.3) obey the same Virasoro algebra

$$[L_n, L_m] = (n - m)L_{n+m} + \frac{c}{12}(n^3 - n)\delta_{n,0} \tag{2.4}$$

The central charge c equals the number of X fields in the matter case (i.e. 26), while it equals -26 in the case of the b, c ghosts. So the total central charge vanishes in D=26. This guarantees the absence of any trace anomaly, and therefore consistency of the bosonic string theory as a gauge theory. From now on we concentrate only on this case.

The previous results about ghosts and critical dimension, can be usefully reformulated in terms of BRST symmetry and its charge Q. Q is defined by

$$Q = \sum_n : c_n \left(L_n^{(X)} + \frac{1}{2} L_n^{(gh)} \right) : \tag{2.5}$$

It is hermitean $Q^\dagger = Q$ and its basic property is nilpotency, $Q^2 = 0$ in critical dimension. The study of the physical spectrum can be reformulated in terms of the cohomology of Q: *the physical states of perturbative string theory are the states of ghost number 1 that are annihilated by* Q, defined up to states obtained by acting with Q on any state of ghost number 0. They can be represented by the old physical states $|\phi, k\rangle$ tensored with the ghost factor $c_1|0\rangle$.

With this at hand we can now turn to string field theory.

3. OSFT

The open string field theory action proposed by Witten, [11], is defined in D=26 by the action

$$\mathcal{S}(\Psi) = -\frac{1}{g_o^2} \int \left(\frac{1}{2} \Psi * Q\Psi + \frac{1}{3} \Psi * \Psi * \Psi \right). \tag{3.1}$$

This action is clearly reminiscent of the Chern–Simons action in 3D. In this expression Ψ is the string field. It can be understood either as a classical functional of the open string configurations $\Psi(x^\mu(z))$, or as a vector in the Fock space of states of the open string theory. In the sequel we will consider for simplicity only this second point of view. In the field theory limit it makes sense to represent Ψ as a superposition of Fock space states with ghost number 1, with coefficient represented by (infinite many) local fields,

$$|\Psi\rangle = (\phi(x) + A_\mu(x) a_1^{\mu\dagger} + \ldots) c_1 |0\rangle. \tag{3.2}$$

The BRST charge Q is the one introduced above for the first quantized string theory.

One of the most fundamental ingredients is the star product. Physically it represents the string interaction, that is the process of two strings coming together to form a third string. More precisely the product of two string fields Ψ_1, Ψ_2 represents the process of identifying the right half of the first string with the left half of the second string and integrating over the overlapping degrees of freedom, to produce a third string which corresponds to $\Psi_1 * \Psi_2$. This can be implemented in different ways, either by using the classical string functional, or by means of the oscillator formalism. Further on we will briefly illustrate the method based on conformal field theory.

Finally the integration in (3.1) corresponds to bending the left half of the string over the right half and integrating over the corresponding degrees of freedom in such a way as to produce a number.

The following rules are obeyed

$$\begin{aligned}
&Q^2 = 0,\\
&\int Q\Psi = 0,\\
&(\Psi_1 * \Psi_2) * \Psi_3 = \Psi_1 * (\Psi_2 * \Psi_3),\\
&Q(\Psi_1 * \Psi_2) = (Q\Psi_1) * \Psi_2 + (-1)^{|\Psi_1|}\Psi_1 * (Q\Psi_2),
\end{aligned} \tag{3.3}$$

where $|\Psi|$ is the Grassmannality of the string field Ψ, which, for bosonic strings, coincides with the ghost number. The action (3.1) is invariant under the BRST transformation

$$\delta\Psi = Q\Lambda + \Psi * \Lambda - \Lambda * \Psi. \tag{3.4}$$

Finally, the ghost numbers of the various objects $Q, \Psi, \Lambda, *, \int$ are $1, 1, 0, 0, -3$, respectively.

Let us now see in more detail how to implement the star product. Let us consider three unit semi-disks in the upper half z_a ($a = 1, 2, 3$) plane. Each one represents the string freely propagating in semicircles from the origin (world-sheet time $\tau = -\infty$) to the unit circle $|z_a| = 1$ ($\tau = 0$), where the interaction is going to take place. We map each unit semi-disk to a 120° wedge of the complex plane via the following conformal maps:

$$f_a(z_a) = \alpha^{2-a} f(z_a)\,,\ a = 1, 2, 3, \tag{3.5}$$

where

$$f(z) = \Big(\frac{1+iz}{1-iz}\Big)^{\frac{2}{3}}. \tag{3.6}$$

Here $\alpha = e^{\frac{2\pi i}{3}}$. In this way the three semi-disks are mapped to non-overlapping (except along the edges) regions in such a way as to fill up a unit disk centered at the origin. The curvature is zero everywhere except at the center of the disk, which represents the common midpoint of the three strings in interaction. Finally the interaction vertex is defined by means of a CFT correlation function on the disk in the following way

$$\int \psi * \phi * \chi = \langle f_1 \circ \psi(0)\, f_2 \circ \phi(0)\, f_3 \circ \chi(0)\rangle \tag{3.7}$$

So, calculating the star product amounts to evaluating a three point function on the unit disk.

The action 3.1 has been quantized with the BV method. Choosing the Siegel gauge, i.e. imposing the condition $b_0|\Psi\rangle = 0$ to fix the enormous gauge symmetry (3.4), the kinetic term becomes particularly simple and can be easily inverted to produce a free propagator $b_0 L_0^{-1}$. This allows one to define the perturbative series and relevant Feynman rules. 0-th and 1-st order amplitudes for tachyons have been computed. Putting the external legs on shell, they reproduce the corresponding first quantized amplitudes, in particular the Veneziano amplitude. This is an important check, but of course now one has an unambiguous way to compute off-shell expressions for the amplitudes, virtually to any perturbative order. What is more important, one should remember that the first quantized amplitudes are integrated over the moduli space of the appropriate Riemann surfaces corresponding to the given perturbative order. It is far from obvious a priori that the perturbative OSFT reproduces the same procedure. However one of the most remarkable results in this context was the proof that it fully 'covers' the moduli space of Riemann surfaces and it does it only once. This is to be contrasted with the analogous problem in closed string field theory, where a third order interaction is not sufficient to cover the full moduli space, and one is obliged to introduce higher order vertices.

In conclusion the OSFT introduced in this section reproduces the results of first quantized string theory. Its added value with respect to the latter is not only that it allows us to compute off-shell amplitudes, but especially that it puts us in the condition to tackle nonperturbative problems. The first and up to now most remarkable result of SFT is the treatment of *tachyon condensation.*

4. The tachyon condensation

Following the rules of the previous section it is possible in favorable cases to explicitly compute the action (3.1). For instance, in the low energy limit, where the string field may be assumed to take the form (3.2), the action becomes an integrated function F of an infinite series of local polynomials (kinetic and potential terms) of the fields involved in (3.2) To limit the number of terms one has to limit once again the gigantic BRST symmetry of OSFT, by choosing a gauge, which is usually the Feynman–Siegel gauge: this means that we limit ourselves to the states that satisfy the condition: $b_0|\Psi\rangle = 0$

Still the action with all the infinite sets of fields contained in Ψ remains unwieldy. As it turns out, it makes sense to limit the number of fields in Ψ, provided we insert all the fields up to a certain level. This is called *level truncation* and turns out to be an excellent approximation and regularization scheme in SFT. Let us see this in more detail for a string field which includes only the tachyon $\phi(x)$. The action turns out to be

$$\mathcal{S}_{(0,0)} = \frac{1}{g_o^2}\int d^{26}x\left(-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\phi^2 - \frac{1}{3}\beta^3\tilde{\phi}^3\right) \tag{4.1}$$

where $\beta = \frac{3\sqrt{3}}{4}$ is a recurrent number in SFT. One can see the kinetic term and the 'wrong' mass term for the tachyon. The fields appearing in the interaction term carry a tilde. This means

$$\tilde{\phi}(x) = e^{(\ln\beta)\partial_\mu\partial^\mu}\phi(x)$$

Incidentally, the fact that the interaction is formulated in terms of tilded fields is a manifestation of the strong (exponential) convergence properties of string theory in the UV.

Let us now consider the potential and study its minimum. We should remember that this theory is supposed to represent the open strings attached to a space–filling D–brane, the D25–brane. In the CFT language such configurations are described by boundary CFT's.

Let us concentrate on the D25–brane and evaluate the total energy of the system brane + string modes. The brane has its intrinsic energy, whose density is the tension τ, which in our conventional units ($\alpha' = 1$), is given by $\tau = \frac{1}{2\pi^2 g_o^2}$. The string modes are represented by the action and, in a static situation, their total energy is given by $-$ the action itself. We precisely wish to study this system in the vacuum. Lorentz invariance requires that only scalars can acquire a VEV. Therefore in (4.1) one must set all the derivatives to 0. Setting $\langle\phi\rangle = t$, what remains of the action (divided by the total volume) can be written in terms of the function $u(t)$ as follows

$$-\frac{S}{V} \equiv \tau\, u(t) = 2\pi^2\left(-\frac{1}{2}t^2 + \frac{1}{3}\beta^3 t^3\right) \tag{4.2}$$

This is the total tachyon potential energy density extracted from the action.

The total energy of the system will be given by the sum of (4.2) and the D25–brane tension

$$U(t) = \tau(1 + u(t)) \tag{4.3}$$

This potential is cubic, it goes to $+\infty$ for positive large t and to $-\infty$ for negative large t and it has a local maximum and and a local minimum, which are easily determined. The former is at $t = 0$, the latter is given by

$$t = t_0 = \frac{1}{\beta^3}, \qquad u(t) \approx -0.684 \tag{4.4}$$

Of course this is a first approximation result. Considering higher level scalar fields (there are infinite many of them) the minimum will be modified. The numerical evaluations performed within the level truncation scheme indicate that the true minimum of the potential corresponds to $u = -1$, i.e. $U = 0$. This coincides with the first conjecture by Sen.

In order to describe the physics of tachyon condensation Sen [8] formulated three conjectures. The first claims that at the minimum of the potential the theory must be stable, so the energy of the space-filling brane must compensate exactly the energy of the strings. The second conjecture concerns the features of the tachyon condensation vacuum: in this vacuum there cannot be open string modes, that is it is the vacuum of an entirely different system, that of closed strings. The third conjecture is a consequence of this statement: one should be able to find in the new vacuum the physics of closed string theory.

The numerical results mentioned above were the first evidence that first Sen's conjecture is correct. Also the other conjectures met support from numerical methods or via cousin theories, such as BCFT. The turning point in this field came in 2005 with the first analytic tachyon vacuum solution found by Schnabl [10].

5. The analytic tachyon vacuum solution

The equation of motion derived from (3.1) is

$$Q\Psi + \Psi * \Psi = 0 \tag{5.1}$$

Its first analytic solution, representing the tachyon vacuum, was found by Martin Schnabl [10]. It allowed to prove Sen's first and second conjectures[5]. In order to describe it some amount of formalism is inevitable. The progress was actually allowed by an improvement in the SFT formalism, made possible by a simple change of variable: $z \to \tilde{z} = \arctan z$. It allows us to introduce objects like

$$\begin{aligned} \mathcal{L}_0 &= L_0 + \sum_{k=1}^{\infty} \frac{2(-1)^{k+1}}{4k^2-1} L_{2k}, \qquad K_1 = L_1 + L_{-1} \\ \mathcal{B}_0 &= b_0 + \sum_{k=1}^{\infty} \frac{2(-1)^{k+1}}{4k^2-1} b_{2k}, \qquad B_1 = b_1 + b_{-1} \end{aligned}$$

and

$$B_1^L = \frac{1}{2}B_1 + \frac{1}{\pi}\left(\mathcal{B}_0 + \mathcal{B}_0^\dagger\right) \qquad K_1^L = \frac{1}{2}K_1 + \frac{1}{\pi}\left(\mathcal{L}_0 + \mathcal{L}_0^\dagger\right)$$

where $c(z), b(z)$ are the ghost fields. All this, at first sight, seems to be a useless complication. However applying these operators to the identity state $|I\rangle$ with respect to the star product:

$$K = \frac{\pi}{2}K_1^L|I\rangle, \qquad B = \frac{\pi}{2}B_1^L|I\rangle, \qquad c = c\left(\frac{1}{2}\right)|I\rangle, \tag{5.2}$$

we obtain a remarkably simple algebra

$$[K, B] = 0, \qquad [K, c] = c\partial c, \qquad \{B, c\} = 1, \qquad \{B, \partial c\} = 0 \tag{5.3}$$

where the product is understood to be the star product and 1 represents $|I\rangle$. In this algebra Q operates as follows

$$QB = K, \qquad Qc = c\partial c \tag{5.4}$$

Instead of writing down the original Schnabl's solution, we will show next an interesting variant of Schnabl's first solution, proposed by T.Erler and M.Schnabl[6], which fully exploit the above algebra and lends itself to be generalized to lump solutions. In terms of this algebra with operator, the new solution[6], is given by

$$\psi_0 = \frac{1}{1+K} c(1+K)Bc = c - \frac{1}{1+K} Bc\partial c, \tag{5.5}$$

The energy of this solution turns out to be the correct one (1st conjecture)

$$E = -\frac{S}{V} = \frac{1}{g_o^2 V}\left(\frac{1}{2}\langle\Psi, Q\,\Psi\rangle + \frac{1}{3}\langle\Psi, \Psi\star\Psi\rangle\right) = -\frac{1}{2\pi^2 g_0^2} \tag{5.6}$$

It is also possible to define a homotopy operator $A = \frac{B}{K+1}$, which satisfies the property $\mathcal{Q}A = 1$, where

$$\mathcal{Q}\psi \equiv Q\psi + \Psi_0 \star \psi - (-1)^{|\psi|}\psi \star \Psi_0 \tag{5.7}$$

is the BRST operator at the tachyon vacuum. This implies that the cohomology around the tachyon vacuum is trivial (2nd conjecture), since

$$\mathcal{Q}\psi = 0 \longrightarrow \mathcal{Q}(A\star\psi) = (\mathcal{Q}A\})\star\psi - A\star(Q\psi) = \psi$$

6. The third conjecture and the lump solutions

The third conjecture predicts in particular the existence of lower dimensional solitonic solutions or lumps, interpreted as Dp–branes, with $p < 25$. These solutions bring along the breaking of translational symmetry and background independence. The evidence for the existence of such solutions collected in the past years is overwhelming. It has been possible to find them with approximate methods or with exact methods in related theories. In the sequel I will present a recently proposed explicit example of analytic lump solution in OSFT.

6.1. Analytic lump solutions

In a recent paper[1], a general method has been proposed to obtain new exact analytic solutions in open string field theory, and in particular solutions that describe inhomogeneous tachyon condensation. The method consists in translating an exact renormalization group (RG) flow generated in a two–dimensional world–sheet theory by a relevant operator, into the language of OSFT. The so-constructed solution is a deformation of the Erler-Schnabl solution described above. It has been shown in [1] that, if the operator has suitable properties, the solution will describe tachyon condensation in specific space directions, thus representing the condensation of a lower dimensional brane. In the following, after describing the general method, we will focus on a particular solution, generated by an exact RG flow first analyzed by Witten[12]. On the basis of the analysis carried out in the framework of 2D CFT in [7], we expect it to describe a D24-brane, with the correct ratio of tension with respect to the starting D25 brane.

Let us see first the general recipe to construct such kind of lump solutions. To start with we enlarge the K, B, c algebra by adding a (relevant) matter operator

$$\phi = \phi\left(\frac{1}{2}\right)|I\rangle. \tag{6.1}$$

with the properties

$$[c,\phi] = 0, \qquad [B,\phi] = 0, \qquad [K,\phi] = \partial\phi, \qquad Q\phi = c\partial\phi + \partial c\delta\phi. \tag{6.2}$$

It can be easily proven that

$$\psi_\phi = c\phi - \frac{1}{K+\phi}(\phi - \delta\phi)Bc\partial c \tag{6.3}$$

does indeed satisfy (formally, see below) the OSFT equation of motion

$$Q\psi_\phi + \psi_\phi\psi_\phi = 0 \tag{6.4}$$

It is clear that (6.3) is a deformation of the Erler–Schnabl solution, which can be recovered for $\phi = 1$.

After some algebraic manipulations one can show that

$$\mathcal{Q}_{\psi_\phi}\frac{B}{K+\phi} = Q\frac{B}{K+\phi} + \left\{\psi_\phi, \frac{B}{K+\phi}\right\} = 1.$$

So, unless the string field $\frac{B}{K+\phi}$ is singular, it defines a homotopy operator and the solution has trivial cohomology, which is the defining property of the tachyon vacuum [5]. On the other hand, in order for the solution to be well defined, the quantity $\frac{1}{K+\phi}(\phi - \delta\phi)$ should be well defined. Moreover, in order to be able to show that (6.3) satisfies the equation of motion, one needs $K + \phi$ to be invertible.

In full generality we thus have a new nontrivial solution if

1. $\frac{1}{K+\phi}$ is singular, but
2. $\frac{1}{K+\phi}(\phi - \delta\phi)$ is regular and
3. $\frac{1}{K+\phi}(K + \phi) = 1$

These conditions seem to be hard to satisfy: for instance, $K + \phi$ may not be invertible, one needs a regularization. It is indeed so without adequate specifications. This problem was discussed in [2, 4], where it was shown that the right framework is distribution theory, which guarantees not only regularity of the solution but also its 'non-triviality', in the sense that if these conditions are satisfied, it cannot fall in the same class as the ES tachyon vacuum solution.

For concreteness we parametrize the worldsheet RG flow, referred to above, with a parameter u, where $u = 0$ represents the UV and $u = \infty$ the IR, and label ϕ by ϕ_u, with $\phi_{u=0} = 0$. Then we require for ϕ_u the following properties under the coordinate rescaling $f_t(z) = \frac{z}{t}$

$$f_t \circ \phi_u(z) = \frac{1}{t}\, \phi_{tu}\left(\frac{z}{t}\right). \tag{6.5}$$

We will consider in the sequel a specific relevant operator ϕ_u and the corresponding SFT solution. This operator generates an exact RG flow studied by Witten in [12], see also [7], and is based on the operator (defined in the cylinder C_T of width T in the arctan frame)

$$\phi_u(s) = u(X^2(s) + 2\, \ln u + 2A) \tag{6.6}$$

where A is a constant. On the unit disk D we have

$$\phi_u(\theta) = u(X^2(\theta) + 2\, \ln \frac{Tu}{2\pi} + 2A) \tag{6.7}$$

If we set

$$g_A(u) = \langle e^{-\frac{1}{2\pi}\int_0^{4\pi} d\theta\, u\left(X^2(\theta)+2\ln\frac{u}{2\pi}+2A\right)} \rangle_D$$

we get

$$g_A(u) = Z(2u)e^{-2u(\ln\frac{u}{2\pi}+A)} \tag{6.8}$$

where $Z(u)$ is the partition function of the system on the unit disk computed by[12]. Requiring finiteness for $u \to \infty$ one gets $A = \gamma - 1 + \ln 4\pi$, which implies

$$g_A(u) \equiv g(u) = \frac{1}{2\sqrt{\pi}}\sqrt{2u}\Gamma(2u)e^{2u(1-\ln(2u))}, \qquad \lim_{u\to\infty} g(u) = 1 \tag{6.9}$$

Moreover, as it turns out, $\phi_u - \delta\phi_u = u\partial_u\phi_u(s)$

The ϕ_u just introduced satisfies all the requested properties. According to [7], the corresponding RG flow in BCFT reproduces the correct ratio of tension between D25 and D24 branes. Consequently $\psi_u \equiv \psi_{\phi_u}$ is expected to represent a D24 brane solution.

In SFT the most important gauge invariant quantity is of course the energy. Therefore in order to make sure that $\psi_u \equiv \psi_{\phi_u}$ is the expected solution we must prove that its energy equals a D24 brane energy.

The energy expression for the lump solution was determined in [1] by evaluating a three–point function on the cylinder C_T. It equals $-\frac{1}{6}$ times the following expression

$$\langle\psi_u\psi_u\psi_u\rangle = -\int_0^\infty dt_1dt_2dt_3\mathcal{E}_0(t_1,t_2,t_3)u^3g(uT)\Bigg\{\Big(-\frac{\partial_{2uT}g(uT)}{g(uT)}\Big)^3$$

$$
+\frac{1}{2}\Big(-\frac{\partial_{2uT}g(uT)}{g(uT)}\Big)\Big(G_{2uT}^2(\frac{2\pi t_1}{T})+G_{2uT}^2(\frac{2\pi(t_1+t_2)}{T})+G_{2uT}^2(\frac{2\pi t_2}{T})\Big)
$$

$$
+G_{2uT}(\frac{2\pi t_1}{T})G_{2uT}(\frac{2\pi(t_1+t_2)}{T})G_{2uT}(\frac{2\pi t_2}{T})\Big\} \tag{6.10}
$$

Here $T = t_1 + t_2 + t_3$ and $g(u)$ is as above while $G_u(\theta)$ represents the correlator on the boundary, first determined by Witten[12]:

$$
G_u(\theta) = \frac{1}{u} + 2\sum_{k=1}^{\infty}\frac{\cos(k\theta)}{k+u}
$$

Finally $\mathcal{E}_0(t_1, t_2, t_3)$ represents the ghost three–point function in C_T.

$$
\mathcal{E}_0(t_1,t_2,t_3) = \langle Bc\partial c(t_1+t_2)\partial c(t_1)\partial c(0)\rangle_{C_T} = -\frac{4}{\pi}\sin\frac{\pi t_1}{T}\sin\frac{\pi(t_1+t_2)}{T}\sin\frac{\pi t_2}{T}
$$

A remarkable property of (6.10) is that it does not depend on u. In fact u can be absorbed in a redefinition of variables $t_i \to ut_i$, $i = 1, 2, 3$, and disappears from the expression.

The integral in (6.10) is well defined in the IR (s very large, setting $s = 2uT$) but has an UV ($s \approx 0$) singularity, which must be subtracted away. Once this done, the expression (6.10) can be numerically computed, the result being ≈ 0.069. This is not the expected result, but this is not surprising, for the result depends on the UV subtraction we have made. Therefore we cannot assign to it any physical significance. To get a meaningful result we must return to the very meaning of third Sen's conjecture, which says that *the lump solution is a solution of the theory on the tachyon condensation vacuum.* Therefore we must measure the energy of our solution with respect to the tachyon condensation vacuum. Simultaneously the resulting energy must be a subtraction-independent quantity because only to a subtraction-independent quantity can a physical meaning be assigned. Both requirements have been satisfied in [2] in the following way.

First a new solution to the EOM, depending on a parameter ε, has been introduced

$$
\psi_u^\varepsilon = c(\phi_u+\varepsilon) - \frac{1}{K+\phi_u+\varepsilon}(\phi_u+\varepsilon-\delta\phi_u)Bc\partial c. \tag{6.11}
$$

and it has been shown that it is gauge equivalent to the tachyon vacuum solution, its energy (after the same UV subtraction as in the previous case) being (numerically) 0. Then, using it, a solution to the EOM at the tachyon condensation vacuum has been obtained. The equation of motion at the tachyon vacuum is

$$
\mathcal{Q}\Phi + \Phi\Phi = 0, \quad \text{where} \;\; \mathcal{Q}\Phi = Q\Phi + \psi_u^\varepsilon\Phi + \Phi\psi_u^\varepsilon. \tag{6.12}
$$

One can easily show that

$$
\Phi_0 = \psi_u - \psi_u^\varepsilon \tag{6.13}
$$

is a solution to (6.12). The action at the tachyon vacuum is $-\frac{1}{2}\langle \mathcal{Q}\Phi, \Phi\rangle - \frac{1}{3}\langle \Phi, \Phi\Phi\rangle$. Thus the energy of Φ_0 is

$$
\begin{aligned}
E[\Phi_0] &= -\frac{1}{6}\langle \Phi_0, \Phi_0\Phi_0\rangle \\
&= -\frac{1}{6}\big[\langle \psi_u, \psi_u\psi_u\rangle - \langle \psi_u^\varepsilon, \psi_u^\varepsilon\psi_u^\varepsilon\rangle - 3\langle \psi_u^\varepsilon, \psi_u\psi_u\rangle + 3\langle \psi_u, \psi_u^\varepsilon\psi_u^\varepsilon\rangle\big].
\end{aligned}
\tag{6.14}
$$

The UV subtractions necessary for each correlator at the RHS of this equation are always the same, therefore they cancel out and the final result is subtraction-independent. A final bonus of this procedure is that the final result can be derived purely analytically and $E[\Phi_0]$ turns out to be precisely the D24-brane energy. With the conventions of [2], this is

$$
T_{D24} = \frac{1}{2\pi^2}
\tag{6.15}
$$

In [3] the same result was extended to Dp-brane lump solutions for any p.

7. Comments

The three conjectures formulated by Ashok Sen about fifteen years ago have been demonstrated beyond doubt in the framework of Witten's OSFT. This is certainly a remarkable result, but from the point of view of OSFT it is only a beginning. The correctness of the three conjectures confirms that open string theory knows about closed string theory. As anticipated in the introduction this was somehow expected. Even the first quantized open string theory contains at one loop information about the closed string spectrum. However what we have learnt from OSFT is much richer information. Even at the classical level (tree level of the perturbative expansion) [9], provided we consider exact analytic solutions (which correspond to specific boundary CFT's, i.e. full expansions in the α' parameter), we can get information about closed string theory. The question is now how rich and complete this information is. The example of AdS/CFT suggests that open and closed strings are two different description of the same underlying physics. On the other hand, the study of string field theory seems to suggest that there is some asymmetry between the two descriptions. If field theory is the right language for physics the open string description is favored. OSFT seems to respond to the best expectations and the exciting questions we are left with at the end of this exposition are: how far can we go in the description of closed string theory by means of open strings? is there a way, for instance, to represent black holes in the open string theory language? Even further, can we recognize in this language bosonized solutions representing fermions?

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