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VIBRATION MITIGATION-BASED MACHINE LEARNING-DRIVEN DESIGN OF METASTRUCTURES

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Abstract. *This research is concerned with the development of a longitudinally excited metastructure, featuring periodically distributed external units, each equipped with internal oscillators functioning as vibration absorbers. Initially, the metastructure designed for vibration attenuation around the first structural resonance, is characterized by uniformity, with all absorbers being identical and consisting of cantilevers integrated into the external components, each cantilever terminating in a concentrated mass block. This study employs a machine learning approach to maximize vibration attenuation efficiency around the second resonance, as well as concurrently at the first and second resonant frequencies in two associated optimality criteria related to the width of the attenuation region and the amplitude reduction, respectively. The new metastructures redesigned based on these criteria are fabricated by 3D printing, and their enhanced vibration mitigation capabilities are verified experimentally.*

Key words: *metastructure, vibration mitigation, auxiliary oscillators, machine learning.*

1. INTRODUCTION

The concept of 'metastructures' has recently emerged in the field of vibration control, evolving from the framework of the concept of metamaterials [1–3]. This approach entails the integration of a series of internal, distributed, and tuned auxiliary oscillators within the external components of a structure, aimed at controlling its vibrational response. Despite its recent introduction, the fundamental principle is rooted in the enhancement of Den

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Hartog's methodology [4], which focuses on managing the response of a main vibrating structure, subjected to external excitation, modelled as a one-degree-of-freedom linear mechanical oscillator. This is achieved by incorporating an auxiliary oscillator that matches the frequencies of the main structure, the external harmonic excitation, and the natural frequency of the auxiliary oscillator itself. Consequently, the auxiliary oscillator must be meticulously designed (tuned) to align with such frequency. As a result, rather than exhibiting resonance characterized by an infinite response at this specific frequency, the undamped main structure, when externally excited, will instead exhibit antiresonance, resulting in a zero-amplitude response [4–6]. The auxiliary oscillator functions as a vibration absorber [4–6], which is, in the context of damping, referred to as a tuned-mass damper [7, 8].

The application of data science and machine learning (ML) has significantly broadened the scope for numerical development and optimization of engineering structures, including vibration absorbers. Nevertheless, this methodology has not been widely adopted in the design of metastructures that exhibit effective vibration control through specifically calibrated auxiliary oscillators. In [9], an Archimedean spiral metastructure was introduced to manage low-frequency flexural waves. The inverse design process, which targeted specific bandgap widths and central frequencies, was successfully executed using ML techniques. The efficacy of this method was substantiated through Finite Element Method (FEM) analysis and experimental validation. The research detailed in [10] utilized numerical simulations alongside ML strategies to implement both forward and inverse design methodologies for a composite metastructure, with the goal of achieving subwavelength and ultrawide bandgaps. The outcomes of the ML approach were corroborated through numerical assessments and experimental tests on 3D-printed prototypes, with distinct excitations applied in both longitudinal and transversal orientations. To derive an optimal metastructure model that incorporates considerations of structural integrity and quasi-zero stiffness properties, a recent study employed a combination of deep reinforcement learning and FEM within an optimization framework [11]. Following this, 3D printing was utilized, and experimental results indicated that the produced metastructures demonstrated exceptional capabilities in vibration reduction, particularly in the low-frequency spectrum. Additionally, hull grillage metastructures were noted for their significant isolation properties concerning low-frequency flexural vibrations [12]. A dataset was generated using the theoretical wave mechanics model of hull grillage metastructures, and a forward prediction neural network model was subsequently applied to assess the vibration transmission characteristics. In [13], a metastructure grounded in phononic crystal principles was examined, and an innovative deep learning approach was proposed to facilitate its optimization through both qualitative and quantitative analyses, thereby reducing the likelihood of errors in judgment. Furthermore, the optimization procedure encompassed metastructures characterized by different periodic constants and filling fractions, yielding significant insights into the equilibrium between spatial efficiency, material usage, and vibration mitigation. The efficiency of the optimized configurations regarding vibration performance was validated through FEM analysis. The recent review article concerning ML-assisted intelligent design of metastructures [14] indicated that while ML has the potential to yield outstanding design outcomes for a range of acoustic or mechanical specifications, the majority of existing studies fell short in terms of manufacturing and experimental validation following the design phase.

The primary aim of this research is to enhance the application of data science, specifically ML, within the domain of the design of metastructures that would yield their desirable behaviour when exposed to an external vibration source. Additionally, the study seeks to qualitatively validate the design through both manufacturing processes and experimental analyses, which is the shortcoming pointed out in [14].

This study is organized as follows. Section 2 elaborates on the primary structure and its structural characteristics. The ML methodology employed, and the results achieved are discussed in Section 3. In Section 4, the vibration responses of the newly developed metastructures are validated through experimental data. Lastly, Section 5 encapsulates the key findings and articulates the relevant conclusions.

2. On the Primary Metastructure

The primary metastructure, labelled herein as MS0, consists of both external units and internal oscillators. This metastructure was first introduced in reference [15] and is also described in detail in [16]. Their basic units are shown in Figure 1.

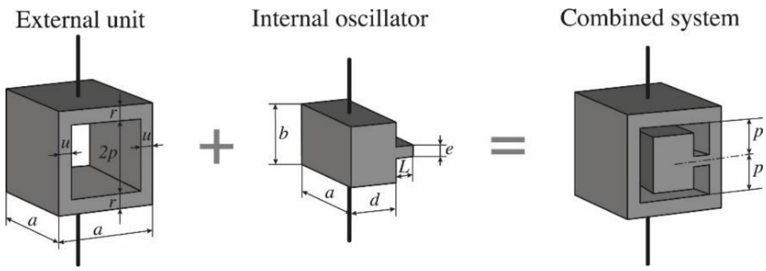

Fig. 1 Units of a metastructure under consideration

The external unit features a hollow square cross-section part, with its essential attributes depicted in Figure 1. In particular, the external dimensions are specified as *a* for both width and length, the wall length is indicated as *u*, and the height of the transverse components, namely the floor and ceiling, is marked as *r*. The internal oscillator depicted in Figure 1 consists of a cantilever characterized by its length *L* and height *e*, with a concentrated mass positioned at its tip, forming a discrete block. The dimensions of the internal oscillator, specifically the length *d*, the width *a*, and the height *b*, are noted in Figure 1. The internal oscillator is symmetrically integrated into the wall of the external unit at a specific location defined by parameter *p* in relation to both the ceiling and the floor, as seen in the combined system in Figure 1. Further, these units can be arranged to get a multi-level structure, which is done in this study to realize a cohesive 10-unit metastructure, achieved by replicating the combined units while excluding the floors. Each absorber within this metastructure is numbered from 1 to 10, beginning from the bottom. Detailed geometric parameters are listed in Table 1, while the MS0 itself is shown in Figure 2a, and its internal oscillators are labelled as the absorber type TA0. To reduce bending, pairs of internal oscillators are attached to the opposing internal walls of the external units, as seen in Figure 2a. The odd-numbered oscillators are secured to one side of the wall, while the even-numbered oscillators are integrated on the opposite side. The internal oscillators exhibit transversal oscillations whose basic axis is aligned with the basic axis of longitudinal oscillations of each individual unit and the overall metastructure, as illustrated by the black tick solid line in Figure 1. This metastructure has demonstrated suitability for 3D printing due to its homogeneous and uniform characteristics, allowing for scalability to a desired number of units while effectively achieving vibration attenuation near the first structural resonance [17]. Nevertheless, the design of absorbers warrants further investigation, particularly in optimizing efficiency for vibration attenuation in the vicinity of the second resonance region, as well as concurrently addressing the first and second resonant response while maintaining a consistent shape. This raises the question of how to redesign MS0 to incorporate such vibration attenuations, which is addressed in the following section.

Table 1 Parameters of MS0

Part	Parameter	Value [mm]
External unit	u	6.5
	r	6.5
	p	14
Internal oscillator	L	6.5
	e	4
	d	17
	b	20
External unit & Internal Oscillator	a	40

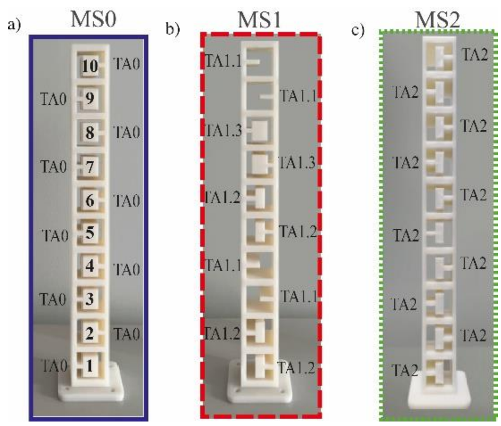

Fig. 2 Metastructures MS with distinct types of absorbers TA: a) MS0; b) MS1; c) MS2.

3. Machine Learning-driven Redesign

The fundamental redesign of the metastructure focuses on the internal oscillators, which influences the dimensions of both the mass block and the cantilever, specifically the parameters d and L. There are two physics-based conditions set for the redesign. First, to maintain the integrity of the longitudinal axis and prevent potential bending, it is essential that the combined lengths adhere to the constraint $L + d/2$ = constant. This condition ensures that the vertical symmetry axis of the structure remains unchanged. The parameter d is evaluated in three distinct scenarios: in the initial configuration of MS0, it serves as the baseline variation; the second scenario features *d* at half its original value; and the third scenario sets *d* to zero, indicating the absence of mass at the cantilever's end. To further mitigate bending, the second condition for redesign necessitates the modification of internal oscillators 1-10 in pairs, with the stipulation that each consecutive pair must be geometrically equivalent (1=2, 3=4, 5=6, 7=8, 9=10). This results in five pairs of internal oscillators that require redesign. By incorporating three variations of parameter d across these pairs, an initial consideration of 3^5=243 unique metastructures is conducted.

The redesigning process encompasses three primary steps, which collectively outline a general methodology for formulating the ML problem, as defined in [16]:

Step 1: Employ simulation software that integrates both geometrical and physical modelling capabilities, along with results assessment, to conduct a comprehensive numerical vibration analysis across all variations of the metastructures. Herein, all 243 parameter variations of the metastructure are generated using Excel. Following this, the variations are imported into the COMSOL Multiphysics software, where the associated metastructures are visualized according to the imported geometrical parameters. More importantly, this step should yield a dataset that encapsulates the Amplitude-Frequency Response Curves (AFRCs) and the associated numerical data for the frequency and the associated amplitude for the point situated on the top of each metastructure for all variations of the metastructure, as being of interest from the viewpoint of further minimization of its amplitude.

Step 2: The discrete numerical dataset generated in Step 1 is used and ML techniques are applied to derive a continuous model of the metastructure, which will serve as the foundation for optimizing the geometry of the absorbers. It is essential to explore multiple ML techniques and select the one that delivers the highest accuracy. Two particular ML techniques, specifically Support Vector Regression (SVR) and Artificial Neural Networks (ANN), are examined in this study. In both methodologies, the numerical dataset generated is utilized to train the metastructure model. This process involves the application of various combinations of geometric parameters corresponding to different metastructure configurations as input data, while the output data is represented by the width of the corresponding attenuation regions. The model incorporates a total of ten input variables, which correspond to the dimensions *d* and *L* for each pair of internal oscillators within the metastructure, while the output consists of four variables. Two of them are linked to the width of the attenuation regions surrounding the first and second modal frequencies, and the other two represent the total 'amount' of vibration attenuation in attenuation regions, defined as:

$$I_i = \int_{B_1}^{B_2} (A_{MS} - A_{MS0})^2 df \ , \ \ i = 1,2 \tag{1}$$

where B_1 and B_2 denote the boundaries of the attenuation region, A_{MS} and A_{MS0} are amplitudes of vibration of the metastructure defined by input geometry parameters and basic metastructure MS0, respectively, and *f* is the frequency.

Although this model interpretation includes redundant features, due to the constraint $L + d/2$ = constant, this representation is adopted to enhance the clarity of the input features. Generally, while the model with redundant features may not be optimal in terms of size and efficiency, it does not compromise the accuracy of the results. The performance and precision of both ML models developed in this manner were evaluated using Root Mean Square Error (RMSE) and Mean Absolute Error (MAE) metrics. Observing the values of these evaluation criteria, it has been concluded that the SVR model outperforms the ANN model significantly. Therefore, only the SVR model is used in further analyses.

Step 3: With the continuous ML model developed in *Step 2*, initiate the optimization process aimed at determining the most effective geometry for the absorbers, while defining precisely the optimality criteria. Two optimality criteria are defined in this study, and both of them are described subsequently.

3.1. First optimality criterion

The first optimality criterion regards the width of a vibration attenuation region. The attenuation region is characterized as the frequency range in which the displacement amplitude of the new metastructure is less than that of MS0, as defined in [17]. Two distinct formulations of the first Optimality Criterion OC1 are introduced:

OC1.1. The extent of the frequency range in which vibration attenuation occurs around the second modal frequency, and

OC1.2. The cumulative width of the frequency ranges exhibiting vibration reduction around both the first and the second modal frequencies.

For both formulations, a singular optimized metastructure is identified, herein referred to as MS1. This metastructure MS1 is shown in Figure 2b. It is evident that it incorporates an original configuration of absorbers, but their arrangement is neither straightforward nor anticipated. In fact, there are three categories of absorbers (TA1) in MS1:

TA1.1: A cantilever-type absorber characterized by its maximum length (specifically absorbers 3, 4, 9, and 10);

TA1.2: An absorber featuring a longer cantilever with a shorter tip mass than in MS0 (absorbers 1, 2, 5, and 6);

TA1.3: An absorber that includes a slightly longer cantilever paired with a slightly shorter tip mass than in MS0 (absorbers 7 and 8).

The specifications for TA1 in terms of their dimensions are detailed in Table 2.

Table 2 Parameters of the metastructure MS1

Parameter of MS1	Internal oscillator number				
	1, 2	3, 4	5, 6	7, 8	9, 10
Types of absorbers (TA1)	TA1.2	TA1.1	TA1.2	TA1.3	TA1.1
L [mm]	9.895	15	9.895	6.6	15
d [mm]	10.21	0	10.21	16.8	0

3.2. Second optimality criterion

Unlike OC1, which regards only the width of the vibration attenuation region, the second optimality criterion OC2 is defined to take into account the amplitude reduction. This criterion is defined as the sum of the integrals of the AFRC around the first and second modal frequency, and it is equal to the sum of the model output parameters I_1 and I_2 defined in Eq. (1).

In this case, the metastructure labelled as MS2 (Figure 2c) is obtained as the optimal one for the maximal reduction of the amplitude in the integral sense described above. MS2 is shown in Figure 2c next to MS0 and MS1 for the sake of their mutual comparison. It is evident that MS2 incorporates uniformly distributed absorbers as in MS0. However, the dimensions of these absorbers (TA2) are different from TA0. Their specification is given in Table 3.

Table 3 Parameters of the metastructure MS2

Parameter of MS1	Internal oscillator number
	1-10
Types of absorbers (TA2)	TA2
L [mm]	9.922
d [mm]	10.156

4. Experimental Results

The original metastructure MS0, along with its redesigned counterparts MS1 and MS2, were produced utilizing Fused Filament Fabrication (FFF) technology, a specific form of 3D printing characterized by a resolution ranging from 0.1 to 0.3 mm (Figure 1). The material employed in this process was Acrylonitrile Butadiene Styrene (ABS), the properties of which are: a Young's modulus of elasticity E=1900 MN/m^2 and a density ρ=1.015 g/cm^3 (these values are obtained from the manufacturer for the metastructures produced). These structures were exposed to base excitation in an experimental setup the details of which are presented in Figure 3. The driving signal was generated using a computer equipped with the SCO-107 software package, designed for sine sweep operations, with the base acceleration amplitude set to 1g, where g represents gravitational acceleration. This signal is initially routed to an LAS200 controller, then to an LPA100 amplifier, and finally to an LDS V408 vibration shaker. The frequency and amplitude of the driving signal are regulated by a lower accelerometer model 4534-B, which has a sensitivity of 10 mV/g. The output from this accelerometer is also sent back to the controller for feedback. Additionally, a second accelerometer of the same model is affixed to the center of the uppermost horizontal plane of the metastructure to measure the response in the vertical direction. This signal is connected to both the controller and the computer for subsequent processing and analysis.

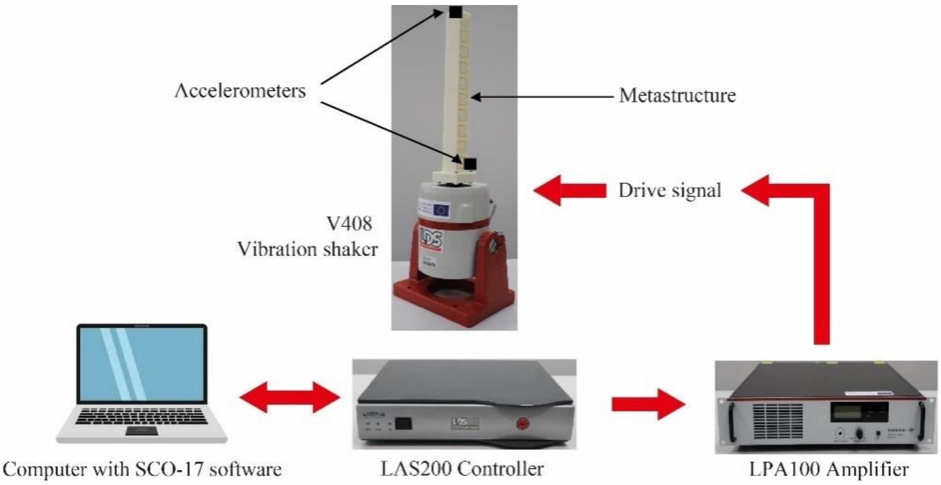

Fig. 3 Experimental setup

The recorded AFRCs for the top of the original metastructure MS0 and the modified metastructure MS1 are presented in Figure 4. The frequency was considered within the frequency range of 400 Hz to 1100 Hz, which is identified as the target frequency band encompassing two significant resonances of MS0 pertinent to this investigation. It is important to note that the subsequent diagrams depict non-dimensional frequency and non-dimensional displacement amplitudes. The non-dimensional frequency is determined by the ratio of the excitation frequency to the frequency of the metastructure MS0 with blocked TA0 absorbers, which is 588 Hz [18]. Meanwhile, the non-dimensional displacement amplitude is calculated as the ratio of the displacement amplitude on the top to the total height of the metastructure, measured at 358 mm. The attenuation region achieved by MS1, which exhibits a reduced amplitude response compared to MS0, is highlighted in grey in this figure, a representation that is consistent across the subsequent figure as well. For comparative purposes, photos of both the original metastructure MS0 and the redesigned metastructure MS1 are displayed on the left and right sides of this figure, respectively, along with the legend for the AFRCs presented. It is seen that the response of both metastructures is similar around the first resonance and that the values of the first antiresonance are close to each other. However, MS1 has more dense resonance frequencies in the frequency region of interest (three of them), while MS0 has two of them. The appearance of the second antiresonance of MS1 decreases the amplitude response and makes it smaller than the one of MS0, which is seen as the second attenuation region starting at the nondimensional frequency of 1.07. It is also seen that the response of MS0 around the second resonance and the response of MS1 around the third resonance are alike, but the one of MS1 is always smaller than the amplitude of MS0.

Analogously, Figure 5 shows the AFRCs for the top of the original metastructure MS0 and the modified metastructure MS2. Their comparison reveals completely distinctive characteristics of the dynamic behaviour of MS2 with respect to MS1 shown in Figure 4. It is seen that the first two resonance peaks of MS2 are shifted to the left, toward higher frequencies. The appearance of the first antiresonance of MS2 is actually the region when its response is of a smaller amplitude than the one of MS0.

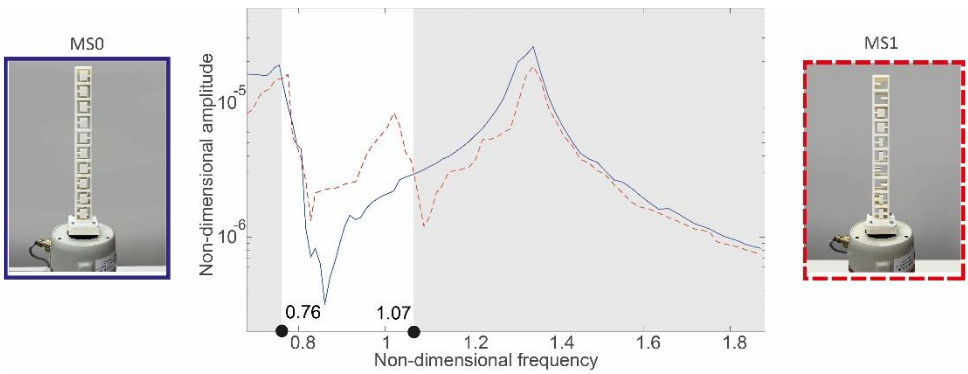

Fig. 4 Measured AFRCs of the point on the top of the redesigned metastructure MS1 compared to the original one MS0 with the attenuation region shaded in grey.

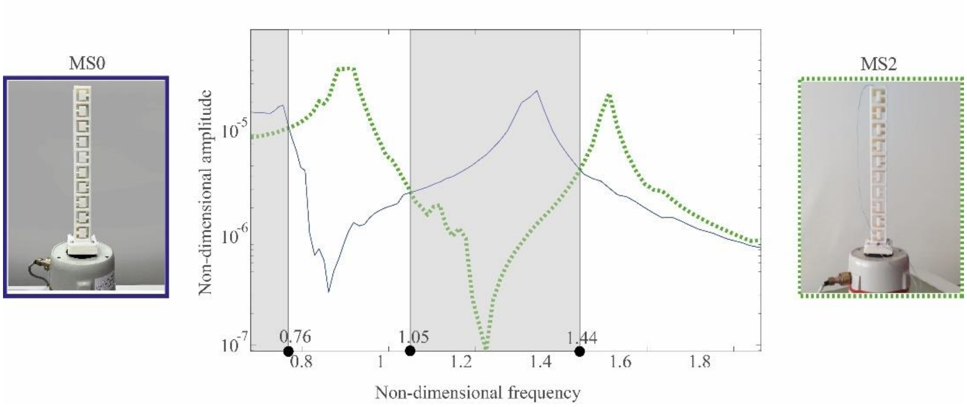

Fig. 5 Measured AFRCs of the point on the top of the redesigned metastructure MS2 compared to the original one MS0 with the attenuation region shaded in grey.

5. CONCLUSIONS

This study has focused on the reconfiguration of integrated internal oscillators within a metastructure composed of ten units that exhibit longitudinal vibrations. The objective has been to attain efficient vibration attenuation at designated resonance frequencies, specifically targeting either the second frequency or both the first and second frequencies concurrently. The initial design of the metastructure MS0 was homogeneous, incorporating uniform absorbers across all units. These absorbers were originally designed as cantilevers that are embedded in the external units of the metastructure, each equipped with a block mass positioned at the end of the cantilever.

The process of redesigning integrated internal oscillators has consisted of three fundamental steps, which together delineate a comprehensive approach to defining the machine learning problem. The initial phase has involved the development of a distinct numerical dataset comprising frequency-displacement amplitude pairs within simulation software that encompasses both geometric and physical modelling functionalities. This dataset has subsequently been utilised, and machine learning techniques have been employed to formulate a continuous model of the metastructure. In this investigation, two specific machine learning techniques, namely Support Vector Regression and Artificial Neural Networks, have been analysed. The efficacy and accuracy of the machine learning models created through this approach have been assessed utilizing the Root Mean Square Error and Mean Absolute Error metrics. Analyzing the results from these evaluation metrics has led to the conclusion that the Support Vector Regression model has significantly surpassed the Artificial Neural Network model in performance. Consequently, only the Support Vector Regression model has been employed in subsequent analyses. In the third phase, utilizing the continuous machine learning model established in the previous step, the optimization process has been initiated to identify the most effective geometry for the absorbers, with a clear definition of the optimality criteria.

This study has delineated two optimality criteria. The first optimality criterion has pertained to the region of vibration attenuation, which has been defined as the frequency spectrum where the displacement amplitude of the newly designed metastructure is lower than that of the original one, surrounding the second modal frequency, as well as the total width of the frequency ranges that demonstrate vibration reduction around both the first and second frequencies. In both cases, a unique optimized metastructure MS1 has been identified, featuring an innovative configuration of three types of absorbers, whose arrangement is neither simple nor predictable. The second optimality criterion has been articulated so that the new metastructure MS2 emerges as the optimal configuration for achieving maximal amplitude reduction in the integral sense. MS2 has been obtained to feature uniformly distributed absorbers of the same shape as those in MS0 but characterised by different dimensions.

The experiments conducted have shown that the newly developed metastructures demonstrate superior vibration attenuation properties in comparison to the original metastructure, as per the established criteria and within the specified frequency range. Thus, it has been confirmed that this study has successfully developed sophisticated and original metastructures through the application of a machine learning approach, which underscores the effectiveness of the data science methodology employed in this research.

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