FACTA UNIVERSITATIS
Series: **Working and Living Environmental Protection** Vol. 21, Nº 4, Special Issue, 2024, pp. 409 - 416
https://doi.org/10.22190/FUWLEP240924039M

Original scientific paper

DYNAMIC ANALYSIS OF A NANOBEAM UNDER THE INFLUENCE OF AN ELECTROMAGNETIC ACTUATOR AND A MECHANICAL IMPACT

UDC 534.01

Bogdan Marinca[1], Nicolae Herisanu[1,2], Vasile Marinca[1,2]

[1]University Politehnica Timisoara, Romania
[2]Center for Advanced and Fundamental Technical Research, Romanian Academy branch of Timisoara, Romania

ORCID iDs: Bogdan Marinca https://orcid.org/0000-0003-2340-1280
Nicolae Herisanu https://orcid.org/0000-0003-2968-5309
Vasile Marinca https://orcid.org/0000-0002-0947-1322

Abstract. *The Optimal Auxiliary Functions Method (OAFM) is applied in the study of nonlinear vibration of a nanobeam, considering the curvature of the beam, the presence of an electromagnetic actuator and a mechanical impact. Our procedure is based on the existence of some auxiliary functions which assure a fast convergence of the approximate solution. The convergence-control parameters present in the auxiliary functions are evaluated by rigorous mathematical procedures.*

Key words: *OAFM, electromagnetic actuator, mechanical impact, nonlinear forced vibration*

1. INTRODUCTION

The study of the vibration of a nanobeam under mechanical impact in the presence of an electromagnetic actuator accounting for the curvature of the beam is interesting for researchers because many structures include the nanobeam. Textile fibers, flexible satellites, paper sheets, oil pipelines, airplane wings, and so on. Nanobeams have attracted considerable attention in the literature. For example, Ghayesh [1] investigated the forced nonlinear vibrations of an axially moving beam fitted with an intra-span spring-support which is solved by the pseudoarclength continuation technique. The thermo-mechanical nonlinear vibration and stability of a hinged-hinged axially moving beam additionally supported by a nonlinear spring-mass support are examined by two numerical procedures

Received September 24, 2024 / Accepted October 10, 2024
Corresponding author: Nicolae Herisanu
University Politehnica Timisoara, Bd. Mihai Viteazu 1, 300222 Timisoara, Romania
E-mail: nicolae.herisanu@upt.ro

by Kazemirad et al. [2]. To obtain static pull-in voltage with fringing field effects in an electrostatically actuated cantilever and clamped-clamped microbeam, Rokni et al. [3] proposed a novel method for converting a governing fourth-order differential equation into a Fredholm integral equation. Peng et al. [4] presented a nonlinear electro-dynamic analysis for a size-dependent microbeam made of materials with nonlinear elasticity by employing the modified behavior of electrically actuated carbon nanotubes-based nano-actuator including the higher-order strain gradient deformation, the geometric nonlinearity, the slack effect and the temperature gradient effects.

The effect of a magnetic field on the nonlinear vibration response of single-walled carbon nanotubes based on nonlocal strain gradient theory is studied by Anh and Hieu [5]. Using the equivalent linearization method with weighted averaging value, expressions of the nonlinear frequencies are obtained in the analytical forms. Yinussa and Sobamovo [6] explored nonlinear internal flow-induced vibration and stability of a pre-tensioned nanotube that rests on an elastic foundation.

In the present work, the nonlinear forced vibration of a nanotube under the influence of mechanical impact and an electromagnetic actuator considering the curvature of the beam is investigated. The nonlinearity of the equation is caused by the curvature of the nanobeam and of the electromagnetic actuator. The governing equation is discretized using the Galerkin-Bubnov procedure. The obtained nonlinear differential equation is solved by using OAFM. A very accurate solution is obtained using a moderate number of convergence-control parameters via auxiliary functions.

2. Formulation of the Problem

A simply supported nanobeam of length L subjected to a mechanical impact by force F and electromagnetic load V_{DC} is presented in Figure 1. The transverse and longitudinal displacements are w(x,t) and u(x,t) respectively.

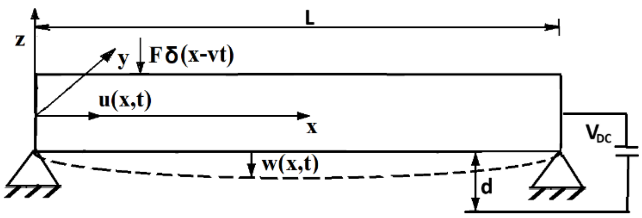

Fig. 1 A simply supported nanobeam subjected to impact force F and electromagnetic actuator

The Euler-Bernoulli beam theory proves that the displacement fields of any point are

$$u_x(x,z,t) = u(x,t) - z\frac{\partial w(x,t)}{\partial x}$$

$$u_y(x,z,t) = 0$$

$$u_z(x,z,t) = w(x,t) \tag{1}$$

The axial strain ε_{xx} and shear strain γ_{xz} of the beam, considering von Karman's nonlinear strain are

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - zk,\;\; \gamma_{xz} = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} = 0 \tag{2}$$

where

$$k = \frac{\partial^2 w}{\partial x^2} \Big/ \left[1 + \left(\frac{\partial w}{\partial x}\right)^2\right]^{3/2} \tag{3}$$

is the curvature of the beam.

The kinetic energy of the beam is

$$K_e = \frac{1}{2}\rho A \int_0^1 \left[\left(\frac{\partial u}{\partial t}\right)^2 + \left(\frac{\partial w}{\partial t}\right)^2\right] dx \tag{4}$$

while the first variation of the strain energy is

$$\delta U_s = \int_V (\sigma_{xx}\delta\epsilon_{xx}) dV \tag{5}$$

where σ_{xx} is the stress: σ_{xx}=E ε_{xx}, E being the elasticity modulus.

Substituting Eq. (2) into Eq. (5) one can get

$$\delta U_s = \int_0^L \left[N\delta\left(\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right) - M\delta\left(\frac{\partial^2 w}{\partial x^2}\right)\right] dx \tag{6}$$

in which N and M are the axial force and bending moment respectively. The stress resultants used in Eq. (6) are defined as

$$N = \int_A \sigma_{xx} dA\,, M = \int_A z\sigma_{xx} dA \tag{7}$$

where A is the area of the cross-section for the nanobeam.

The strain energy U_s can be written as

$$U_s = \frac{1}{2}\int_0^L \left[N\left(\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right) - M\left(\frac{\partial^2 w}{\partial x^2}\right)\right] dx \tag{8}$$

The virtual work by the external mechanical impact and electromagnetic actuator is given by

$$\delta W = \int_0^L q\delta W dx \tag{9}$$

where

$$q = F\delta(x - vt) + \frac{1}{2}\frac{C_0 V_{DC}^2}{[d-w(x,t)]^2} - \frac{1}{2}\frac{C_0 V_{DC}^2}{[d+w(x,t)]^2} \tag{10}$$

in which C_0 is the capacitance of the actuator, d is the gap width and V_{DC} is the voltage. The expression of the electromagnetic actuation can be simplified as

$$\frac{1}{2}\frac{C_0 V_{DC}^2}{[d-w(x,t)]^2} - \frac{1}{2}\frac{C_0 V_{DC}^2}{[d+w(x,t)]^2} = \frac{2C_0 V_{DC}^2}{L}\left[\frac{W}{d} + 2\left(\frac{W}{d}\right)^3 + 3.0925\left(\frac{W}{d}\right)^5\right] \tag{11}$$

The variational form of the equation of motion can be obtained by Hamiltonian principle

$$\delta \int_{t_1}^{t_2} [K_e - U_s + W)] dt = 0 \tag{12}$$

From Eqs. (4), (8), (9) and (12) and integrating by parts, and then collecting the coefficients of δ_u and δ_w we obtain the following equations of motion

$$\frac{\partial N}{\partial x} - \rho A \frac{\partial^2 u}{\partial t^2} \tag{13}$$

$$\frac{\partial^2 M}{\partial x^2} - \frac{\partial}{\partial x}\left(N \frac{\partial w}{\partial x}\right) + \frac{2C_0 V_{DC}^2}{d^2}\left[\frac{W}{d} + 2\left(\frac{W}{d}\right)^3 + 3.0925\left(\frac{W}{d}\right)^5\right] + F\delta(x - vt) =$$

$$= \rho A \frac{\partial^2 w}{\partial t^2} - \rho I \frac{\partial^4 w}{\partial t^2 \partial x^2} \tag{14}$$

where N and M from Eq. (7), become by integration:

$$N = EA\left[\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right] \tag{16}$$

$$M = -EI \frac{\partial^2 w}{\partial x^2} \Big/ \left[1 + \left(\frac{\partial w}{\partial x}\right)^2\right]^{3/2} \tag{17}$$

Usually, the longitudinal inertial term $\frac{\partial^2 u}{\partial t^2}$ into Eq. (13) can be neglected, such that from Eq. (13) it is clear that N is a constant N=C and therefore from Eq. (16) it holds that

$$\frac{\partial u}{\partial x} = \frac{C}{EA} - \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \tag{18}$$

By integrating the last equation with the boundary conditions

$$u(0, t) = u(L, t) = 0 \tag{19}$$

the constant C is given by

$$C = N = -\frac{EA}{2L}\int_0^L \left(\frac{\partial w}{\partial x}\right)^2 dx \tag{20}$$

According to the Eq. (17), the term which defines the curvature of the beam can be written in the form:

$$\frac{\partial^2 w}{\partial x^2} \Big/ \left[1 + \left(\frac{\partial w}{\partial x}\right)^2\right]^{3/2} \cong \frac{\partial^2 w}{\partial x^2}\left[1 - \frac{3}{2}\left(\frac{\partial w}{\partial x}\right)^2\right] \tag{21}$$

such that the nonlinear equation of motion for the nanobeam can be obtained by substituting Eqs. (20), (21) and (17) into Eq. (14) as follows:

$$EI\left[\frac{\partial^4 w}{\partial x^4} - \frac{3}{2}\frac{\partial^4 w}{\partial x^4}\left(\frac{\partial w}{\partial x}\right)^2 - 3\left(\frac{\partial^2 w}{\partial x^2}\right)^3 - 9\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x^2}\frac{\partial^3 w}{\partial x^3}\right] + \rho A \frac{\partial^2 w}{\partial t^2} + \frac{2C_0 V_{DC}^2}{d^2}\left[\frac{W}{d} + 2\left(\frac{W}{d}\right)^3 +\right.$$

$$\left. 3.0925\left(\frac{W}{d}\right)^5\right] + \rho I \frac{\partial^4 w}{\partial t^2 \partial x^2} - \frac{EA}{2L}\frac{\partial^2 w}{\partial x^2}\int_0^L \left(\frac{\partial w}{\partial x}\right)^2 dx = F\delta(x - vt) \tag{22}$$

The following nondimensional quantities are considered:

$$\bar{x} = \frac{x}{L}, \bar{w} = \frac{w}{d}, \bar{t} = \frac{t}{L^2}\sqrt{\frac{EI}{\rho A}}, \bar{\alpha} = \frac{I}{AL^2}, \bar{\beta} = \frac{2C_0 L^4 V_{DC}}{d^3 EI}, \bar{v} = vL\sqrt{\frac{\rho A}{EI}}, \bar{f} = \frac{FL^4}{EI} \tag{23}$$

Omitting the bars, the nondimensional form of the Eq. (22) can be written as

$$\frac{\partial^2 w}{\partial t^2}+\frac{\partial^4 w}{\partial x^4}-\frac{3}{2}\frac{\partial^4 w}{\partial x^4}\left(\frac{\partial w}{\partial x}\right)^2-3\left(\frac{\partial^2 w}{\partial x^2}\right)^3-9\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x^2}\frac{\partial^3 w}{\partial x^3}+\beta(w+2w^3+3.0925w^5)-\frac{1}{\alpha}\frac{\partial^4 w}{\partial t^2\partial x^2}-\frac{\alpha}{2}\frac{\partial^2 w}{\partial x^2}\int_0^1\left(\frac{\partial w}{\partial x}\right)^2 dx=f\delta(x-vt) \quad (24)$$

The solution of Eq. (24) can be assumed, according to the Galerkin-Bubnov procedure to be of the form

$$w(x,t)=X(x)T(t) \quad (25)$$

By substitution of Eq. (25) into Eq. (24) and then multiplying Eq. (24) by X(x) and integrating on the domain [0,1], using the expression

$$\int_0^1 f(x)\delta(x-vt)dx=f(vt) \quad (26)$$

one can obtain the following nonlinear differential equation of motion

$$\ddot{T}+\omega^2 T+aT^3+bT^5=f_0(vt),\ f_0(vt)=fX(vt) \quad (27)$$

where the dot denotes the derivative with respect to time and the parameters which appear into Eq. (27) are

$$q=\int_0^1\left[X^2(x)dx+\alpha X(x)\frac{d^2X(x)}{dx^2}\right]dx$$

$$\omega^2=\frac{1}{q}\left[\int_0^1\frac{d^4X(x)}{dX^4}X(x)\,dx+\beta\int_0^1 X^2(x)dx\right]$$

$$a=-\frac{3}{q}\int_0^1\left[X(x)\frac{d^4X(x)}{dx^4}\left(\frac{dX(x)}{dx}\right)^2+X(x)\left(\frac{d^2X(x)}{dx^2}\right)^5+3X(x)\frac{dX(x)}{dx}\frac{d^2X(x)}{dx^2}\frac{d^3X(x)}{dx^3}\right]dx-\frac{1}{2\alpha q}\left[\int_0^1\frac{dX(x)}{dx}dx\right]\left[\int_0^1 X(x)\frac{d^2X(x)}{dx^2}dx\right]-\frac{\beta}{q}\int_0^1 X^4(x)dx,\quad b=3.0925\int_0^1 X^5(x)\,dx \quad (28)$$

In the present study, we consider the case of a simply supported beam, and therefore the boundary conditions are

$$w(0,t)=\frac{\partial^2 w(0,t)}{\partial x^2}=0,\ \ w(1,t)=\frac{\partial^2 w(1,t)}{\partial x^2}=0 \quad (29)$$

The eigenfunction X(x) can be expressed from Eq. (29) as

$$X(x)=sin\pi x \quad (30)$$

For the nonlinear differential Eq. (27), where $f_0(t)=fsin\pi vt$, the initial conditions are

$$T(0)=A,\ \dot{T}(0)=0 \quad (31)$$

For nonlinear Eq. (27) and (31) we will apply OAFM [7-11].

3. APPLICATION OF OAFM TO THE NONLINEAR EQUATION OF NANOBEAM

To find an analytical approximate solution for nonlinear differential Eq. (27) and (31) near the primary resonance ω≈πv, we make the transformation

$$\tau = \Omega t, \;\; T(t) = A\theta(\tau) \tag{32}$$

Eqs. (27) and (31) can be rewritten as

$$\theta'' + \left(\frac{\omega}{\Omega}\right)\theta + \frac{aA^2}{\Omega^2}\theta^3 + \frac{bA^4}{\Omega^2}\theta^5 = \frac{f_0}{A\Omega^2} sin\frac{\omega\tau}{\Omega}\;,\;\; \theta(0) = 1, \theta'(0) = 0 \tag{33}$$

where the prime denotes the derivative with respect to τ and Ω is the frequency of the system. The linear and nonlinear operators corresponding to Eq. (33) are respectively:

$$L[\theta(\tau)] = \theta'' + \theta \;\;, N[\theta(\tau)] = \left(\frac{\omega^2}{\Omega^2} - 1\right)\theta + \frac{aA^2}{\Omega^2}\theta^3 + \frac{bA^4}{\Omega^2}\theta^5 - \frac{f_0}{A\Omega^2} sin\frac{\omega\tau}{\Omega} \tag{34}$$

The approximate solution of Eq.(34) can be written as

$$\bar{\theta}(\tau) = \theta_0(\tau) + \theta_1(\tau) \tag{35}$$

The initial approximate solution $\theta_0(\tau)$ is determined from the linear differential equation

$$\theta_0''(\tau) + \theta_0(\tau) = 0, \quad \theta_0(0) = 1, \theta_0'(0) = 0 \tag{36}$$

whose solution is

$$\theta_0(\tau) = cos\tau \tag{37}$$

Inserting Eq. (37) into the second expression of Eq. (34), it holds that

$$N[\theta_0] = N_1 cos\tau + N_3 cos3\tau + N_5 cos5\tau - N_6 sin\frac{\omega\tau}{\Omega} \tag{38}$$

where

$$N_1 = \frac{\omega^2}{\Omega^2} - 1 + \frac{3aA^2}{4\Omega^2} + \frac{5bA^4}{8\Omega^2},\;\; N_2 = \frac{aA^2}{4\Omega^2} + \frac{5bA^4}{46\Omega^2}, N_5 = \frac{bA^4}{16\Omega^2}\;\; N_6 = -\frac{f_0}{A\Omega^2} \tag{39}$$

From Eq. (38) we propose the following linear equation for the first approximation:

$$\Theta_1'' + \theta_1 = (C_1 + 2C_2 cos2\tau + 2C_3 cos4\tau)(N_1 cos\tau + N_3 cos3\tau),$$
$$\theta_1(0) = \theta_1'(0) = 0 \tag{40}$$

After some manipulations, Eq. (40) can be written as

$$\theta_1'' + \theta_1 = [(C_1 + C_2)N_1 + (C_2 + C_3)N_3]cos\tau + [(C_2 + C_3)N_1 + C_1 N_3]cos3\tau + [C_2 N_3 + C_3 N_1]cos5\tau + C_3 N_3\; cos7\tau \tag{41}$$

Avoiding the secular term in the last equation, we can find the frequency of the system:

$$\Omega^2 = \omega^2 + \frac{3aA^2}{4} + \frac{aA^2}{4}\frac{C_2 + C_3}{C_1 + C_2} \tag{42}$$

The solution of Eq. (40) becomes

$$\Theta_1(\tau) = \frac{(C_2 + C_3)N_1 + C_1 N_3}{8}(cos\tau - cos3\tau) + \frac{C_2 N_3 + C_3 N_1}{24}(cos\tau - cos5\tau)$$

$$\frac{C_3N_3}{48}(cos\tau - cos7\tau) \tag{43}$$

The approximate solution of Eqs. (27) and (31) becomes:

$$\bar{T}(t) = Acos\Omega t + \frac{A[(C_2+C_3)N_1+C_1N_3}{8}(cos\Omega t - cos3\Omega t) + \frac{A[C_2N_3+C_3N_1}{24}(cos\Omega t - cos5\Omega t) + \frac{AC_3N_3}{48}(cos\Omega t - cos7\Omega t) \tag{44}$$

where Ω is given by Eq. (42). The convergence control parameters are determined by minimizing the residual of the initial equation.

4. Numerical Application

The efficiency of OAFM can be proved through the following particular case: ω= 1.5, A = 1, a = 0.88, b = 1.23, f = 0.001, v = 0.478.

The obtained values of the convergence-control parameters are: C_1 = 0.750030876926, C_2 = -1.069177153794, C_3 = 0.013599617044

Fig.2 shows the comparison between the approximate solution (44) and the numerical solution obtained by a fourth-order Runge-Kutta approach.

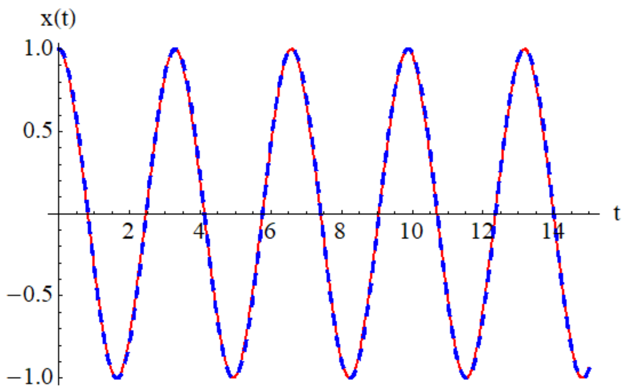

Fig. 2 Comparison between the analytical solution (44) and numerical integration results for Eqs. (27) and (31), numerical; _ _ _ _ Eq.(44)

It can be observed that our approximate solution for the nanobeam obtained through OAFM is nearly identical to the numerical integration results, which proves the efficiency of our analytical technique.

4. Conclusions

According to the present results, the oscillatory behavior of simply supported uniform nanobeam, taking into consideration the curvature of the beam, is studied. The Bernoulli-Euler beam is subjected to a mechanical impact using the Dirac-delta function and the electromagnetic actuation. The OAFM procedure was applied to solve the complex nonlinear differential equation introducing so-called auxiliary functions and some convergence-control parameters, without supplementary hypothesis. These parameters are optimally determined by rigorous mathematical procedures. Our technique leads to a very accurate solution using only one iteration. It should be emphasized that any nonlinear dynamical system is reduced to only two linear differential equations.

REFERENCES

1. Ghayesh M., Stability and bifurcation of an axially moving beasm with an intermediate spring support, *Nonlinear Dynamics*, 69, pp.193-200, doi:10.1007/s11071-011-0257-2, https://link.springer.com/ article/10.1007/s11071-011-0257-2
2. Kazemirad S., Ghayesh M., Amabili M., Thermal effects on nonlinear vibration of an axially moving beam with an intermediate spring-mass support, *Shock and Vibration*, 20, pp.387-399, doi:10.1155/2013/621295, https://onlinelibrary.wiley.com/doi/10.3233/SAV-120752
3. Rokni H., Seethaler R.J., Milani A.S., Hashemi S.H., Li X.F., Analytical closed-form solutions for size-dependent pull-in behavior in electrostatic micro-actuators via Fredholm integral equation, *Sensors and Actuators A: Physical*, 190, pp.32-43, doi:10.1016/j.sna.2012.10.035, https://www.sciencedirect.com /science/article/abs/pii/S0924424712006528?via%3Dihub
4. Peng J., Yang L., Lin F., Yang J., Dynamic analysis of size-dependent microbeams with nonlinear elasticity under electrical actuation, *Applied Mathematical Modeling*, 43, pp.441-453, doi: 10.1016/j.apm.2016.11. 025, https://www.sciencedirect.com/science/article/pii/S0307904X16306308
5. Ahn N.D., Hieu D.V., Nonlinear vibration of nonlocal strain gradient nanotubes under longitudinal magnetic field, *Vietnam Journal of Mechanics*, 43, pp.55-77, doi:10.15625/0866-7136/15467, https://vjs.ac.vn/index.php/ vjmech/article/view/15467
6. Yinusa A.A., Sobamowo M.G., Mechanics of nonlinear internal flow-induced vibration and stability analysis of a pre-tensioned single-walled carbon nanotube using classical differential transformation method with CAT and SAT after-treatment techniques, *Forces in Mechanics*, 7, 100083 doi:10.1016/j. finmec.2022.100083,https://ww.sciencedirect.com/science/article/pii/S2666359722000130
7. Marinca V., Herisanu N., Marinca B., *Optimal Auxiliary Functions Method for Nonlinear Dynamical Systems*, Springer, Cham, 2021
8. Herisanu N., Marinca V., An efficient analytical approach to investigate the dynamics of a misalignment multirotor system, Mathematics, 8, 1083, doi:10.3390/math8071083, https://www.mdpi.com/2227-7390/8/ 7/1083
9. Marinca B., Bogdan C., Marinca V., Dynamical SEIR epidemic model by OAFM, *Chaos, Solitons,Fractals* 14, 2020, 110949, doi:10.1016/j.chaos.2021.11094, https://www.sciencedirect.com/science/article/ pii/S0960077921003039?via%3Dihub
10. Herisanu N., Marinca V., A solution procedure combining analytical and numerical approach to investigate a two degree of freedom vibroimpact oscillator, *Mathematics*, 9, 1374, doi:10.3390/math9121374, https://www.mdpi.com/2227-7390/9/12/1374
11. Herisanu N., Marinca B., Marinca V., Nonlinear vibration of double walled carbon nanotubes subjected to mechanical impact and embedded on Winkler-Pasternak foundation, *Materials* 15, 8599, 10.3390/ma15238599, https://www.mdpi.com/1996-1944/15/23/8599