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THE EFFECTS OF DISCOVERY-BASED LEARNING OF DIFFERENTIATED ALGEBRA CONTENT ON THE LONG-TERM KNOWLEDGE OF STUDENTS IN EARLY MATHEMATICS EDUCATION

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Abstract. *One of the essential characteristics of knowledge that determines its quality is its durability. The durability of students' knowledge depends, among other things, on the quality of organization and implementation of teaching in the process of acquiring new knowledge, as well as reviewing and repeating old knowledge. More specifically, the durability of students' knowledge depends on the applied methodological approach and the students' activities in the classroom. The aim of this study was to examine the effects of discovery-based learning on differentiated algebra content on the long-term knowledge of students in early mathematics education. To achieve this goal, an experiment was conducted with parallel groups consisting of a sample of 261 fourth-grade students from primary schools. The experiment aimed to investigate whether the methodological approach to algebra instruction based on the principles of discovery learning and content differentiation yields better effects on the durability of students' knowledge compared to traditional learning methods. The results of the research showed that discovery-based learning on differentiated algebra content contributes to better knowledge durability overall and at each of the three achievement levels (basic, intermediate, and advanced).*

Key words: *knowledge durability, algebra instruction, discovery-based learning, content differentiation*

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1. INTRODUCTION

The accelerated development of science and technology in the 21st century has led to the emergence of the concept of a "knowledge society" or a "learning society," which has further driven global changes in the educational system with the aim of transforming traditional teaching methods to equip students with the necessary skills and abilities to successfully face the challenges of modern society (Mirkov, 2011). In traditional education, frontal instruction is the prevalent one, with the focus on the teacher's lecturing role, which reduces interaction between the teacher and students and leaves little room for independent student activities. This mode of instruction inadequately stimulates students to participate in problem-solving and is not effective in preparing students for lifelong learning (Bognar and Matijević, 2002). Another shortcoming of traditional instruction today is its uniformity. This means that "regardless of individual differences, students of the same age should master the same curriculum objectives, acquire knowledge of the same extent and intensity, engage in tasks of equal difficulty, and reason and conclude in the same way. In other words, they should progress at the same or approximately the same pace" (Milovanović, 2008, p. 470). "As a consequence of such a methodological approach, students who are below or above average are left aside because their active participation in teaching is hindered by instruction that is not adapted to their needs" (Janković, 2016, pp. 269-270).

On the other hand, there is an emphasis on the importance of active learning in education today, viewing the student as an active participant in the learning and teaching process. In such circumstances, "the student is a carrier, initiator, critic, researcher, interpreter. However, the student is not only the carrier of teaching but also its goal, which is why teaching is adapted to the needs and abilities of students in order to achieve their self-realization" (Stevanović, 2002, p. 25).

"Throughout the history of education, various types, teaching strategies, organizational models, and conceptual solutions have emerged in an effort to create conditions in which teaching would be more aligned with the needs, interests, possibilities, and abilities of students" (Maričić and Milinković, 2015, p. 63). Didactic literature emphasizes that different forms of active learning (teaching), including discovery-based learning, support different learning styles and students' abilities. In these forms of learning, differentiation is achieved by allowing students to access content at different levels.

When it comes to mathematics instruction, researchers worldwide (Kieran, 1981, 2004; Filloy & Rojano, 1989; Sfard & Linchevski, 1994; Knuth, Stephens, McNeil, and Alibali, 2006; Vergnaud, 1988, cited in Carraher, Schliemann, Brizuela & Earnest, 2006, and others) have confirmed that young students face numerous difficulties in learning algebra. These difficulties arise from the abstract nature of the content itself and the limited cognitive abilities of younger students. This implies the necessity of changing the approach to teaching these contents and finding suitable instructional methods for successful learning of basic algebraic concepts. Considering that students acquire the highest quality knowledge through independent activities, where the content and learning requirements are adapted to their individual capabilities, we have devised a methodological approach based on functional connection through discovery learning and content differentiation in the early algebra instruction. Differentiation of program content has been performed at three levels of complexity.

Since the durability of students' knowledge significantly determines the quality of acquired knowledge, the aim of this study is to evaluate, through experimental means, the effects of the aforementioned methodological approach on the durability of students' knowledge in the early algebra instruction. This research represents a part of a comprehensive empirical study in which various effects of discovery-based instruction with differentiated content were examined.

The research should demonstrate whether the implementation of this methodological instructional organization in algebra teaching can make acquired knowledge more durable compared to knowledge acquired through conventional instructional methods. This can provide a positive impetus for teachers to increasingly engage in instruction organized in this manner.

1.1. Discovery-based learning with differentiated content used in algebra instruction in elementary mathematics education

The content in the field of algebra constitutes an important part of the curriculum in the teaching and learning of elementary school mathematics. At this age, special attention should be given to the proper formation of early algebraic concepts. However, it is important to consider the limitations associated with algebraic content, arising from its abstract nature on the one hand, and the cognitive limitations of young students in terms of symbolic thinking, representation, and acquisition of algebraic content on the other hand. Researchers worldwide have identified numerous difficulties that students face in understanding and mastering algebraic content at a younger age. Some of the most common problems include: misunderstanding the meaning of letters representing unknowns or variables, limited interpretation of the equals sign, difficulties in understanding equations and inequalities and comprehending the procedures for solving them, and challenges in developing the idea of functions.

The aforementioned points lead us to the conclusion that besides introducing algebraic content into the curriculum, it is necessary to find adequate approaches for students to acquire this content. Therefore, there has been increased interest among researchers and theorists in studying the problem of methodological approaches to aligning algebraic content with children's cognitive abilities.

The passivity of students in the process of learning algebraic content, which involves mere memorization of formulas without a genuine understanding of the concepts being learned, prevents students from applying what they have learned and significantly diminishes their interest in mathematics. Since one of the primary intentions of mathematics education is to enable and encourage students to actively participate in the learning process, to explore independently, discover, mentally grasp content, and apply what they have learned in practice and everyday life, discovery-based learning should hold a significant place in mathematics education, particularly in the learning of algebra.

Discovery learning is commonly understood as a form of learning in which students independently discover the content they need to acquire. The process of discovery is typically not completely independent but guided by the teacher through "Socratic questioning" or other techniques because it is not expected for the student to fully discover scientific concepts that would require years of scientific work (Hammer, 1997). Discovery learning involves teachers creating instructional situations that allow students to assume the role of scientist-researchers and satisfy their (natural) curiosity by actively constructing mental models that adequately explain their experiences (Driver et al., 2000, as cited in Kalathaki, 2015). This form of learning stems from constructivist learning theories and is defined as an interactive process in which the student, as a subject, learns with understanding, aiming to enhance and transform existing knowledge, feelings, attitudes, and meaning (Gazibara, 2018).

During discovery learning, students are fully engaged in the instruction process, motivated to explore and present their ideas. The role of the teacher in discovery-based instruction is to create problem situations for students and familiarize them with facts, conditions, and examples that illustrate the concepts and principles to be discovered. The teacher guides students to independently generate ideas and concepts within the instructional content and apply them to problem-solving. "The activities of students in discovery-based instruction resemble the research activities undertaken by experts in science" (Kistian et al., 2017, p. 9). In this way, the student is in a position to independently perceive the connections between elements of the problem situation, between known and unknown quantities, gradually transferring them to the realm of symbolism, all the while understanding them. Throughout the process of discovery, the dominant aspect is the student's activity and their creative and exploratory act, as opposed to passive reception of information, receiving ready-made rules, symbolic generalizations, and so on. The student can grasp rules and generalizations only if they have integrated all the parts into a whole, firmly connected them, and understood them well.

The role of the teacher in the process of discovery learning in mathematics instruction is significant. Among other things, the teacher is expected to possess the ability to create instructional situations that encourage students to be active and creative, increasing their motivation for learning (Kistian et al., 2017). The teacher's role as a source of information is considerably reduced compared to the role of a teacher who guides and directs students on the path of discovery of new knowledge.

Discovery learning in algebra instruction must be well-prepared and tailored to the current abilities and capacities of the students. Simply presenting mathematical problems and encouraging students to independently seek solutions will not necessarily lead to learning (Garelick, 2009). Garelick emphasizes that problem tasks that are not adapted to the students' abilities do not lead to the acquisition of active and applicable knowledge. The author emphasizes that in mathematics instruction based on discovery, tasks must be well-formulated, not confusing to students, and selected in a way that gradually increases their difficulty, allowing students to use their knowledge from simpler tasks to solve more complex ones. According to this author's understanding, this structured discovery-based instruction plays a significant role in development and education as it provides the necessary support to the student (Garelick, 2009).

The studies conducted by author Malešević has shown that discovery-based learning in primary mathematics education has the following positive effects: "this form of learning is stimulating because it emphasizes the learning goal at the beginning, thus activating students' motivation; students grasp the meaning of the content; knowledge is more lasting, of higher quality, and transferable; it develops proper reasoning; students become capable of self-education and develop according to their individual abilities; learning becomes part of internal motivation, learning for its own sake rather than for grades; students develop self-awareness of their abilities" (Malešević, 2011, p. 10).

On the other hand, it is known that in practice there are numerous individual differences among students in terms of prior knowledge, mathematical abilities, motivation for learning, interests, and so on. Therefore, it is believed that optimal results in mathematics education, including learning algebra, can be achieved through discovery-based learning that is differentiated according to students' level of knowledge, abilities, and interests.

Furthermore, the generality, abstractness, and symbolism characteristic of algebraic content create the need for differentiated approaches, where content and learning requirements are adapted to students' individual capabilities. Differentiating algebraic content allows these contents to be presented at different levels of complexity to different categories of students. This creates conditions for active student participation in tailored instruction, where they explore independently, mentally master the content, and actively apply discovered solutions. Hence, the concept of teaching based on respecting individual characteristics and differences among students, as well as activating students in the learning process through discovery, provides a good foundation for more effective acquisition of algebraic content.

The differentiation of program content for the purposes of this research was performed at three levels of complexity, based on prescribed educational achievement standards for the end of the first cycle of compulsory education for the subject Mathematics (*General Achievement Standards - Educational Standards for the End of the First Cycle of Compulsory Education - Mathematics, 2011*). Through analysis of these standards, it was determined that they do not define outcomes for all three cognitive levels for certain algebraic contents. Therefore, operationalization of requirements was carried out, and minimum, optimal, and maximum demands for algebraic content specified in the curriculum for teaching and learning mathematics in the fourth grade of primary school were precisely determined for the following areas: *Dependency of results of arithmetic operations on component changes, Equations, Inequalities, Expressions with variables*. Thus, for each of the mentioned algebraic contents, outcomes were precisely operationalized at three levels of achievement.

Differentiated instruction, as well as discovery-based learning, have their foundation in the constructivist theory of learning, in which the student is seen as an active creator of their knowledge. Respect for the prior knowledge and experiences of the student, as well as the understanding of the teacher's role as a facilitator in the learning process, are common characteristics of differentiation and discovery-based learning. Discovery-based learning provides a good context for the differentiation and individualization of teaching and learning. Educational literature emphasizes that different forms of active learning, including discovery-based learning, support different learning styles and student abilities. In these forms of learning, differentiation is achieved by enabling students to access content at different levels.

Advocates of discovery-based learning believe that students fully understand only those contents that they have discovered themselves, and that the essential characteristic of this type of learning is the mental mastery of educational content. The early mathematics instruction should be organized in such a way that the student acquires knowledge through constant discovery of concepts, rules, properties, etc. "Therefore, discovery-based learning should be present in the implementation of the entire mathematics curriculum (at all levels of mathematical education), regardless of the form of teaching used and regardless of the type of class applied" (Vuković, 1998, p. 206).

However, as already emphasized, in teaching practice, not all students can solve all tasks simultaneously. In such conditions, individual characteristics of students, their prior knowledge, and pace of work come to the forefront. Therefore, it is advisable to create learning material based on levels of complexity and difficulty so that each student can respond to part of the requirements. In this regard, discovery-based learning is most productive in individualized and differentiated teaching through prepared written materials where the requirements are differentiated according to levels of complexity. Despite the fact that in this type of teaching the student learns on their own, they are guided by the teacher throughout the entire learning process, using specially prepared written materials (programmed, semi-programmed, problem-based workbooks, etc.) and orally, following a secure path. This way, students gain a clear insight into the structure of the content and acquire knowledge characterized by functionality, applicability, and operability (Vuković, 1998).

Considering all of this, discovery-based learning should have a dominant role in the implementation of the model of differentiated mathematics instruction. Considering that independent student activity in the learning process is crucial for acquiring the highest quality knowledge, and that the effects of this learning are significantly better when the content and learning requirements are adapted to individual student capabilities, the basis of this work is a methodical approach based on the idea of functionally connecting discovery-based learning and three-level content differentiation in the acquisition of algebraic content.

2. Methodological Framework Of Research

It is a well-known fact that along with the process of learning a subject, there is also a process of forgetting it. Among the numerous factors influencing the speed of forgetting learned content is the methodological approach applied during the learning process (Vučić, 1991). Therefore, students will be more motivated and interested in the content they learn, and consequently, their knowledge will be more enduring if teachers provide optimal conditions in which students are mentally engaged, create teaching situations that require a certain intellectual effort from students, and enable them to discover connections between given data coherently. On the other hand, the primary indicator of the effectiveness and success of teaching is the permanence of acquired knowledge, skills, and habits, and one of the primary teaching principles is the principle of knowledge, skills, and habits' permanence.

Since the basis of this study is a methodical procedure based on a differentiated approach that allows all students to actively acquire knowledge through the process of independent discovery, we were interested in whether such organization of teaching and activities in the process of learning algebraic content contributes to the acquisition of student knowledge that withstands the process of forgetting for a longer period.

The main objective of the research is to examine the impact of discovery-based learning of differentiated algebraic content on the permanence of students' knowledge in the early mathematics education. Additionally, we wanted to investigate whether the applied methodological approach in learning algebraic content contributes to better knowledge permanence at each level of learning achievement (basic, intermediate, and advanced).

The general hypothesis from which we proceeded in this research is that discovery-based learning of differentiated algebraic content contributes to an increase in the permanence of students' knowledge in the early mathematics education.

The research used the experimental method. We employed the experimental method in the form of an experiment with parallel groups. By introducing the experimental variable, discovery-based learning of differentiated content, into the experimental group, we aimed to determine its effects on the permanence of students' knowledge in algebra instruction. For this purpose, we formed two groups: the experimental group, in which algebraic content was taught with the created experimental lesson models, and the control group, in which the mentioned instructional content was implemented in the usual manner. The experimental program was implemented over the course of 30 regular mathematics classes. Students in the experimental group actively learned algebraic content through the process of independent discovery using teaching worksheets with differentiated requirements at three levels of complexity. Due to the impossibility of equalizing the examined groups by transferring students from one class to another, the dependent variable was statistically controlled using the analysis of covariance.

Data collection was conducted using the testing technique, with knowledge tests as instruments for students (initial, final, and retest) created by the researchers for the purposes of this research. The participants individually solved assignments on the test. The initial knowledge test was administered to both groups of participants before the start of the experimental program to determine students' prior knowledge of algebra. The final knowledge test was conducted after the implementation of the experimental program in both groups of participants to determine the effects of learning through discovery on differentiated algebraic content in algebra instruction. The repeated test or retest to assess the effects of the experimental program on knowledge permanence was conducted three months after the final testing. The tests consisted of three subtests (for each level of achievement), with six tasks each, totaling 18 tasks. The tasks were scored based on their difficulty (with 4, 5, or 6 points, depending on the task's complexity level), and the maximum number of points that could be obtained by correctly solving the tasks was 90.

A mini-pilot study preceded the final version of the knowledge tests to determine the appropriate metric characteristics. The metric characteristics of the tests were examined on a sample of 102 fourth-grade students from the "Jovan Jovanović Zmaj" Elementary School in Vranje. The calculated values of the Cronbach's alpha coefficient for each task (on all three tests) ranged from 0.72 to 0.86, indicating the reliability of the constructed tests. All tasks (on all three tests) were sufficiently discriminative, as confirmed by the task discriminative values greater than 0.12, ranging from 0.17 to 0.25.

The research sample was selected from the population of fourth-grade students in elementary schools in the Pčinja District, consisting of 261 students from the "Radoje Domanović" and "Jovan Jovanović Zmaj" Elementary Schools in Vranje. The sample structure is presented in Table 1.

Table 1 The sample structure according to the group affiliation of the participants

Experimental group				Control group			
School	Class	N	%	School	Class	N	%
“Radoje	IV/1	24	9.2	“Radoje	IV/3	25	9.6
Domanović”	IV/2	27	10.3	Domanović”	IV/4	27	10.3
“Vuk Karadžić”	IV/3	26	10	“Vuk Karadžić”	IV/1	25	9.6
	IV/5	26	10		IV/2	26	10
	IV/6	29	11.1		IV/4	26	10
Total		132	50.6	Total		129	49.4

The data obtained in the research were statistically analyzed using the IBM SPSS Statistics 21 software package.

3. The Research Results And Discussion

Before the start of the experimental program, an initial testing was conducted, which determined that there were no differences in prior knowledge of algebraic content between the students in the experimental and control groups. Immediately after the implementation of the experimental program, a final testing was conducted, followed by a retesting of the students from both groups three months after the completion of the experimental program in order to determine the effects of the experimental model on the durability of students' knowledge. These effects were initially observed through the overall score of the students on the retest.

Table 2 provides an overview of the composite score on the final test and retest for the experimental and control groups, as well as for all participants together. It can be observed that students in the experimental group achieved significantly higher average scores (M = 51.42, SD = 22.87) on the final measurement compared to the students in the control group (M = 41.87, SD = 19.16). The retesting of the students from both groups shows a decline in performance compared to the final testing, which is expected given that the retesting was conducted three months after the completion of the experimental program, resulting in some forgetting of the learned material. However, when comparing the results between the groups, the experimental group outperforms the control group.

Table 2 The performance of students in the experimental (E) and control (C) groups on the final test and retest - descriptive statistics

		N	M	SD	Std. Error	95% Confidence interval	
						Lower bound	Upper bound
Final test	E-group	132	51.42	22.87	1.99	47.49	55.36
	K- group	129	41.87	19.16	1.69	38.53	45.21
	Total	261	46.70	21.62	1.34	44.07	49.34
Retest	E- group	132	49.848	22.597	1.967	45.958	53.739
	K- group	129	38.938	18.899	1.664	35.646	42.230
	Total	261	44.456	21.517	1.332	41.833	47.079

The determined value of variance (F (1.259) = 17.862; p = 0.000) indicates that there is a statistically significant difference between the students in the experimental and control groups in the average achievement on the retest as a whole (Table 3). More specifically, the students in the experimental group achieved significantly better results compared to the students in the control group even three months after the implementation of the experimental program, which supports the confirmation of the initial assumption

Table 3 The difference in overall achievement between the experimental and control groups on the final test and retest (ANOVA)

		Sum of Squares	df	Mean Square	F	p
Final test	Between groups	5 957.688	1	5 957.688		
	Within a group	115 535.002	259	446.081	13.356	0.000
	Total	121 492.690	260			
Retest	Between groups	7 766.270	1	7 766.270		
	Within a group	112 608.474	259	434.782	17.862	0.000
	Total	120 374.743	260			

regarding the durability of knowledge among students in the experimental group under the influence of the experimental factor.

In order to confirm that the statistical significance between the experimental and control groups on the retest is a result of the implemented experimental program and not a result of group differences, we conducted an analysis of covariance (ANCOVA). The results of the initial testing were used as a covariate in both groups. The covariate was measured before the implementation of the experimental program, and the value of the Cronbach's alpha coefficient of 0.79 (Table 4) indicates sufficient reliability.

Table 4 Cronbach's alpha coefficient for the initial test

Reliability Statistics		
Cronbach's Alpha	Cronbach's Alpha Based on Standardized Items	N of Items
0.79	0.77	18

Table 5 The difference between the experimental and control groups on the retest (ANCOVA test)

Source	Type III Sum of Squares	df	Mean Square	F	p	Partial Eta Squared
Corrected Model	112 209.405[a]	2	56 104.703	1 772.739	0.000	0.932
Intercept	259.500	1	259.500	8.199	0.005	0.031
Inici_total	104 443.136	1	104 443.136	3 300.087	0.000	0.927
Group	8 073.332	1	8 073.332	255.093	0.000	0.497
Error	8 165.338	258	31.649			
Total	636 197.000	261				
Corrected Total	120 374.743	260				

a. R Squared = 0.932 (Adjusted R Squared = 0.931)

The calculated value of covariance (F (1.258) = 255.093; p = 0.000) confirms that the difference between the experimental and control groups on the retest is statistically significant (Table 5). This indicates that the observed difference between the groups is a result of the applied methodological model rather than group disparities. The value of partial eta squared (0.497) confirms a substantial impact of discovery-based learning on differentiated content. This means that even three months after the implementation of the experimental program, 49.7% of the variance in the retest scores can be explained by the independent variable's influence.

Previous research studies examining the effects of discovery-based learning (Minner, Levy & Century, 2010; Balim, 2009; Malesević, 2003) and differentiation in teaching (Vulović, 2011) on student achievement have highlighted the durability of knowledge as one of the main advantages of such instructional approaches. Our findings are in line with these results. This outcome can be explained by the fact that in this approach, concepts that need to be learned are not presented in their final form; instead, students discover and apply them independently in new situations. Therefore, the acquired knowledge is based on construction, making it more meaningful and enduring for students.

To examine the effects of the experimental program on the durability of student knowledge, we also analyzed the results on individual subtests of the retest. These subtests were structured into three levels of complexity, corresponding to different levels of students' educational achievements: basic, intermediate, and advanced.

Descriptive indicators of student success in solving tasks at the three levels of achievement on the retest are presented in Table 6.

Table 6 The results of the students in the experimental and control groups on the retest at each of the three levels of achievement (descriptive indicators)

		N	M	SD	Std. Error	95% Confidence interval		Min	Max
						Lower bound	Upper bound		
Basic level	Experimental	132	20.67	4.74	0.41	19.85	21.48	8.00	24.00
	Control	129	18.45	4.98	0.44	17.58	19.32	0.00	24.00
	Total	261	19.57	4.98	0.31	18.96	20.18	0.00	24.00
Intermediate level	Experimental	132	19.81	9.34	0.81	18.20	21.42	0.00	30.00
	Control	129	16.40	10.04	0.88	14.65	18.14	0.00	30.00
	Total	261	18.12	9.82	0.61	16.93	19.32	0.00	30.00
Advanced level	Experimental	132	9.36	11.66	1.01	7.36	11.37	0.00	36.00
	Control	129	4.09	6.67	0.59	2.93	5.25	0.00	24.00
	Total	261	6.76	9.86	0.61	5.56	7.96	0.00	36.00

The average achievement of students in the control group at the basic level of the retest is M = 18.45; SD = 4.98, while the average achievement of students in the experimental group is higher, with M = 20.67; SD = 4.74. At the intermediate level of the retest, students in the control group achieved an average of M = 16.40; SD = 10.04, while students in the experimental group achieved an average of M = 19.81; SD = 9.34 points. The average achievement of students in the control group at the advanced level of the retest is M = 4.09; SD = 6.67, while students in the experimental group achieved a higher average score of M = 9.36; SD = 11.66. Looking at the differences in the average number of points achieved between the E and K groups on the three levels of achievement, we can conclude that the mentioned difference is largest at the advanced level of achievement.

By conducting an analysis of variance on different groups, the effect of the implemented instructional approach on the durability of students' knowledge at the three levels of achievement was examined.

Table 7 The difference between the experimental and control groups at the three levels of achievement (*ANOVA test)*

		Sum of Squares	df	Mean Square	F	p
Basic level	Between Groups	320.683	1	320.683	13.555	***0.000***
	Within Groups	6 127.256	259	23.657		
	Total	6 447.939	260			
Intermediate level	Between Groups	760.974	1	760.974	8.104	***0.005***
	Within Groups	24 319.102	259	93.896		
	Total	25 080.077	260			
Advanced level	Between Groups	1 812.364	1	1 812.364	19.984	***0.000***
	Within Groups	23 489.429	259	90.693		
	Total	25 301.793	260			

The experimental and control groups significantly differ in achievement at the basic level of the retest ($F(1.259) = 13.555$, $p = 0.000$), with the experimental group outperforming the control group. Statistically better performance of the experimental group compared to the control group was also observed at the intermediate level of the retest ($F(1.259) = 8.104$, $p = 0.005$). The experimental group achieved better results even at the advanced level of the retest compared to the control group ($F(1.259) = 19.984$, $p = 0.000$).

Summarizing the effects of discovery learning on the long-term knowledge retention of students, it can be concluded that there are differences between the experimental and control groups at all levels of the retest, three months after the implementation of the experimental program. The difference lies in the longer retention of acquired knowledge in the experimental group. The long-term knowledge retention of the experimental group was achieved through their independent discovery activities during the processing of algebraic content, where they actively reached understanding through their own effort. Additionally, well-designed practice and reinforcement activities, including tasks beyond their achievement level, contributed to the long-term retention of knowledge in the experimental group. Furthermore, the tasks within the experimental program that required the application of acquired knowledge in real-life situations also contributed to the longevity of knowledge retention in the experimental group. These findings align with literature indicating that "the longevity of acquired knowledge depends not only on the organization of the lesson but also on the type of knowledge that the student acquires" (Trnavac and Đorđević, 1998, p. 64).

4. CONCLUSIONS

The previously presented and analyzed research results indicate that the teaching of algebraic content through discovery learning using differentiated materials in the experimental group has an impact on better student achievement even after a certain period of time. In other words, the results suggest that the experimental model contributes to the acquisition of more enduring knowledge at all three levels of achievement compared to the traditional model of learning. This means that if the methodological approach to acquiring algebraic content is based on receptive acquisition of content, i.e., the teacher's direct instruction and memorization by students with or without understanding, such acquired knowledge is not resistant to the process of forgetting. On the other hand, independent discovery of concepts, rules, etc., accompanied by intensive mental activation, contributes to all students, even those with the weakest performance, acquiring more enduring knowledge.

Considering the long-term effects of this methodological model, it is necessary to apply it more frequently in the implementation of elementary mathematics content. For this purpose, teachers need to be continuously informed and equipped with ways to conceptualize this didactic instruction model. In doing so, they will not only become implementers of education but also successful creators and organizers. Teachers who participated in this research willingly embraced and positively evaluated the new approach to work, thereby assuming a new role in the instructional process, which implies replacing the teacher's lecturing function with an instructional one.

REFERENCES

Balim, A., G. (2009). The Effects of Discovery Learning on Students' Success and Inquiry Learning Skills. *Eurasian Journal of Educational Research, 35*, 1-20.

Bognar, L., i Matijević, M. (2002). *Didaktika*. Zagreb: Školska knjiga.

Brizuela, B. M., & Schliemann, A. D. (2003). Fourth Graders Solving Equations. International Group for the Psychology of Mathematics Education, 2, 137-144. https://files.eric.ed.gov/fulltext/ED500917.pdf

Blanton, L. M., and Kaput, J. J. (2011). *Functional Thinking as a Route Into Algebra in the Elementary Grades*. In: J. Chai, & E. Knuth (Eds.): Early Algebraization, Advances in Mathematics Education (pp. 5-23). Berlin: Springer.

Blanco, L. J., & Garrote, M. (2007). Difficulities in learning inequalities in students of the first year of pre-university education in Spain. *Eurasia Journal of Mathematics Science and Technology Education, 3*, 221-229. https://www.ejmste.com/download/difficulties-in-learning-inequalitiesin-students-of-the-first-year-ofpre-university-education-in-4070.pdf

Carracher, W. D., Schliemann, D. A., Brizuela, M. B., & Earnest, D. (2006). Arithmetic and Algebra in Early Mathematics Education. *Journal for Research in Mathematics Education, 37*(2), 87-115. https://www.researchgate.net/publication/298917525_Arithmetic_and_algebra_in_early_mathematics_education

Carraher, D. W., Schliemann, A. D., & Brizuela, B. (2000). Early algebra, early arithmetic: Treating operations as functions. Plenary address presented at the Twenty-second Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. Tucson: Arizona.

Carraher, D. W., Schliemann, A. D., & Schwartz, J. (2008). Early algebra is not the same as algebra early. In J. Kaput, D. Carraher, & M. Blanton (Eds.), *Algebra in the Early Grades*. (235–272). Mahwah, NJ: Lawrence Erlbaum Associates/Taylor & Francis Group and National Council of Teachers of Mathematics.

Carpenter, T. P., & Levi, L. (2000). *Developing conceptions of algebraic reasoning in the primary grades*. Madison, WI: National Center for Improving Student Learning and Achievement in Mathematics and Science.

Filloy, E., & Rojano, T. (1989). Solving equations: The transition from arithmetic to algebra. *For the learning of mathematics, 9*(2), 19–25.

Garelick, B. (2009). Discovery learning in math: Exercises versus problems. *Nonpartisan Education Review / Essays, 5*(2), 1-17. https://nonpartisaneducation.org/Review/Essays/v5n2.htm

Gazibara, S. (2018). Constructivist active learning environments from the students'perspective. In *5th International Multidisciplinary Scientific Conference on Social Sciences and Arts SGEM 2018* (pp. 183-190). Albena, Bugarska, 26. 8 – 1. 9. 2018. https://www.researchgate.net/publication/327797978_Constructivist_Active_Learning_Environments_from_the_Students'_Perspective

Hammer, D. (1997). Discovery learning and discovery teaching. *Cognition and instruction,*15 (4), 485–529.

Јанковић, С. (2016). Индивидуализација наставе математике применом проблемске наставе. *Методичка пракса, 13*(3-4), 269-282.

Kalathaki, M. (2015). Evaluation Tool for the Application of Discovery Teaching Method in the Greek Environmental School Projects. *World Journal of Education, 5*(2), 40-51. https://files.eric.ed.gov/fulltext/EJ1158417.pdf

Kieran, C. (1981). Concepts associated with the equality symbol. *Educational Studies in Mathematics, 12*, 317-326.

Kieran, C. (2004). *The Equation / Inequality Connection in Constructing Meaning for Inequality Situations*. In M. J. Høines, A. B. Fuglestad (Eds.), *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education* (pp. 143–147). Bergen: Bergen University College.

Kistian, A., Armanto, D., & Sudrajat, A. (2017). The Effect Of Discovery Learning Method On The Math Learning Of The V Sdn 18 Students Of Banda Aceh, Indonesia. *British Journal of Education, 5*(11), 1-11. http://www.eajournals.org/wp-content/uploads/The-Effect-of-Discovery-Learning-Method-on-the-Math-Learning-of-the-V-Sdn-18-Students-of-Banda-Aceh-Indonesia.pdf

Knuth, E., Stephens, A., McNeil, N., & Alibali, M. W. (2006). Does Understanding the Equal Sign Matter? Evidence from Solving Equations. *Journal for Research in Mathematics Education, 37*(4), 297-312. https://www.researchgate.net/publication/234007126_Does_understanding_the_equal_sign_matter_Evidence_from_solving_equations

Küchemann, D. (1978). Children's understanding of numerical variables. *Mathematics in school, 7*(4), 23-26.

Маричић, С., и Милинковић, Н. (2015). Диференцирана настава и ученици потенцијално даровити за математику. У А. Михајловић (ур.), *Зборник радова са III међународног научног скупа Методички основи наставе математике III* (pp. 61-74). Јагодина: Педагошки факултет.

Малешевић, Д. (2011). *Моделовање садржаја наставе математике за учење откривањем у разредној настави* [Необјављена докторска дисертација]. Врање: Учитељски факултет у Врању.

Малешевић, Д. (2003). *Утицај учења откривањем на развој продуктивног мишљења ученика у почетној настави математике* [Необјављена магистарска теза]. Врање: Учитељски факултет у Врању.

Mirkov, S. (2011). Konstruktivistička paradigma i obrazovanje za društvo znanja: progresivni diskurs u nastavi. U *Tehnologija, informatika i obrazovanje za društvo učenja i znanja. 6. međunarodni simpozijum*, (pp. 3-5). Čačak: Tehnički fakultet. http://www.ftn.kg.ac.rs/konferencije/tio6/radovi/2)%20Pedagoske%20dimenzije%20drustva%20ucenja%20i%20znanja/PDF/201%20Snezana%20Mirkov.pdf.
Миловановић, Ј. Б. (2008). Математички задаци с обележјем стандарда као модели индивидуализоване и диференциране наставе математике. *Настава и васпитање*, *57*(4), 469-482. https://www.pedagog.rs/2008/05/26/matematicki-zadaci-s-obelezjem-standarda-kao-modeli-individualizovane-i-diferencirane-nastave-matematike/?_rstr_nocache=rstr4126484f6b67ce5f
Minner, D., Levy, A. J., & Century, J. (2010). Inquiry-Based Science Instruction:What Is It and Does It Matter?. *Journal of Research in Science Teaching*, *47*(4), 474-496.
Правилник о образовним стандардима за крај првог циклуса обавезног образовања за предмете српски језик, математика и природа и друштво (2011). Просветни гласник, Службени гласник РС, бр. 5/2011.
Sfard, A., & Linchevski, L. (1994). Between arithmetic and algebra: In the search of a missing link the case of equations and inequalities. *Rendiconti Del Seminario Matematico*, *52*(3), 279-307. http://www.seminariomatematico.polito.it/rendiconti/cartaceo/52-3/279.pdf
Stevanović, M. (2002). *Škola i stvaralaštvo.* Labin: MediaDesign.
Stacey, K., & MacGregor, M. (1999). Learning the algebraic method of solving problems. *The Journal of Mathematical Behavior*, *18*(2), 149–167.
Trnavac, N., i Đorđević, J. (1998). *Pedagogija.* Beograd: Naučna knjiga komerc.
Вуковић, В. (1998). *Савремено учење математике.* Јагодина: Учитељски факултет у Јагодини и Студио СЦ Јагодина.
Вуловић, Н. (2011). *Примена метода активног учења на диференцираним садржајима геометрије у почетној настави математике* [Необјављена докторска дисертација]. Јагодина: Педагошки факултет у Јагодини.
Vučić, L. (1991). *Pedagoška psihologija-Učenje.* Beograd: Centar za primenjenu psihologiju Društva psihologa Srbije.
Vergnaud, G. (1985). Understanding mathematics at the secondary-school level. In A. Bell, B. Low, & J. Kilpatrick (Eds.), *Theory, Research & Practice in Mathematical Education* (pp. 27-45). University of Nottingham, UK: Shell Center for Mathematical Education.
Зељић, М. (2014). *Методички аспекти ране алгебре.* Београд: Учитељски факултет.

EFEKTI UČENJA PUTEM OTKRIĆA NA DIFERENCIRANIM SADRŽAJIMA ALGEBRE NA TRAJNOST ZNANJA UČENIKA U POČETNOJ NASTAVI MATEMATIKE

Jedno od bitnih svojstava znanja koje određuje wegov kvalitet jeste njegova trajnost. Trajnost učeničkog znanja, između ostalog, zavisi od kvaliteta organizacije i izvođenja nastavnog rada u procesu usvajanja novog, ali i utvrđivanja i ponavljanja starog znanja. Tačnije, trajnost znanja učenika zavisi od primenjenog metodičkog pristupa i od aktivnosti učenika na času. Cilj ovog rada bio je da se ispitaju efekti učenja putem otkrića na diferenciranim sadržajima algebre na trajnost znanja učenika u početnoj nastavi matemaatike. U tom cilju sproveden je eksperiment sa paralelnim grupama na uzorku od 261 učenika četvrtog razreda osnovne škole, kako bi se ispitalo da li metodički pristup nastavi algebre zasnovan na principima učenja putem otkrića i diferencijacije sadržaja daje bolje efekte na trajnost znanja učenika u odnosu na tradicionalni način učenja. Rezultati istraživanja pokazali su da učenje putem otkrića na diferenciranim sadržajima doprinosi boljoj trajnosti znanja ukupno i na svakom od tri nivoa postignuća (osnovni, srednji i napredni).

Ključne reči: *trajnost znanja, nastava algebre, učenje putem otkrića, diferencijacija sadržaja.*