FACTA UNIVERSITATIS
Series: **Working and Living Environmental Protection** Vol. 21, N° 4, Special Issue, 2024, pp. 293 - 303
https://doi.org/10.22190/FUWLEP241003028S

Original scientific paper

THE POLAR CHARACTERISTIC OF AN ACOUSTIC PARABOLIC REFLECTOR

UDC 534

Violeta Stojanović[1], Zoran Milivojević[2]

[1]Academy of Applied Technical and Preschool Studies, Niš Department, Niš, Serbia
[2]MB University Belgrade, Serbia

ORCID iDs: Violeta Stojanović https://orcid.org/0009-0009-4877-6696
Zoran Milivojević https://orcid.org/0000-0002-2240-3420

Abstract. *The first part of the paper presents the geometry of the acoustic parabolic reflector. After that, analytical formulas for calculating the amplification and polar characteristics of the reflector are shown. The second part of the paper describes the experiment in which measurements were made and the acoustic characteristics of the parabolic reflector located in the yard of the Academy of Applied Technical and Preschool Studies, Department of Niš, in Niš were calculated. First, acoustic impulse responses for acoustic excitation at angles from -90° to 90° were measured. After that, the polar characteristic of the radiation was calculated. Finally, the beamwidth of the radiation, beam solid angle of the radiation, and directivity of the acoustic parabolic reflector were determined. The results of the experiment are presented numerically and graphically.*

Key words: *impulse response, polar characteristic, directivity.*

1. INTRODUCTION

The concentration and behavior of waves on curved surfaces have been studied since ancient times. The Greek mathematician Diocles, somewhere between 190 and 180 BC, wrote about the “surface of a mirror which, when positioned towards the Sun, reflects the sun's rays that converge to a single point, thus causing ignition” [1]. This concept was applied in 1896 during the lighting of the first Olympic torch relay using sunlight, concentrated by a parabolic mirror. A. W. Love noted in his articles published before 1978 that as early as 1888, Heinrich Hertz, who experimentally proved the existence of electromagnetic waves, constructed the first antenna from a metal plate that was a curved,

Received October 3, 2024 / Accepted October 10, 2024
Corresponding author: Violeta Stojanović
Academy of Applied Technical and Preschool Studies, Niš Department, Aleksandra Medvedeva 20, 18000 Niš, Serbia
E-mail: violeta.stojanovic@akademijanis.edu.rs

cylindrical parabolic surface ensuring that the focal area was a line, not a point. J. W. S. Rayleigh's first insights into the reflection of sound waves on curved surfaces are associated with the cave of Dionysius' Ear in Syracuse, Sicily (Dionysius ruled Syracuse between 432 and 376 BC). He also highlights St. Paul's Cathedral in London and its Whispering Gallery inside the dome, built in 1710, for its unique acoustic effects.

Kremer and Miller, referencing the impressions of Reverend John Blackburn in 1828, state that the first use of acoustic parabolic reflectors occurred in churches in the 19^{th} century [2]. They emphasize the necessity of an adequately sized acoustic parabolic reflector and note that the priest should not be positioned at the reflector's focus during the liturgy, as the noise from the congregation would significantly disturb him. V. C. Sabine, who studied room reverberation time, also provided theoretical assumptions and explanations related to this issue [3].

Acoustic parabolic reflectors were used before the development of radar, between the First and Second World Wars, for detecting airplanes [4]. There were: a) small reflectors mounted on the listener's head, b) large ones transported by trucks, and c) gigantic stationary concrete structures, some of which still exist near the coast of Great Britain, aimed at the English Channel. In 1930, Olson and Wolff published two papers proposing the use of parabolic constructions that function as a horn at low frequencies and as a reflector at higher frequencies [5]. In 1932, Kellogg constructed a parabolic reflector for recording wildlife sounds, publishing his research in 1938.

Today, acoustic parabolic reflectors are used in various applications: a) in engineering research for the detection of acoustic signals in non-contact testing methods, b) with parabolic microphones for sound recording in nature (such as bird recording, locating and rescuing lost people and animals), field sound for sports broadcasting and eavesdropping on conversations. They are also used in education and entertainment, such as telescopes in astronomy and sound reflectors in auditoriums and amphitheaters.

This paper presents the polar characteristic of an acoustic parabolic reflector located in the yard of the Academy of Applied Technical and Preschool Studies, Department of Niš, in Niš. After discussing the geometry and analytical formulas for calculating the amplification and polar characteristics of the acoustic reflector, the experiment is explained. The basis of the experiment consists of impulse responses h_θ for acoustic excitation at angles θ = -90°: 15°: 90°. Based on the impulse responses, the amplitude characteristics of sound waves as a function of angle θ are presented. The following were calculated: a) the amplification of the reflector, b) the polar radiation characteristic, c) beamwidth and the beam solid angle of radiation, and d) directivity. The results of the experiment are presented both numerically and graphically.

The organization of the paper is as follows: Section 2 presents the geometry and characteristics of the acoustic parabolic reflector, Section 3 explains the experiment, presents and analyzes the results of the experiment, and Section 4 presents the conclusion.

2. ACOUSTIC PARABOLIC REFLECTOR

2.1. Geometry and characteristics of the reflector

A parabolic surface or paraboloid is formed by rotating a parabolic curve around its axis of symmetry [6]. The main characteristic of a paraboloid is that all incident rays parallel to the axis of rotation correspond to reflected rays that pass through and intersect at a single common point, the focus. When a section is cut from the surface of the paraboloid (which is mathematically infinite), a parabolic reflector is obtained. If the plane cutting the paraboloid is perpendicular to the axis of symmetry, a circular section of the paraboloid is formed, with its focus at the center – on the axis of symmetry at a certain distance from the geometric center of the surface. In most parabolas, the focus is located outside the parabola's aperture because the cutting plane is usually closer than the focal distance. The section obtained in this manner is a classic parabolic reflector.

In Fig. 1, a model of a parabolic reflector is given where: h – the depth of the reflector, R – the aperture radii, h_v – the distance from the vertex O of the parabola and the wavefront, z_s – the distance of a point located on the surface of the reflector (P_1) from the line passing through the vertex O, z_F – the distance from the focus F to the vertex (FO) [7]. The movement of the acoustic wavefront is represented by rays (1) and (2). The path that a wave takes from any position in the plane of the parabola's aperture (P_0, P_2) to the focus F is constant, i.e., the equation holds: $P_0F = P_2F$. This means that the energy of the wavefront from the plane of the aperture will be concentrated at the focus F. The equation of the parabolic reflector is:

$$r^2 = 4 \cdot z_F \cdot z \,. \tag{1}$$

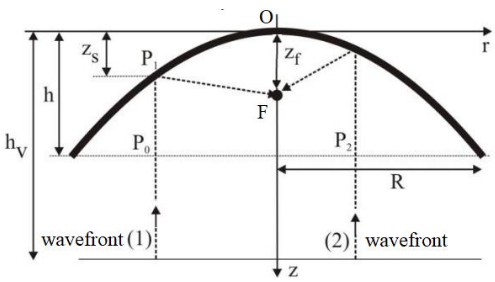


Fig. 1 Model of an acoustic parabolic reflector

The ratio of sizes z_F/h determines the angle at which the edges of the parabola are seen from the focus. The smaller the ratio z_F/h, the deeper the reflector, and the larger this angle. Conversely, if the ratio z_F/h is larger, the reflector is shallower, and the angle is smaller. Fig. 2 shows the geometry of parabolic reflectors with equal radii R but different z_F/h ratios.

2.2. Amplification and polar characteristic of the reflector

In the plane of the reflector's aperture, acoustic waves have an acoustic pressure, P_i. After reflection, the waves are concentrated at the focus of the reflector, where they generate a pressure P_z [8]. The amplification of the reflector is defined as $F_z = P_z / P_i$. The formula for calculating the theoretical amplification of the reflector is:

$$F_z = \begin{pmatrix} 1 + \left[4\pi \dfrac{z_F}{\lambda} \ln\left(1 + \dfrac{h}{z_F}\right)\right] + \\ 8\pi \dfrac{z_F}{\lambda} \ln\left(1 + \dfrac{h}{z_F}\right) \sin\left(4\pi \dfrac{z_F}{\lambda}\right) \end{pmatrix}^{\frac{1}{2}} , \tag{2}$$

where λ is the wavelength of the incident acoustic wave. When sin $(4\pi z_F/\lambda) = \pm 1$, the following equation is obtained:

$$F_z = 4\pi \frac{z_F}{\lambda} \ln\left(1+\frac{h}{\lambda}\right) \pm 1 . \tag{3}$$

For high amplification, Eq. (3) becomes:

$$F_z = 4\pi \frac{z_F}{\lambda} \ln\left(1+\frac{R^2}{4{z_F}^2}\right) . \tag{4}$$

The amplification of an acoustic parabolic reflector depends on its dimensions h, R and z_F as well as the wavelength λ (or frequency) of the acoustic wave. The amplification of an acoustic parabolic reflector, expressed in dB, can be represented as follows:

$$G = 20\log_{10}\left(F_z\right) . \tag{5}$$

The increase in amplification per octave can be analyzed as a function of the z_F/λ: a) the amplification increases by 6 dB/octave in the frequency range where $z_F/\lambda \geq 1$ (Figs. 2a and 2b), b) the amplification is equal to 0 in the low-frequency region where $z_F/\lambda < 1/64$ and c) the amplification is variable for $1/64 < z_F/\lambda < 1$, which is a result of the sine term in Eq. (2). The last enhancement in strength is more pronounced in flat reflectors where $h/z_F < 1$ (Fig. 2c). Specifically, when the direct and reflected sound waves are approximately equal in strength, the reflected sound wave initiates the formation of a standing wave.

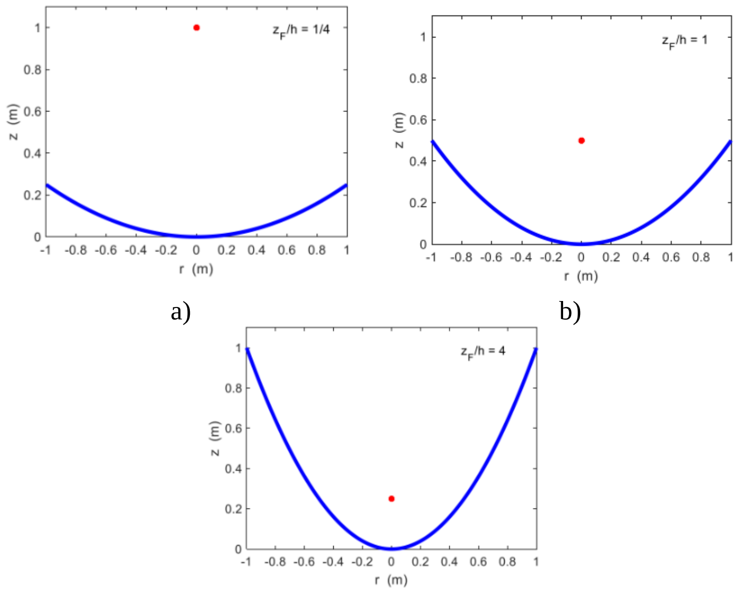


Fig. 2 Geometry of parabolic reflectors with equal radii R and: a) $z_F/h = 1/4$; b) $z_F/h = 1$; c) $z_F/h = 4$

The polar characteristic of an acoustic parabolic reflector represents a graphical representation of the distribution of acoustic amplification of sound waves in different directions (after their reflection and directing) in a polar diagram [9].

Normalized amplification of the acoustic parabolic reflector represents the ratio of the reflector's gain G (dB) (given Eqs. (2) and (5)), and the maximum gain G_{max} (dB) (given by Eqs. (4) and (5)) [8]. The beamwidth of the reflector, $\Delta\theta_B$, represents a parameter measured at half of the radiation power [9]. It is defined as the angle between two points on the main radiation beam of the reflector that is 3 dB lower in value than the level of maximum emission. The beam solid angle can be calculated as a function of the beamwidth as follows:

$$\Delta\Omega = \frac{\pi}{4}\left(\Delta\theta_B\right)^2 . \tag{6}$$

Directivity represents a measure of the directionality of the radiation pattern of the reflector. Since the acoustic parabolic reflector has a highly directional radiation diagram, directivity can be expressed by the following formula:

$$D_{\max} = \frac{16}{\Delta\theta_B^{\,2}} . \tag{7}$$

Directivity in dB is:

$$D = 10\log_{10} D_{\max} . \tag{8}$$

The beam width and the beam solid angle of the radiation are illustrated in Fig. 3.

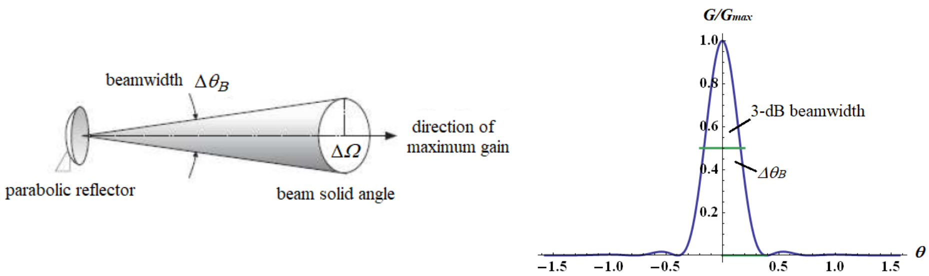


Fig. 3 Beamwidth and beam solid angle of the radiation

3. Experimental Results and Analysis

3.1. Experiment

For the purpose of determining the polar characteristics of the radiation and the characteristics of the acoustic parabolic reflector, an experiment was conducted. The experiment included: a) measurement of acoustic impulse responses for acoustic excitation as a function of the angular displacement of the sound source relative to the axis of the reflector $\theta = (-90^o \div 90^o)$ with a step of $\Delta\theta = 15^o$, b) calculation of the reflector's amplification (G (dB) and G_{max} (dB)) at the focus as a function of angle θ, c) calculation of the polar characteristic of the reflector ($G/G_{max} = f(\theta)$), d) amplitude characteristics of sound waves for different angles θ and f) calculation of reflector parameters: beamwidth $\Delta\theta_B$, beam solid angle $\Delta\Omega$ and maximum directivity, D_{max}.

For the experiment, the measurement was conducted on the acoustic parabolic reflector located in the courtyard of the Academy of Applied Technical and Preschool Studies, Department of Niš, in Niš (Fig. 4a). The geometry of the reflector is shown in Fig. 2a. The dimensions of the reflector are a) depth h = 0,3571 m, b) aperture radii R = 1 m, c) focal length z_F = 0,7 m. The reflector is made of concrete. The surface of the reflector opening was considered ideally smooth (the reflection coefficient is equal to 1).

The measurement procedure consists of the following: the excitation signal was emitted by a sound source whose position was changed along a semicircle (with the center in the vertex of the reflector) with a radius r = 5 m for angles θ with a step of $\Delta\theta$. The receiver was positioned at the focus of the reflector (Fig. 4b). The ambient temperature was t = 32^{o} and the speed of sound was c=350.39 m/s. The results of the experiment are presented graphically and numerically. The software support used was EASERA and Matlab.

The equipment used for the experiment as follows: a) an omnidirectional microphone (PCB 130D20), having a diaphragm diameter of 7 mm; b) B&K omnidirectional sound source type 4295 (dodecahedron loudspeaker); c) B&K audio power amplifier, rated at 100W RMS, stereo, type 2716-C; d) a laptop, incorporating a Soundmax Integrated Digital Audio sound card from Analog Devices.

a) b)

Fig. 4 a) The acoustic parabolic reflector in the yard of the Academy of Applied Technical and Preschool Studies, Department of Niš, in Niš and b) the position of the receiver during the experiment

3.2. The Base

The basis of the experiment consists of impulse responses. In order to determine the impulse responses h_θ from the transmitting side, a sweep signal was emitted with the following parameters: $f_d = 20$ Hz, $f_g = 20$ kHz, (that is $\lambda_g = 17.52$ m, $\lambda_d = 0.018$ m), $t = 5$ s, sampling frequency $f_S = 44.1$ kHz and 16 bps. Impulse responses were recorded in 13 measuring points for $\theta = -90^o : 15^o : 90^o$.

3.3. The Results

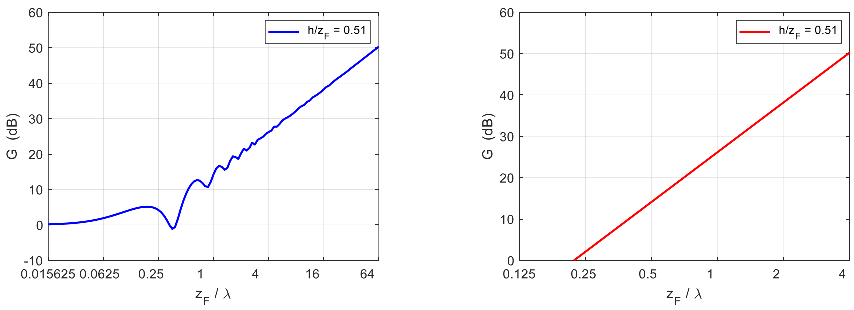
Table 1 presents the calculated values for the considered parameters of the acoustic parabolic reflector: a) amplification G for sound waves of frequencies: $f = \{20, 100, 1000, 10000, 20000\}$ Hz, b) beamwidth of the radiation $\Delta\theta_B$, c) beam solid angle of the radiation $\Delta\Omega$ and d) maximum directivity of the reflector D_{max}.

Table 1 The calculated values of the parameters of the acoustic parabolic reflectors

f (Hz)	20	100	1000	10000	20000
G (dB)	-13.681	0.298	20.298	40.298	46.319
$\Delta\theta_B$ (°)			17.46		
$\Delta\theta_B$ (rad)			0.305		
$\Delta\Omega$ (sr)			0.073		
D_{max}			172.336		
D_{max} (dB)			22.363		

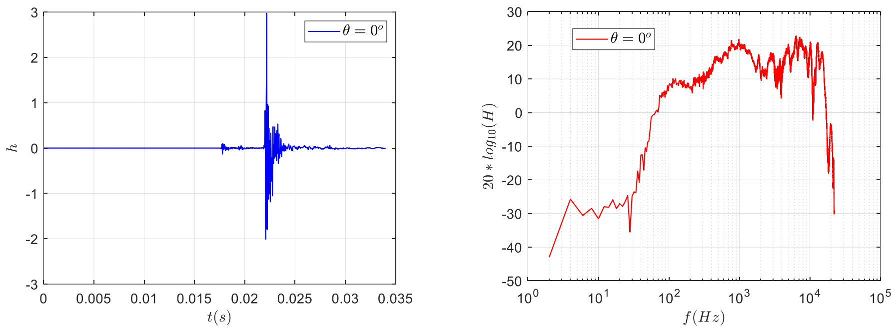
Figure 5 shows: a) the theoretical amplification of the analyzed reflector at the focus as a function of the ratio z_F/λ and b) the theoretical amplification of the reflector at the focus as a function of the ratio z_F/λ calculated without the sine term in Eq. (2) [8].

The acoustic impulse responses h are shown in: a) Fig. 6a ($\theta = 0^0$), b) Fig. 7a ($\theta = 45^o$) and c) Fig. 8a ($\theta = 90^o$). The amplitude characteristics $20{\cdot}\log_{10}(H)$ are shown in: a) Fig. 6b ($\theta = 0^o$), b) Fig. 7b ($\theta = 45^o$) and c) Fig. 8b ($\theta = 90^o$). The normalized acoustic amplification $|G/G_{max}|$ as a function of the angle θ is shown in Fig. 9a. The polar characteristic is presented in Fig. 9b.



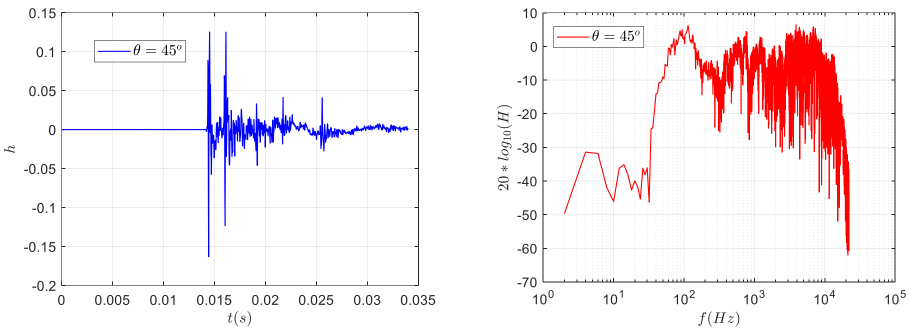
a) b)

Fig. 5 a) The theoretical amplification of the analyzed reflector at the focus as a function of the ratio z_F/λ and b) the theoretical amplification of the reflector at the focus as a function of the ratio z_F/λ calculated without the sine term in Eq. (2)



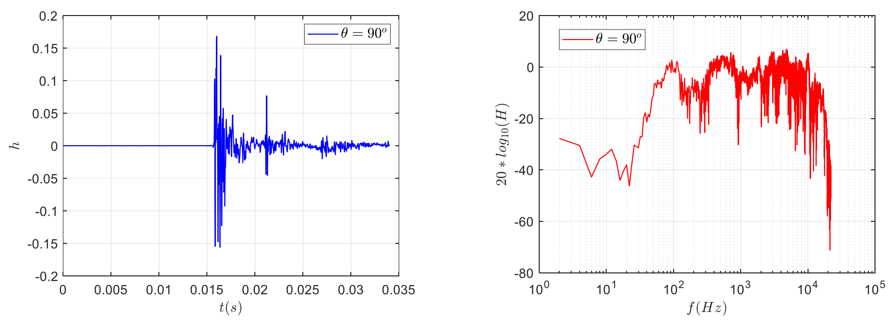
a) b)

Fig. 6 Acoustic impulse response for $\theta = 0^o$: a) time domain and b) amplitude characteristic



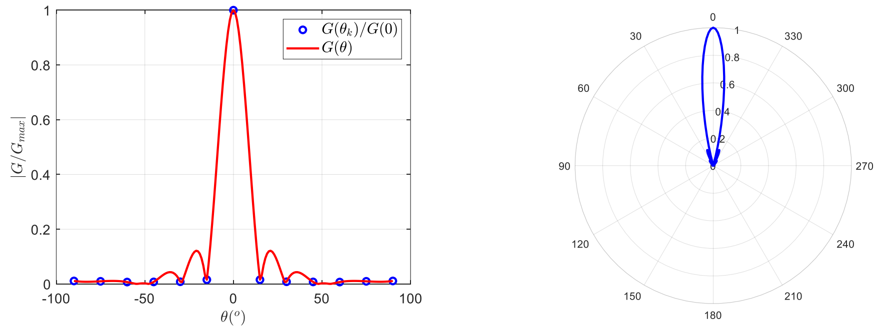
a) b)

Fig. 7 Acoustic impulse response for $\theta = 45^o$: a) time domain and b) amplitude characteristic



a) b)

Fig. 8 Acoustic impulse response for $\theta = 90^o$: a) time domain and b) amplitude characteristic



a) b)

Fig. 9 a) The normalized acoustic amplification as a function of the angle θ, b) polar characteristic of the acoustic parabolic reflector

3.4. The Results Analysis

Based on the results presented in Tab. 1 and in Figs. 5÷9, the following conclusions can be drawn:

1) Regarding the amplification of the acoustic parabolic reflector:
   a) based on the theoretical assumptions, the amplification at the focal point of the reflector was calculated as a function of the ratio z_F/λ [8]. For sound waves with frequencies f = 20 Hz ÷ 20 kHz the amplification values are: G = -13.681 ÷ 46.319 dB. For f = {20, 100} Hz, there is no amplification, as this reflector has limited efficiency at lower frequencies. With increasing frequency, the amplification value also rises. Amplification depends on the dimensions of the reflector,
   b) this reflector demonstrates high amplification for waves with frequencies f = {1000, 10000} Hz and a ratio z_F/λ = 1.998 ÷ 19.978: G = 20.298 ÷ 40.298 dB. For waves with frequencies f = {10, 20} kHz and for z_F/λ = 19.978 ÷ 39.956 the amplification is G = 40.298 ÷ 46.319 dB.
   These values indicate that the reflector has extreme amplification, and exceptional efficiency, meaning that it enables high directionality of sound waves with minimal losses [10].
2) Regarding the parameters of the acoustic parabolic reflector:
   a) the values for the beamwidth, $\Delta\theta_B = 17.46^o$ and the solid angle $\Delta\Omega = 0.073$ sr indicate that the reflector focuses the reflected sound waves into a relatively narrow beam and that there is greater efficiency in transmitting sound energy to a specific point or in a particular direction [11]. This allows for better sound clarity, but it also means that the coverage of the sound space is limited and
   b) the maximum directivity of the reflector has a value of D_{max} = 172.336, or, D_{max} = 22.363 dB. This value categorizes the analyzed acoustic reflector among those with excellent directivity and confirms the reflector's efficiency in focusing the sound signal.
3) Regarding the amplitude characteristics of the acoustic parabolic reflector:

a) the acoustic reflector, with the sound source positioned on its axis, $\theta = 0^o$, is most efficient for transmitting sound signals with frequencies $f = 10^3$ Hz and $f \approx 10^4$ Hz and sound level values of 21 dB and 23 dB, respectively. The peaks in Fig. 6b, with steep drops on both sides, indicate that the reflector is very precise in its directionality within this narrow range of frequencies [11]. This is also a result of the high amplitude being maintained within the narrow bands of the mentioned frequencies,
b) when changing the position of the sound source in relation to the axis of the reflector, for positions $\theta = 45^o$ and $\theta = 90^o$, the maximum amplitudes of the sound signals significantly decrease. Their levels are 5 dB and from 1 ÷ 3 dB, respectively, in the frequency range $f = 10^2 \div 5 \cdot 10^3$ Hz. In Figs. 7b and 8b, broader maxima can be observed with a slight drop on both sides. This indicates that as the source moves away from the axis, the directivity of the reflector decreases, and the reflector does not efficiently focus the sound, resulting in scattering and a loss of sound quality,
c) the maximum amplitudes for the position of the sound source: a) $\theta = 0^o$ are found in a higher frequency range, and b) $\theta = 45^o$ and $\theta = 90^o$ focus on lower frequencies. This indicates that the reflector performs better at higher frequencies when the source is directed straight toward it.

4) Regarding the polar characteristics of the acoustic parabolic reflector:
   a) the normalized acoustic amplifications $|G/G_{max}|$ as a function of the sound source position, with respect to the angle θ, confirm the maximum directionality of the reflector along its axis and the maximum sound intensity at the focus. This enhances sound quality and reduces noise from other directions [10]. The smaller peaks in Fig. 9a for $|G/G_{max}| = \{0.01, 0.05\}$ indicate that the reflector slightly directs sound in the directions $\theta = \{20^o, 35^o\}$, but this is much weaker than the main beam. This is due to the reflection of the waves [10],
   b) Figure 9 a is presented in a polar coordinate system (Fig. 9b). The normalized power is depicted as the radius r relative to the center. The main peak (leaf-shaped) indicating the maximum power of the reflector for the sound source position $\theta = 0^o$ has the largest radius, $r = 1$. The other peaks, which are symmetric with respect to the main peak (corresponding to the sound source positions $\theta = \{20^o, 35^o\}$), have much smaller radii. The uniform symmetry around the main axis indicates that the reflector evenly directs sound on both sides of the reflector's axis. This means that the reflector's performance remains consistent whether sound waves spread to the left or right of the main axis.

4. CONCLUSION

This paper analyzes the polar characteristics of the acoustic parabolic reflector located in the courtyard of the Academy of Applied Technical and Preschool Studies, Department of Niš, in Niš.

Based on the experiment, which involved recording acoustic impulse responses for acoustic excitation at angles $\theta = -90^o: 15^o: 90^o$, the following results were obtained:

1) the amplification of the reflector is:
   a) $G = 20.298 \div 40.298$ dB for waves whose frequencies are $f = \{1000, 10000\}$ Hz and for $z_F/\lambda = 1.998 \div 19.978$ and

   b) G = 40.298 ÷ 46.319 dB for waves whose frequencies are f = {10, 20} kHz and for z_F/λ = 19.978 ÷ 39.956,
2) the beamwidth of the radiation, $\Delta\theta_B = 17.46^o$ and the solid angle of the radiation $\Delta\Omega = 0.073$ sr,
3) the maximum directivity of the reflector D_{max} = 22.363 dB,
4) the maximum amplitudes for the position of the sound source:
   a) $\theta = 0^o$ are found in a higher frequency range where the sound signal levels are 21 ÷ 23 dB, and
   b) $\theta = 45^o$ and $\theta = 90^o$ focus on lower frequencies where the sound signal levels are 1 ÷ 5 dB,
5) the normalized acoustic amplification: a) $|G/G_{max}| = 1$ for the position of the sound source along its axis and b) $|G/G_{max}|$ = {0.01, 0.05} for the positions of the sound source θ = {20^o, 35^o} and
6) the polar characteristic shows the maximum power of the reflector for the position of the sound source $\theta = 0^o$.

Based on the obtained results, it can be concluded that the analyzed acoustic parabolic reflector shows reduced efficiency at lower frequencies, indicating limitations in sound transmission in that range. However, in the direction of the main axis, it exhibits extreme amplification at higher frequencies, exceptional efficiency, excellent directivity, and maximum normalized acoustic amplification, which is also confirmed by the polar characteristics of the reflector.

Thanks to these characteristics, the analyzed acoustic parabolic reflector is very suitable not only for further scientific research but also for the educational activities of students.

REFERENCES

1. Cormack J. M, Hamilton M. F. (2019), *Transient Solution for the Directional Response at the Focus of a Paraboloidal Reflector*, Acoustical Society America, Proceedings of Meetings on Acoustics, Vol. 35, 045002, pp. 1 – 10.
2. Cremer L., Muller H. A. (1978), *Principles and Applications of Room Acoustics*, Vol. 1, Applied Science, New York, pp. 92 – 96.
3. Sabine W. C. (1927), *Collected Papers on Acoustics*, Harvard.
4. Heifer M. (2009), *Sound Source Localisation with Acoustic Mirrors*, NAG/DAGA, Rotterdam.
5. Wahlstrom S. (1985), *The Parabolic Reflector as an Acoustical Amplifier*, J. Audio Eng. Soc., Vol 33, No. 6, pp. 418 – 429.
6. Dobričić D. (2007), *Efficient feed for offset parabolic antennas for 2.4 GHz*, AntenneX, pp. 1 - 20.
7. Y. T. Tsai, J. Zhua, (2013), *Transient Axial Solution for Plane and Axisymmetric Waves Focused by a Paraboloidal Reflector*, The Journal of the Acoustical Society America, Vol. 133 (4), pp. 2025-2035.
8. Milivojević Z. (2018), *Parabolični reflektor kao akustički pojačavač*, Zbornik radova VTŠSS Niš, pp. 36 – 39.
9. Balanis C. A. (2005), *Antenna Theory and Design – 3rd Edition*, John Wiley &Sons, Inc., Hoboken, New Jersey.
10. Pierce A. D. (2019), *Acoustics: An Introduction to ITS Phynciples and Applications*, Springer.
11. Kraus J. D., Marhefka R. J. (2017), *Antennas for All Applications – 2nd Edition*, Internet Archive HTML5 Uploader 1.6.3.