FACTA UNIVERSITATIS
Series: **Teaching, Learning and Teacher Education** Vol. 6, Nº2, 2022, pp. 155 - 167
https://doi.org/10.22190/FUTLTE221205011A

Original research paper

STUDENTS' ACHIEVEMENTS IN PRIMARY SCHOOL MATHEMATICS ACCORDING TO THE VAN HIELE MODEL OF THE DEVELOPMENT OF GEOMETRIC THINKING

UDC 371.3: 514; 514:: 371.382; 371.26:51

Sanja Anđelković, Nela Malinović-Jovanović

Pedagogical Faculty in Vranje, University in Niš, Serbia

Abstract. *A theoretical framework that provides a foundation for understanding the process of forming geometric concepts is the van Hiele's model of the development of geometric thinking, which follows the students' progress with a hierarchically arranged series of levels characterized by an increase in abstractness. The model is aligned with the development of children's cognitive structures from an early age, when they can see objects only as a whole, and until the age when they are capable of formal, axiomatic geometric thinking. The paper presents the results of the research of student achievements in relation to the first three levels of development of geometric thinking according to van Hiele's model in primary school mathematics classes. The research was conducted on a sample of 118 primary school students in the Pčinja district. Using the theoretical analysis method, based on the description of the level (visualization, analysis and informal deduction) and the curriculum of the primary geometry lessons, appropriate criteria were set as indicators to measure if a level was reached. A knowledge test was created that contained tasks corresponding to each of the abovementioned levels, and it was used as an instrument. The results obtained indicate that students' geometric thinking, in relation to their educational stage, develops in accordance with van Hiele's levels. Thus, first-grade students reached the visualization level, a third of the second-grade students and half of the third-grade students reached the analysis level, while 16.1% of the fourth-grade students reached the informal deduction level, which was expected for children of this age.*

Key words: *van Hiele's model, geometric thinking, student achievement, primary school geometry*

Received December 05, 2022/Accepted December 31, 2022
Corresponding author: Sanja Anđelković
Pedagogical Faculty in Vranje, University of Niš, Partizanska 14, 17500 Vranje, Serbia
Phone: +381 17 422 962 • E-mail: sanjaa@pfvr.ni.ac.rs

1. Introduction

There is more and more need to understand geometry in the modern world, therefore, it is necessary to focus on a more detailed study of its understanding by students. Numerous cognitive theories contribute to the understanding of geometric thinking and mental processes that occur while forming geometric concepts. In addition to familiarizing students with geometric thinking, these theories allow teachers to adapt geometry teaching to students' abilities, in order to reach a higher level of thought as soon as possible. One of those theories is van Hiele's model, which describes the levels of geometric thinking.

The van Hiele's model of the development of geometric thinking follows the progress of students through a hierarchical sequence of five levels characterized by increasing abstraction. The model is aligned with the development of children's cognitive structures from the level when they perceive objects only as a whole, to the level when they are capable of formal, axiomatic geometric thinking. Progression through the levels does not happen with age, but largely depends on learning. "Each level is characterized by a special language, symbols and structure" (Đokić and Zeljić, 2017: 626). The levels describe how one thinks in a geometrical context, and the key difference between the levels is represented by the objects which we are able to think about geometrically. By progressing to a higher level, the object of geometric thinking changes.

1.1. Van Hiele's model of the development of geometric thinking

A significant contribution to a better understanding of the development of geometric thinking was made by the research of the Dutch married couple van Hiele. This research has resulted in the van Hiele's model of the levels of understanding geometry. This theory explained the causes of students' problems in learning geometry and offered suggestions on how to overcome them.

Van Hiele's model of the development of geometric thinking indicates five levels of children's development of geometric knowledge, from recognizing geometric shapes to the axiomatic foundation of geometry (Van Hiele, 1986). The van Hiele's marked the levels of understanding of geometry with numbers zero through four. Researchers from America later renumbered these levels and labelled them with numbers one through five, with zero indicating the level of thought at which the student does not recognize shapes at all (Ma, Lee, Lin & Wu, 2015).

In the text that follows, we list van Hiele's levels of geometric knowledge and their properties (Van Hiele, 1986; Clements & Battista, 1992; Gutiérrez, 2014; Romano, 2009; Ma et al., 2015).

1. *Visualization level*. At this level, students visually recognize geometric shapes such as triangle, quadrilateral, circle. This recognition takes place according to the rough shape of geometric figures, and not according to geometric properties. In the same way, they name geometric figures, based on their shape, and not based on their properties. The visualization level is characterized by a very simple language. This is how the expressions: "this object is shaped as...", „it looks like – doesn't look like..."are commonly used. Students are able to recognize that a given shape is a rectangle because it looks like a door, for example. If that rectangle were rotated by e.g., 45˚ students would not recognize this geometric figure. At the level of visualization, students classify figures based on overall perception, thus putting all figures that "look similar"in the same category.

2. *Analysis level*. At this level, students identify figures by their geometric properties, and learn the terms used to describe them. However, the properties are not logically arranged, so children still cannot see the basic relationships between them. For example, children do not understand that if all three sides of an equilateral triangle are of equal length, it means that all three angles must be equal. Also, children notice that all four sides of a square are of equal length and all four angles are right angles, and that the opposite sides of a rectangle are of equal length and all four angles are right angles, but they still cannot connect that a square is actually a rectangle because it has all the properties characteristic of a rectangle. Students who think at the level of analysis understand that the shape of a figure does not depend on its position and size. Thus, the child knows that, for example, the rotation of a square will not change that square. When describing a geometric figure, they list all its properties, but they cannot identify which of them are necessary and which are sufficient to describe it. They can draw conclusions inductively, based on a few examples, but still cannot use deduction.

3. *Abstraction level* (informal deduction). Geometric reasoning at this level is reflected in the generalization abilities when students begin to understand the necessary and sufficient properties to describe a geometric figure. The beginnings of deductive reasoning appear with the use of logical argumentation, but students still do not understand the meaning of formal deduction, that is, they cannot fully understand the role of definitions, axioms, theorems and proofs. Students notice the relationships between the properties of geometric figures, and based on that, the logical connections between the geometric figures themselves, which allows them to make a hierarchical classification of them. For example, they can understand that every square is a rectangle because it has all the properties of a rectangle, but also that a rectangle is not a square (Hoffer, 1983).

4. *Level of formal deduction*. At this level of geometric thinking, students understand the meaning and role of definitions, theorems, axioms and proofs within the axiomatic system. Moreover, students can independently derive proofs from previously known statements, use abstract concepts and draw conclusions relying more on logic than on intuition.

5. *The level of rigor*. At this level of thought, students can study different axiomatic systems and understand the relationships between them. Their reasoning is based on axioms, theorems and definitions. Also, students can understand indirect proofs, that is, proof by contrapositive and can understand non-Euclidean geometry.

These levels of geometric thinking occur in a hierarchical order, whereby progress through the levels is successive. At each next level, new knowledge is acquired, and in order to reach the next level, it is necessary to master the previous one. Reaching a higher level of thought is not conditioned by age or maturity, but exclusively by learning (Van Hiele, 1986). The effectiveness of learning is achieved by the active participation of students in the classroom. On the other hand, lectures, which are characterized by frontal work and passivity of students who are required to memorize the material without essential understanding, do not ensure progress towards a higher level of thought. That is why teachers should align the process of learning geometry with the level of geometric thinking of students, and to ensure that they acquire appropriate geometric experiences that will allow them to actively participate in the learning process.

"Each level of thought, apart from having a special interpretation of the same term, also has a special language" (Romano, 2009, p. 98). Language, as well as carefully selected materials, plays an important role in the development of geometric thinking (Crowley, 1987). With that in mind, the teacher must be mindful to use words that belong to the language that corresponds to the students' level of thinking, because, otherwise, they will not understand it.

Since they do not understand what is being taught to them, students will try to memorize the material. However, such material, like everything that is learned without understanding, will be quickly forgotten, and the students will not be able to apply it.

Later analysis of van Hiele theory resulted in its considerable modification over the years. Thus, Clements & Battista (1992) proposed a modification of the model in such a way as to introduce a new level that occurs before the visual level, which they called pre-recognition. Geometric thinking at this level is characterized by the fact that children can identify a subset of figures based on visual properties, but cannot recognize many common shapes and do not distinguish between figures in the same class.

Burger and Shaughnessy (1986) pointed out that the levels are not discrete, but should be seen as mutually integrated. The level of students' geometric thinking may differ depending on the topic they are studying, and some students may find themselves at the transition between the two levels. An important precondition for progressing to a higher level of thinking is the active adoption of geometric content and learning it with understanding, for which some students need more time, and some less time. Therefore, it is difficult to specify the time required to move from a lower to a higher level of thinking.

The hierarchical development of cognition in the field of geometry was also confirmed in the study by De Villers and Njisane (1987), which was conducted with high school students in South Africa. These authors singled out eight different categories of geometric thinking: recognition and representation of figure types, visual recognition of properties; use and understanding of terminology; verbal description of figures; one-step deduction; longer deduction; hierarchical classification and reading and interpreting given definitions. According to the authors' findings, the listed categories correspond to van Hiele's model of geometric thinking, namely the first two of these eight categories correspond to van Hiele's first level, the next two to the second, and the last three to van Hiele's third level. Thus, the study has supported the van Hiele model.

1.2. Indicators of achievement of students' geometric thinking level

The van Hiele's theoretical framework allows the teacher to set the boundaries within which he should implement geometric activities and adapt the instruction of geometry to the students' characteristics. That is why it is necessary for the teacher to know well the specifics of students' thinking at each individual level.

Based on the theoretical analysis of the relevant literature (Van Hiele, 1986, Crowley, 1987, Romano, 2009, De Villiers, 1987, Vlasnović and Cindrić, 2014), based on the description of the level and curriculum of the primary school geometry lessons, appropriate criteria were set as indicators to measure the achievement of students' geometric thinking level. Since in this paper we are dealing with the development of geometric thinking of students in the lower grades of primary school, indicators have been singled out only for the first three levels of geometric thinking.

At the level of visualization, the students:

- recognize geometric figures by their shape;
- they use basic names for shapes without an explanation for such naming;
- they do not know the main properties of geometric figures;
- geometric figures are experienced based on overall perception;
- put figures that look similar in the same class;
- do not recognize rotated geometric figures.

At the level of analysis, the students:
- identify and distinguish geometric figures through essential properties;
- describe geometric figures by means of all observable visual properties;
- do not recognize the relationships between geometric figures;
- use language that fits formal geometric concepts;
- recognize rotated geometric figures.

At the level of informal deduction, the students:
- understand the relationships between the properties of a specific geometric figure;
- understand and explain the relationships between different geometric figures and classify them hierarchically;
- mental connections between geometric properties are expressed by using specific definitions;
- understand the necessary and sufficient condition for providing a definition;
- “proofs” are not rigorously deductive.

1.3. Geometry-related content in curricula for lower grades of primary school

The study of geometry content in primary school mathematics classes enables the development of students' geometrical thought and intuition and prepares them for constructive-geometrical activities.

The Guidelines on the curriculum for the first grade of primary school (*Правилник о програму наставе и учења за први разред основног образовања и васпитања*, 2017) outlines the initial geometry lessons related to the perception of space, the position of objects and beings, comparing objects and beings by size and classifying them according to their common properties, noticing and naming geometric bodies and figures, but also content related to line, point and line segment, identifying and drawing straight, curved, broken, closed and open lines.

In the second grade, the concepts of line, half-line and line segment are introduced and it is continued with the drawing of curved and broken lines, where the students are introduced graphically and calculationally to the concept of the length of a broken line. The curriculum also specifies drawing rectangles, squares and triangles on a square and dotted grid, as well as calculating the volume of geometric figures. Also, second-grade students are introduced to the idea of symmetry and congruence of geometric figures on a perceptual level (*Правилник о програму наставе и учења за други разред основног образовања и васпитања*, 2018).

The study of geometry in the third grade is based on the geometry concepts already introduced. The third-grade students can identify and draw parallel lines, normal lines and intersecting lines, they learn about the concept of circle and disc, angle and types of angles. The third-grade curriculum envisages the expansion of students' knowledge of the concepts of rectangle, square and triangle, while analyzing the main properties of these geometric figures and classifying them with regard to the observed properties. Students draw a rectangle and a square, as well as a triangle and a circle, and the concept of volume (of a rectangle, square and triangle) is introduced, which is calculated using a formula (*Правилник о програму наставе и учења за трећи разред основног образовања и васпитања*, 2019).

The geometry of space is taught in the fourth grade. More precisely, students learn about the elements and properties of the cube and cube, draw grids and make models of these geometric bodies. The program envisages that students understand the connections between the properties of squares and rectangles introduced in previous grades with the properties of cubes and cuboids, and therefore also the connections that exist between these geometric figures (*Правилник о програму наставе и учења за четврти разред основног образовања и васпитања*, 2019).

By taking into account the curriculum requirements for the primary school instruction of geometry, we estimate that, from the aspect of the education age of students, the following goals should be reached:

- first and second grade – level of visualization (students aged 6.5-7.5 and 7.5-8.5, respectively),
- third grade – level of analysis (students aged 8.5-9.5 years) and
- fourth grade – level of analysis and in one part the level of informal deductions (students aged 9.5-10.5 years).

Theories on the understanding of geometric thinking, one of which is van Hiele theory, in addition to enabling teachers to learn about the development of geometric thinking in students, also enable them to adapt geometry lessons to the developmental level of students and undertake appropriate activities so that students' progress to a higher level of thinking as soon as possible.

Van Hiele's theory describes five levels of geometric thinking in which students first learn to recognize an object as a whole and then analyze its parts and essential properties. Later, students understand the relationships between shapes and their properties and can make simpler deductive conclusions. Advancement to a higher level depends primarily on learning and how geometric content is acquired. In most studies that have examined Van Hiele's theoretical framework as well as the original theory (Van Hiele, 1986), the sample has consisted of high school students. Some studies that dealt with younger children (Vlasnović and Cindrić, 2014) advocate the idea that younger students' geometric thinking is predominantly "visual." However, in order to identify the specific and original ideas that students from first to fourth grade of primary school develop in relation to geometric figures, it is necessary to analyze whether these students progress in their development of geometric thinking according to the levels of van Hiele theory. This paper presents a research aimed at determining whether students progress according to van Hiele's levels during the first four grades of primary school. This should allow teachers to better understand students' geometric thinking, which in turn can help them design instructional activities to achieve students' desired geometric competencies.

3. Methodological Framework Of Research

The main goal of the research was to analyze the academic achievements of junior primary school students in regards to the first three levels of development of geometric thinking from the van Hiele's theory.

We were interested in whether the organization of geometry classes and activities contribute to the development of students' geometric thinking and progress along the levels of the van Hiele's model.

The general objective from which we started in this research is that students' geometric thinking during the first four grades of primary school develops in line with the characteristics of van Hiele's levels of thought.

The *independent* variable of the research is the educational age of students - first, second, third and fourth grade of primary school. The *dependent* variable is the level of students' geometric thinking according to van Hiele's theory.

The descriptive method was used for the collection, analysis and interpretation of data. The theoretical analysis method was used in order to analyze the existing knowledge about the research problem, i.e., when analyzing and interpreting scientific and professional literature in order to create a theoretical framework for the research and set up an appropriate methodological framework for the research.

Data collection was done with a test, where a knowledge test for students was used as an instrument. The instrument was created for the purposes of this research. The test consisted of 15 open-ended and closed-ended questions. The assignments were classified into three groups that measure students' geometry knowledge at the first three levels according to van Hiele's theory. The test was created based on the described indicators of achievement of students' level of geometric thinking. The scoring criterion is based on the criterion from Usiskin's study (1982), according to which a student has reached the appropriate level of geometric thinking if he correctly completed at least 60% of the tasks (three, four or five tasks) from the given level and reached the previous level. The respondents solved the tasks on the test on their own.

The final version of the test was preceded by a pilot study on a sample of 74 primary school students, where it was determined that the Cronbach's alpha for each task ranges from 0.74 to 0.87. These values confirm the reliability of the designed test. Discriminative values for all tasks are greater than 0.12 and range from 0.15 to 0.24, which shows that the tasks have sufficient discriminability.

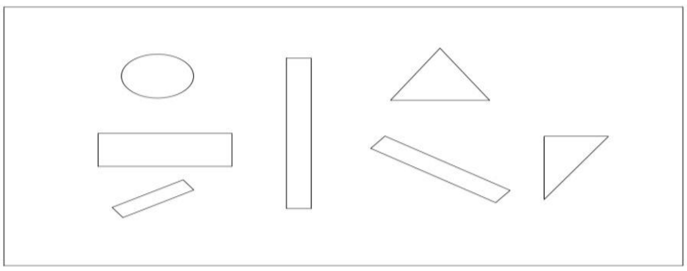
The research sample consisted of first, second, third and fourth-grade students attending primary schools in the Pčinja district, in the 2020/2021 school year.

The available/convenient sample of the reserch consisted of 118 students attending the first four grades of the primary school “Radoje Domanović” and “Jovan Jovanović Zmaj” in Vranje.

4. RESULTS – ANALYSIS AND INTERPRETATION

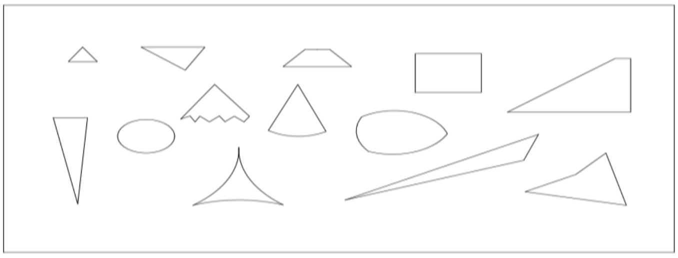
The first group of tasks was aimed at classifying students whose geometric thinking is at the level of visualization from students with higher level geometric thinking. This group consisted of five open-ended tasks where the students were asked to recognize and circle the appropriate figure among the offered figures. In addition to the figures in standard position, the group of figures also included the rotated ones (Example 1). When analyzing the answers to these tasks, we separated the students who identified only the figures in standard position from those students who circled all the figures.

Example 1 Circle all the rectangles in the picture



Within the first group of tasks, there were also examples in which, in addition to characteristic figures, there were also figures with curved, rounded or recessed sides, which generally resemble a characteristic figure (Example 2). When analyzing the answers to these tasks, we separated students who identified only characteristic figures from students who circled all figures that looked "similar" to them.

Example 2 Circle all the triangles in the picture



A number of students could not be classified in any group because we could not see any pattern in their selection of figures. These students were not even at the level of visualization and we did not consider them. Table 1 shows the students' results for the first five tasks by grade. Students who identified only the figures in their standard position and who circled all figures that visually looked "similar" are students who are at the visualization level in terms of geometric thinking. Students who correctly circled all figures are at least at the level of analysis in terms of geometric thinking, and some may be at a higher level. Among these students, there are also those who have reached the level of visualization, but in terms of understanding geometry, they are still not at the level of analysis. This means that they are at the transition between the level of visualization and the level of analysis. The answers to the first five tasks show us that the number of students, whose geometric thinking is at a higher level than the level of visualization, tends to increase, with the increase of the grade the students attend.

Table 1 Student scores for the first five test tasks

Grade	Students who do not select all the shapes correctly				Students who select all the shapes correctly			
	1 st grade	2 nd grade	3 rd grade	4 th grade	1 st grade	2 nd grade	3 rd grade	4 th grade
First task	20 (74%)	14 (48.3%)	3 (11.5%)	2 (6%)	7 (26%)	15 (51.7%)	23 (88.1%)	29 (93.6%)
Second task	22 (81.%)	12 (41.3%)	5 (19.2%)	2 (6%)	5 (18.6%)	17 (58.6%)	21 (80.8%)	29 (93.6%)
Third task	22 (81.%)	13 (44.8%)	6 (23%)	4 (13%)	5 (18.6%)	16 (55.2%)	20 (77%)	27 (87%)
Fourth task	23 (85.%)	12 (41.4%)	5 (19.2%)	1 (3%)	4 (14.8%)	17 (58.6%)	21 (80.8%)	30 (96.7%)
Fifth task	21 (78%)	12 (41.3%)	5 (19.2%)	2 (6%)	6 (22.2%)	16 (58.6%)	21 (80.8%)	29 (93.6%)

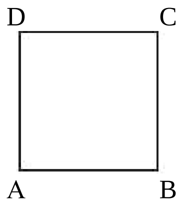
The answers indicate that the majority of students who chose ellipses (which visually resemble circles) among the offered figures for circles are actually first graders. These students circle the ellipses because they look "round" to them, which is an indicator of visual level. Third and fourth graders correctly identified all the circles.

Many first-graders and a certain number of second-graders use the term rectangle only for rectangles in a standard position, and do not identify a "rotated" square as a square. This is a clear indicator that their geometric thinking is at the visualization level.

The analysis of the answers shows that going from the first and getting to the fourth grade, the number of students who correctly circle all the triangles increases. A significant number of junior primary school students, use the shape property to circle non-convex triangles (they generally resemble a triangle in the standard position) in addition to the characteristic ones, because they look "triangular". Those students even believe that a triangle side is also the one that is "recessed", "bent" or "rounded". Therefore, we conclude that geometric thinking of these students is at the level of visualization.

The second group of tasks on the test aimed to identify the number of students who reached the level of analysis in terms of geometric thinking. This group consisted of five tasks in which the students were asked to circle among the offered geometric properties those that correspond to the given figure (Example 3). Within this group of tasks, there were also those in which students were required to identify a given geometric figure based on a picture.

Example 3 ABCD is a square. Circle the sentences that are true for all the squares.



a) a square has four right angles
b) a square has four sides
c) sides AD and BC are normal
g) sides AB and BC are not equal in length

Based on the scoring criteria we set, the student was believed to have reached the analysis level if he reached the visualization level and completed 60% of the tasks from the analysis level correctly. The achieved results of the tested students are shown in Table 2.

Table 2 Students who have reached the van Hiele level of analysis

Grade	Number of students who reached the level of analysis	
	f	%
First grade	0	0.0%
Second grade	7	24.1%
Third grade	13	46.4%
Fourth grade	25	80.0%

We notice that no first-grade student has reached the level of analysis in terms of geometric thinking, while 24.1% of second-grade students are at this level. Such results were expected, taking into account the geometry lesson plans provided by the Curriculum for the first grade of primary school. We estimate that these students could not reach the level of analysis according to van Hiele, i.e., they are required to know and understand geometry only at the level of visualization.

When it comes to the third and fourth-grade students, the data supports the initial objective, i.e., that students' progress in geometric thinking according to van Hiele's theory. A total of 80% of fourth graders at the level of analysis is a satisfactory percentage, while we expected more from third graders.

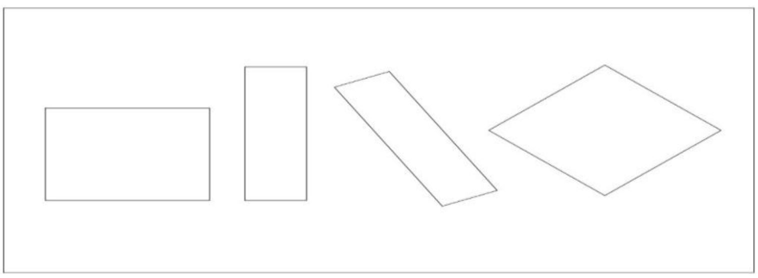
Although the third-grade mathematics course plan includes the level of analysis of triangle, square, rectangle and circle, it was observed that students were more successful in analyzing rectangle and square than analyzing circle and triangle. This leads us to the conclusion that the lessons about these geometric concepts did not offer students adequate activities that would encourage the development of their geometric thinking.

It is also interesting to note that in the ninth question, the students mostly preferred their own definitions of an isosceles triangle, which were based on a picture and listed all the properties they observed. This, in fact, is one of the indicators of understanding geometry at the level of analysis.

A certain number of students are at the transition between the level of visualization and the level of analysis. These are students who reached the lowest level, but did not complete a sufficient number of tasks from the analysis level based on the set criteria, and therefore did not reach this level of thought. This transition is not strict and it is not possible to determine the moment when a certain student moved from the level of visualization to the level of analysis.

The third group of tasks aimed to determine the number of students who have reached the level of informal deduction, but only one segment of it which is possible for them given the lesson materials provided for lower grades of primary school. These are students who have reached the level of analysis and correctly completed 60% of the tasks from the level of informal deduction. The tasks from this group helped us to find out whether students understand the relationships between the properties of geometric figures and whether they can express these observed relationships with characteristic definitions, as well as whether they understand the hierarchical relationship between geometric figures (Example 4).

Example 4 Which of these shapes is a rectangle?



A **B** **C** **D**

a) all
b) B and C only
c) only B
d) only A and D

The analysis of the answers shows that only 16.1% of students reached the level of informal deduction – all of them in the fourth grade. These students correctly state the characteristic definition of a cube. They can classify figures and conclude that rectangles are not squares, but also that squares are rectangles.

5. DISCUSSION

In this study, an analysis was conducted to determine the academic performance of elementary school students in regards to the first three developmental levels of geometric thinking according to van Hiele's theory. The results indicate that students' geometric thinking develops according to van Hiele's levels depending on their educational level. First grade students reached the visualization level, second and third grade students reached the analysis level, while few of the fourth grade students reached the informal deduction level. These results are consistent with those from previous studies (Md. Yunus, Mohd Ayub, & Hock, 2019; Wu, et al., 2015; Abdul Halim, 2013; Noraini, 2007; Ding & Jones, 2006; Gary, 2007; Abu et al., 2012; Usiskin, 1982; Wu & Ma, 2006). For example, Wu and Ma (2015, 2006), who examined the distributions of Van Hiele levels of geometric thinking among first through sixth graders, found that most students in grades 1-2 were at the visualization level, grades 3-6 were at the analysis level, no students in grades 1-4 were at the informal deduction level, and no students in grades 1-2 were at the analysis level. In the preliminary analysis of Md. Yunus, Mohd Ayub, and Hock (2019), most students appeared to be working at the visualization level and managed to reach the analysis level when learning geometry. Noraini (2007) found that a large proportion of Malaysian elementary school students were working at the lower levels of van Hiele's levels of geometric thinking. This was similar to the level of other students in countries such as the United States, United Kingdom, China, and Taiwan (Md. Yunus, et al., 2019).

Accordingly, the results of this study are consistent with other research findings and highlight the importance of students' understanding of geometric concepts and development of geometric thinking.

6. CONCLUSIONS

The general conclusion obtained by this research is that, according to van Hiele's classification, level 0 is the most frequent one among first-grade primary school students. A third of the second, and a half of the third-grade students are at the level of analysis, while only 16.1% of the fourth-grade students reached the level of informal deduction in only one segment of it which is possible for them given the lesson materials provided for lower grades of primary school. This means that as they progress through the grades, they also advance in the development of geometric thinking through the levels described by van Hiele's theory.

We find small deviations among the third-grade students, which leads us to the conclusion that geometry lessons may not have offered students adequate activities that would encourage the development of their geometric thinking. We found that for a certain number of students, their geometric thinking is at the transition between two levels. This transition is gradual and partial, and it is very difficult to recognize the moment of transition from one level to another.

The results confirm our initial objective that students in lower grades of primary school progress over time through the levels of geometric thinking. The only question is whether a sufficient number of students are progressing, given that a certain number of students have not even reached the basic level of geometric thinking.

As for the answer to the question: Why is that so? We can answer that this is probably due to the fact that geometry learning material is insufficiently represented in the curricula, insufficient attention is paid to these contents in the textbooks, and so in general, geometry instruction is not given much importance. Another reason is that the instruction process is more oriented towards memorizing the material, and less towards the understanding of geometric concepts and the development of students' geometric thinking.

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POSTIGNUĆA UČENIKA U POČETNOJ NASTAVI MATEMATIKE PREMA VAN HILEOVOJ TEORIJI RAZVOJA GEOMETRIJSKOG MIŠLJENJA

Jedan od teorijskih okvira koji pruža osnovu za razumevanje procesa formiranja geometrijskih pojmova jeste Van Hileova teorija razvoja geometrijskog mišljenja koja napredak učenika prati kroz hijerarhijski uređen niz nivoa koje karakteriše porast apstraktnosti. Model je usklađen sa razvojem kognitivnih struktura dece od ranog uzrasta, kada su u mogućnosti da objekte sagledavaju samo kao celinu, pa do uzrasta kada su sposobni za formalno, aksiomatsko zasnivanje geometrije. U radu su prikazani rezultati istraživanja postignuća učenika u odnosu na prva tri nivoa razvoja geometrijskog mišljenja prema Van Hileovoj teoriji u početnoj nastavi matematike. Istraživanje je sprovedeno na uzorku od 118 učenika osnovnih škola u Pčinjskom okrugu. Metodom teorijske analize, na osnovu opisa nivoa (vizuelizacija, analiza i neformalna dedukcija) i programskih sadržaja početne nastave geometrije, postavljeni su odgovarajući kriterijumi kao indikatori za merenje postignutosti nivoa. Kao instrument, konstruisan je test znanja koji je sadržao zadatke koji odgovaraju svakom od navedenih nivoa. Dobijeni rezultati ukazuju na to da se geometrijsko mišljenje učenika, u odnosu na njihov obrazovni uzrast, razvija u skladu sa Van Hileovim nivima. Tako su učenici prvog razreda postigli nivo vizuelizacije, trećina učenika drugog i polovina učenika trećeg razreda postigla je nivo analize, dok je 16,1% učenika četvrtog razreda dostiglo nivo neformalne dedukcije što je bilo očekivano kada su u pitanju deca ovog uzrasta.

Ključne reči: *Van Hileova teorija, geometrijsko mišljenje, postignuća učenika, početna nastava geometrije*