FACTA UNIVERSITATIS
Series: **Teaching, Learning and Teacher Education** Vol. 7, Nº1, Special Issue, 2023, pp. 95 - 102
https://doi.org/10.22190/FUTLTE220926010D

Original research paper

A COMPOSED MATHEMATICS GRADE OF ACADEMIC KNOWLEDGE, PROJECT WORK, AND HOMEWORK: A FUZZY LOGIC APPROACH

UDC 371.322.1:51-057.874; 37.016:510.6

Daniel Doz, Darjo Felda, Mara Cotič

Faculty of Education, University of Primorska, Slovenia

Abstract. *Student knowledge assessment is a key element of the pedagogical process, as it provides students, parents, and educators with important feedback information on students' knowledge and skills. Assessment of students' mathematical knowledge is complex, as several factors are normally included in students' final grades, and simply calculating the average of students' achievements may not provide a complete picture of their knowledge. Therefore, in this paper, we aimed to investigate the possibility of using fuzzy logic to assess students' knowledge and competencies by considering (1) students' overall academic performance, (2) the quality of students' project work on a topic from the history of mathematics and (3) the regularity of handing in homework. The study was conducted considering 22 Italian high school students. The results show that students' academic performance is similar to student grades obtained with fuzzy logic.*

Key words: *mathematics, project work, homework, fuzzy logic*

1. INTRODUCTION

Assessing students' knowledge is an important part of the education process (Tosuncuoglu, 2018), which has the aim of informing students, parents, and educators about the quality of the learnt material (Felda, 2018). Assessment of students' knowledge and abilities is very important in mathematics since it is useful to solve everyday problems (Ojose, 2011; Putnam, 1992) and permits adults to properly function in today's society (Felda & Cotič, 2012).

Recently, several studies have explored the possibility of using the mathematical theory of fuzzy logic to assess students' knowledge and competencies (Barlybayev et al., 2016; Ingoley & Bakal, 2012; Namli & Şenkal, 2018; Petroudi et al., 2013; Varghese et

Received September 26, 2022/Accepted December 25, 2022
Corresponding author: Daniel Doz
Faculty of Education, University of Primorska, Cankarjeva 5, 6000 Koper, Slovenia
Phone: +386 05 663 1260 • E-mail: daniel.doz@upr.si

al., 2017; Voskoglou, 2013). Fuzzy logic is a part of mathematical logic, which was developed by the mathematician Lofti A. Zadeh in 1965 (Zadeh, 1965). This kind of logic permits dealing with uncertain quantities, which are often an expression of verbal descriptions (Bai & Wang, 2006).

A membership function is a function that determines the level of membership of an element in a set. In classical logic, an element might be part of a set or not, and there is no other possibility. In fuzzy logic, however, an element might only partially belong to a set, and the level of "belonging" to a set is expressed as a real number between 0 and 1. Formally, the membership function associated with each element of a universal set U is defined as the function $\mu_A: U \to [0,1]$. A fuzzy set is then the set defined by $A = \{(x, \mu_A(x)): x \in U\}$. In real-life applications, the membership functions are defined by the user, based on his/her experience and based on the aims of the project (Yadav, Soni & Pal, 2014). In practice, among the most used membership functions, we consider the trapezoidal function (Yadav et al., 2014), which is defined as follows ($a < b < c < d$):

$$\text{Trap}(x,a,b,c,d) = \begin{cases} 0 & x \le a, x \ge d \\ \dfrac{x-a}{b-a} & a < x \le b \\ 1 & b < x \le c \\ \dfrac{d-x}{d-c} & c < x \le d \end{cases}$$

Overall, the application of fuzzy logic to solve real-world problems requires three steps, known as the "fuzzy process" (Yadav et al., 2014):

1. the fuzzification of crisp data: i.e., the conversion of real data into fuzzy values through the membership functions;
2. the inference process: using the set of inference rules of the form "IF *X* THEN *Y*", which are set by the user, permits us to convert fuzzy input information into output fuzzy values;
3. the defuzzification of fuzzy values: fuzzy information is then converted into crisp output information via the defuzzification method.

The fuzzification of crisp input values represents the first step of the fuzzy process and it is comprised of two steps (Bai & Wang, 2006):

- the determination of the membership functions for input and output information;
- all crisp input data is converted into fuzzy values, i.e. into verbal variables.

Membership functions have usually different shapes and are chosen by the users, based on the input and output data.

After the fuzzification of the input data, inference rules are used to produce the fuzzy output information. Such rules are based on the everyday experience of the user (Bai & Wang, 2006).

The defuzzification of the data represents the last step of the fuzzy process. The results obtained via inference are converted into real data (crisp information) once again. Among the defuzzification methods, a popular choice is the method of the center of gravity (COG) or centroid method. The COG is defined as the following quantity:

$$COG(A) = \frac{\int \mu_A(x) \cdot x dx}{\int \mu_A(x)\, dx}.$$

In the last two decades, several works have explored the possibility of assessing students' knowledge and skills through fuzzy logic. In particular, students' grades are fuzzy variables, since they describe students' knowledge, competencies, and abilities through verbal, imprecise variables. For instance, assigning a grade on a numerical scale means assigning a numerical value to an imprecise descriptor. Also, if teachers want to consider several other factors, which are difficult to assess with standardized tests or, at least, written and oral examinations, fuzzy logic represents a valid alternative that might be used to assess students' knowledge. Petroudi, Pirouz, and Pirouz (2013) presented a model for determining students' final grades by considering their achievements on two written exams and a practical one. The researchers used triangular membership functions and firstly combined students' grades on two written exams and, secondly, the found grade with the grade on the practical exam. The findings suggest that the novel method produced lower grades for better-achieving students, while it produced higher grades for lower-achieving students.

Yadav et al. (2014) used students' achievements at the end of the middle and end term (i.e., the first and second semesters), while Meenakshi and Pankaj (2015) considered students' attendance of lectures, internal examination, and external examination. The latter research found no statistically significant differences in students' assessment with the novel fuzzy method and the traditional one, which considers only teacher-given grades.

2. METHODS

2.1. Aims of the research

Despite the literature has examined the possibility of using fuzzy logic to assess students' knowledge in various ways, much less is known about the possibility of using fuzzy logic to assess students' mathematical knowledge considering (1) students' academic knowledge; (2) students' project work; and (3) the regularity of turning in homework. Therefore, the main aim of the present research is to examine this novel method of assessing students' mathematical knowledge. Based on the abovementioned literature, we hypothesize that the novel method would produce grades similar to teacher-given grades (Yadav et al., 2013; Meenakshi & Pankaj, 2015).

2.2. Methodology

The present study is a small-scale research. The non-experimental quantitative research method was applied.

2.3. Sample

The participants were 22 grade-9 students from a lyceum in North-Eastern Italy. Among them, 7 were girls (31.8%) and 15 were boys (68.2%). The average age of the participants was $M = 14.4$ years ($SD = .503$; $min = 14$; $max = 15$), with a median of 14.

2.4. Data collection

Students' grades were retrieved from the official school records after obtaining their and their parents' signed informed consent. Students' grades in mathematics are expressed on a scale from 1 to 10, where 6 is the first passing grade: all grades lower than 6 are failing grades, while 10 represents the excellent grade (D.Lgs. 62/2017). Students' grades are based on their achievements on written tests and oral examinations of the topics studied in the first year of high school (MIUR, 2010).

Students' project work evaluations were retrieved from the teacher's official records. Those assessments are measured on a scale from 1 to 10, i.e. in the same way mathematics grades are presented. Students' project work was individual work on the topic of the history of mathematics. Students were required to write a report about one of the following topics:

- Babylonian mathematics;
- Egyptian mathematics;
- Mayan mathematics;
- Euclidean geometry;
- George Boole and Boolean algebra.

Students' projects were assessed considering the correctness of the contents, the used language and terminology, and the efficacy of communication. Students had 6 months to complete the project work.

Students' homework was assessed based on the regularity of turning in the assignments. The assessment of the regularity is presented in the Procedure section.

2.5. Procedure

The model we used is depicted in Figure 1.

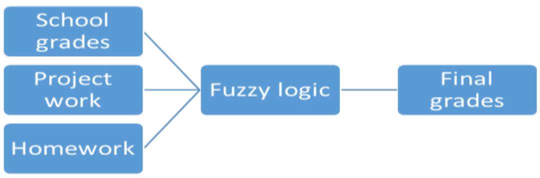

Fig. 1 The proposed model.

Students' mathematics grades and project work are fuzzified using the membership functions presented in Table 1 and Figure 2.

Table 1 The Membership functions used to fuzzify raw students' grades.

Level	Membership function
Extremely low (EL)	Trap(x,1,1,3,4)
Low (L)	Trap(x,3,4,5,6)
Middle (M)	Trap(x,5,6,7,8)
High (H)	Trap(x,7,8,10,10)

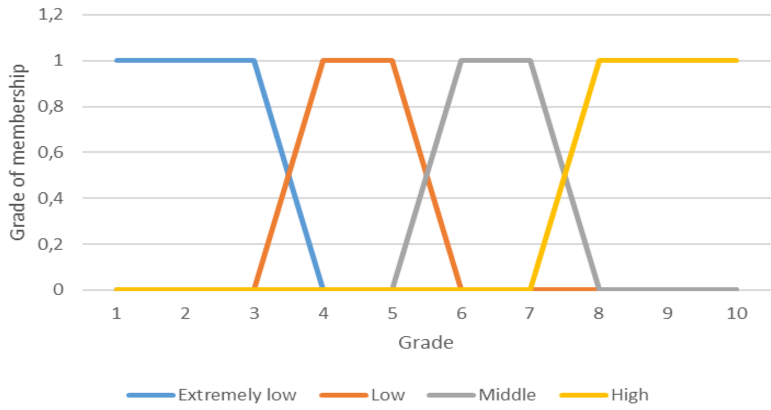

Fig. 2 The fuzzification membership function.

The regularity of turning in the homework is assessed as presented in Table 2 and depicted in Figure 3.

Table 2 The evaluation of students' regularity of turning in homework.

Description	Membership function
Good (G)	Trap(x,0,0,2,3)
Middle (M)	Trap(x,2,3,5,6)
Poor (P)	Trap(x,5,6,50,50)

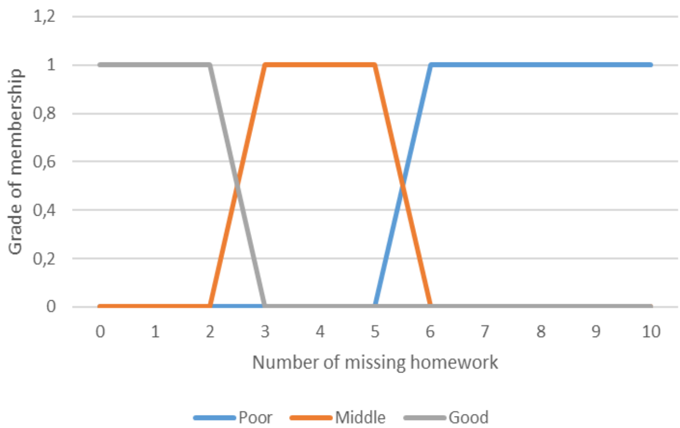

Fig. 3 The membership function for the number of missing homework.

We defined the inference rules as those presented in Table 3. Since there are 4 possible grades both for students' project work and their mathematics attainment, and there are three levels for homework, we had $4 \cdot 4 \cdot 3 = 48$ possibilities.

Table 3 The inference rule

		Project work											
		EL			L			M			H		
	Homework	P	M	G	P	M	G	P	M	G	P	M	G
Mathematics grade	EL	EL	EL	EL	EL	EL	L	L	L	L	M	M	M
	L	EL	L	L	L	L	L	L	M	M	M	M	M
	M	L	L	M	L	M	M	M	M	M	M	M	H
	H	L	M	M	M	M	J	M	M	H	M	H	H

As the defuzzification method, we used the centroid method. The defuzzification membership functions are the same as presented in Table 2 and Figure 2.

2.6. Data analysis

The collected data was analyzed using descriptive and inferential statistical tools with the software *Jamovi*. To compute the final grades with fuzzy logic, we used the MATLAB R2020b software (MathWorks, 2020). In particular, we used the Fuzzy Logic Toolbox (Sivanandam et al., 2007) application which is available in MATLAB.

3. Results

There were no significant differences between boys and girls in the tested variables. In Table 4 we present the descriptive statistics of students' mathematics grades, project works, and homework. The Wilcoxon signed rank test has shown that there are no statistically significant differences between students' mathematics grades (M = 7.98; SD = 1.69; Mdn = 8.60) and fuzzy grades (M = 7.87; SD = 1.33; Mdn = 8.76), W = 135; p = .808.

Table 4 Descriptive statistics of students' grades, project work, and homework.

Variable	M	SD	Mdn	min	max
Mathematics grade	7.98	1.69	8.60	4.20	9.80
Project work	8.66	1.25	9	5	10
Homework regularity	5.27	10.37	1	0	43
Fuzzy grade	7.87	1.33	5.76	4.50	8.76

In Table 5 we present Spearman's correlation coefficients among the variables. As it might be seen, fuzzy grades do positively and strongly correlate to students' mathematics grades and achievements on the project work. On the other hand, fuzzy grades are negatively and strongly correlated to students' homework regularity: students that are more regular in turning in their assignments, get higher fuzzy grades.

Table 5 Spearman's correlation coefficients

Variable	1.	2.	3.	4.
1. Mathematics grade	–	$.687^{***}$	$-.481^{*}$	$.842^{***}$
2. Project work		–	-.476*	$.755^{***}$
3. Homework regularity			–	$-.566^{**}$
4. Fuzzy grade				–

Note: $^{*}p < .05$; $^{**}p < .01$; $^{***}p < .001$.

4. DISCUSSION AND CONCLUSIONS

Literature has vastly examined the possibility of using fuzzy logic to assess students' knowledge. In the present paper, we tested a new method of assessing students' mathematical knowledge using three factors, i.e. (1) students' academic knowledge, given by teachers' grades, (2) students' achievement on the project work, which comprised individual research on a topic of the history of mathematics, and (3) students' regularity of turning in assignments and homework. All three achievements were combined using the method of fuzzy logic, following the inferential rules defined in by the researchers (cf. Bai & Wang, 2006).

In order to test the assessment method, we adopted a small-scale research, comprising N = 22 Italian high school students. Due to the small number of participants, we suggest future research replicate our results using larger samples. Nevertheless, our research has highlighted that students' fuzzy grades do not differ significantly from teacher-assigned grades via classical methods. A deeper analysis of fuzzy grades has shown that they are positively and strongly correlated to students' mathematics grades and their achievements on the project work. On the contrary, fuzzy grades are negatively and strongly correlated with the number of missing assignments and homework.

The novel assessing method with fuzzy logic thus produces grades that are similar to those that are assessed by teachers in a classical way, however, it does include more information about students' knowledge, work, and abilities than regular mathematics grades. In particular, the proposed method includes information about students' communication and research skills, and the regularity of turning in their homework. In future research, different results can be expected.

The present work is not without limitations. As an obvious limitation is the number of participants in the research. As expressed above, future research might enlarge the sample and use a similar method of assessing students' mathematical knowledge. Secondly, an intrinsic limitation of applying fuzzy logic, is that the used membership functions and inference rules have been determined by the researchers, based on their experience as educators (cf. Bai & Wang, 2006). Therefore, a slight difference in the definition of the used membership functions or different inference rules might lead to completely different results. Additional research is needed to determine the most suitable definition of membership functions and inference rules for educators and future studies might investigate how different membership functions or inference rules influence the final students' grades. Thirdly, in the present research, only homework regularity was considered; future research might integrate our results also with the quality of homework by adding to the model an additional variable.

REFERENCES

Bai, Y., & Wang, D. (2006). Fundamentals of fuzzy logic control-fuzzy sets, fuzzy rules and defuzzifications. In *Advanced fuzzy logic technologies in industrial applications* (pp. 17-36). Springer, London.

Barlybayev, A., Sharipbay, A., Ulyukova, G., Sabyrov, T., & Kuzenbayev, B. (2016). Student's performance evaluation by fuzzy logic. *Procedia Computer Science, 102*, 98-105. https://doi.org/10.1016/j.procs.2016.09.375

D.Lgs. 62/2017. https://www.gazzettaufficiale.it/eli/id/2017/05/16/17G00070/sg (15th May 2022).

Felda, D. (2018). Preverjanje matematičnega znanja. *Journal of Elementary Education, 11*(2), 175-188. https://doi.org/10.18690/rei.11.2.175-188.2018

Felda, D., & Cotič, M. (2012). Zakaj poučevati matematiko. *Journal of Elementary Education, 5*(2/3), 107-120.

Ingoley, S., & Bakal, J. W. (2012). Use of fuzzy logic in evaluating students' learning achievement. *International Journal on Advanced Computer Engineering and Communication Technology, 1*(2), 47-54.

MathWorks (2020). https://it.mathworks.com/products/new_products/release2020b.html (19th May 2022).

Meenakshi, N., & Pankaj, N. (2015). Application of fuzzy logic for evaluation of academic performance of students of computer application course. *International Journal for Research in Applied Science & Engineering Technology, 3*(10), 260-7.

MIUR (2010). Indicazioni nazionali per i licei. https://www.istruzione.it/alternanza/allegati/NORMATIVA%20ASL/INDICAZIONI%20NAZIONALI%20PER%20I%20LICEI.pdf (15th May 2022).

Namli, N. A., & Şenkal, O. (2018). Using the fuzzy logic in assessing the programming performance of students. *International Journal of Assessment Tools in Education, 5*(4), 701-712. https://doi.org/10.21449/ijate.429123

Ojose, B. (2011). Mathematics literacy: Are we able to put the mathematics we learn into everyday use. *Journal of mathematics education, 4*(1), 89-100.

Petroudi, S. H. J., Pirouz, M., & Pirouz, B. (2013). Application of fuzzy logic for performance evaluation of academic students. In *2013 13th Iranian Conference on Fuzzy Systems* (IFSC) (pp. 1-5). IEEE.

Putnam, R. T. (1992). Teaching the" hows" of mathematics for everyday life: A case study of a fifth-grade teacher. *The Elementary School Journal, 93*(2), 163-177.

Sivanandam, S. N., Sumathi, S., & Deepa, S. N. (2007). *Introduction to fuzzy logic using MATLAB* (Vol. 1). Berlin: Springer.

Tosuncuoglu, I. (2018). Importance of Assessment in ELT. *Journal of Education and Training Studies, 6*(9), 163-167.

Varghese, A., Kolamban, S., Shreedhar, J. P., & Nayaki, S. (2017). Outcome based assessment using fuzzy logic. *International Journal of Advanced Computer Science and Applications, 8*(1), 103-106. http://dx.doi.org/10.14569/IJACSA.2017.080115

Voskoglou, M. G. (2013). Fuzzy logic as a tool for assessing students' knowledge and skills. *Education sciences, 3*(2), 208-221. https://doi.org/10.3390/educsci3020208

Yadav, R. S., Soni, A. K., & Pal, S. (2014). A study of academic performance evaluation using Fuzzy Logic techniques. In *2014 International Conference on Computing for Sustainable Global Development* (INDIACom) (pp. 48-53). IEEE.

Zadeh, L. A. (1965). Fuzzy sets. *Information and Control, 8*(3), 338-353. https://doi.org/10.1016/S0019-9958(65)90241-X

PROVERA MATEMATIČKOG ZNANJA UČENIKA KROZ PROJEKTNI RAD I DOMAĆE ZADATKE SA FUZZI LOGIKOM

Provera znanja učenika je ključni element pedagoškog procesa jer učenicima, roditeljima i nastavnicima pruža važne povratne informacije o znanjima i veštinama učenika. Procena matematičkog znanja učenika je složena, pošto je nekoliko faktora obično uključeno u konačnu ocenu učenika, a jednostavno izračunavanje prosečnog uspeha učenika možda neće dati potpunu sliku njihovog znanja. Zbog toga smo u ovom radu želeli da istražimo mogućnost korišćenja fuzzi logike za procenu znanja i kompetencija učenika, uzimajući u obzir (1) ukupan akademski uspeh učenika, (2) kvalitet projektnog rada učenika na temu iz predmeta istorija matematike i (3) redovno izvođenje domaćih zadataka. Istraživanje je sprovedeno na uzorku od 22 italijanska srednjoškolca. Rezultati pokazuju da je akademski uspeh učenika sličan ocenama učenika dobijenim fuzzi logikom.

Ključne reči: *matematika, projektni rad, domaći zadatak, fuzzi logika*