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ACOUSTIC METAMATERIALS: AN OVERVIEW ON VIBRATION PROPERTIES

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Abstract. *In the paper the theoretical consideration of the acoustic metamaterials is given. Metamaterials, which are usually composite, are artificial materials whose properties differ from those observed in nature or in the constituent materials. Metamaterials which are suitable for acoustic wave absorption are presented. Acoustic absorber is a beam made of solid material connected with spring-mass subunits. The purpose of the subunits is to give a band gap where some frequencies of acoustic wave are stopped. Mathematical models for various types of connection of subunits in the metamaterial and absorber are discussed. Based on the analogy between electromagnetic and acoustic waves the concept of negative effective mass is introduced as a basic principle for modeling. Acoustic metamaterial beams based on one, two or multi-frequency vibration absorbers are discussed. Depending on connection of absorbers in the beam, the structure may absorb wave in one-direction (for example the longitudinal one) or waves in two directions (transversal and longitudinal). In the paper an overview of mathematical models and suggestions for further investigation are given.*

Key words: *mass-in-mass subunit, metamaterial structure, resonance properties, frequency band gap.*

1. INTRODUCTION

Recently, various artificial structures such as acoustic metamaterials have been extensively studied to control the transmission of acoustic waves. Their potential application is in the field requiring the manipulation of group velocities of acoustic waves, for example, in acoustic filters, in precise spatial-spherical and microscopic control, but the main use is in noise reduction systems and acoustic cloaks. Namely, the ‘conservative’ noise absorbers have some limitation of noise reduction due to their dimensions and weight.

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It is shown that at a given frequency the level of sound transmitted through a partition will be reduced by 5-6 dB for every doubling of the mass of the partition, which implies that to improve the transmission loss by 30 dB the partition must become 32-64 times heavier.

To control, direct and manipulate sound waves acoustic metamaterials are designed. The idea for production of acoustic metamatirials issues from the analogy between sound and electromagnetic waves and the properties of mechanical and electromagnetic systems. Using the fact that there is an analogy between the mass density and the bulk modulus with electromagnetic permittivity and permeability, the researchers started a project of acoustic material in 2000. Acoustic metamaterials have to be designed to achieve similar phenomena as electromagnetic metamaterials. As the electromagnetically induced transparency can control optical responses, the acoustic metamaterial is created based on a series of mechanical resonators arranged in a medium to realize slow sound wave at a frequency between the resonant frequencies of the resonators.

Investigation in metamaterials has two main directions: one, in forming the conception of new structures and the other, in designing new types of units which form the material. However, both are based on the concept of conventional vibration absorber [1].

2. CONCEPT OF VIBRATION ABSORBER

As it is well known, the conventional vibration absorber consists of a lumped mass m_2 attached with a linear spring k_2 to the mechanical system with mass m_1 excited with a harmonic force (see Fig. 1). Equations of motion of the two mass system is

$$m_1\ddot{u}_1 + k_2(u_1 - u_2) = F_0 \exp(i\omega t), \tag{1}$$

$$m_2\ddot{u}_2 + k_2(u_2 - u_1) = 0, \tag{2}$$

and ω are amplitude and frequency of the excitation force, while u_1 and u_2 are generalized coordinates. The solutions of equations are in general

$$u_1 = a_1 \exp(i\omega t), \quad u_2 = a_2 \exp(i\omega t) \tag{3}$$

where amplitudes of vibration are

$$\begin{aligned} a_1 &= \frac{F_0(k_2 - m_2\omega^2)}{(k_2 - m_1\omega^2)(k_2 - m_2\omega^2) - k_2^2}, \\ a_2 &= \frac{F_0 k_2}{(k_2 - m_1\omega^2)(k_2 - m_2\omega^2) - k_2^2}, \end{aligned} \tag{4}$$

and $i=\sqrt{-1}$ is the imaginary unit. For this model only one local resonance frequency exists. The vibration absorber uses the 1:1 external resonance between the forcing frequency on the main system ω and the local resonance frequency of the absorber $\omega_2 = \sqrt{k_2/m_2}$ to transform the vibration energy to the absorber and stop the main system's motion (u_1=0).

Let us transform the two-mass system to a single-mass system with effective mass m_{eff} whose motion is the same as that of m_1. The effective mass is defined by treating this two degree-of-freedom system as a one degree-of-freedom one by assuming the internal absorber being unknown to the observer. In other words, the identity of the internal mass m_2 would be ignored and its effect would be absorbed by the introduction of an effective mass m_{eff}.

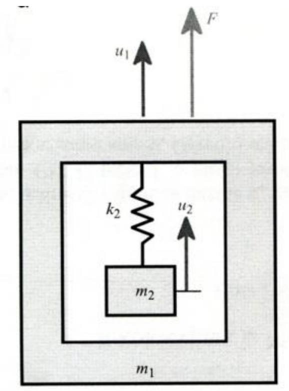

Fig. 1 Mass-in-mass model

If the motion of the mass m_1 is u_1, the effective mass has also the motion u_1. The linear momentums for the both models have to be equal, i.e.,

$$m_{eff}\dot{u}_1 = m_1\dot{u}_1 + m_2\dot{u}_2. \tag{5}$$

Substituting the assumed solution (3) it is

$$m_{eff}a_1 = m_1a_1 + m_2a_2. \tag{6}$$

Motion of the mass m_2 is given with the equation (2). Substituting the assumed solutions (3) it is

$$-m_2\omega^2 a_2 + k_2(a_2 - a_1) = 0. \tag{7}$$

After some modification the equations (6) and (7) yield the effective mass

$$m_{eff} = m_1 + m_2 \frac{k_2}{k_2 - m_2\omega^2}. \tag{8}$$

Introducing $\omega_2 = \sqrt{k_2 / m_2}$ into (8) it is

$$m_{eff} = m_1 + m_2 \frac{\omega_2^2}{\omega_2^2 - \omega^2}. \tag{9}$$

Analyzing the relation (9) it is obvious that the effective mass depends on the ration between the excitation frequency and natural frequency of the system

$$m_{eff} = m_1 + m_2 \frac{\omega_2^2}{\omega_2^2 - \omega^2}, \tag{9}$$

i.e.,

$$m_{eff} = m_1 + m_2 \frac{1}{1 - \frac{\omega^2}{\omega_2^2}}. \tag{10}$$

Three modes of motion are evident: 1) acoustic mode when $\omega<\omega_2$, 2) resonant mode when $\omega=\omega_2$ and 3) optic mode when $\omega>\omega_2$.

For the acoustic mode the effective mass is positive. In the resonant mode the effective mass is theoretically infinite. In the optical mode the effective mass is negative for

$$m_1 + m_2 \frac{1}{1 - \frac{\omega^2}{\omega_2^2}} < 0. \tag{11}$$

Otherwise, it is positive. For a parameter values which satisfy the condition (11) the relation for variation of the effective mass as the function of the frequency is plotted in Fig. 2.

Differentiating the relation (5) we have

$$(m_{eff} - m_1)\ddot{u}_1 = m_2\ddot{u}_2. \tag{12}$$

Substituting (12) into (2) we obtain

$$k_2(u_2 - u_1) = -(m_{eff} - m_1)\ddot{u}_1. \tag{13}$$

Equation (1) and (13) give

$$F_0 \exp(i\omega t) = -m_{eff}\ddot{u}_1. \tag{14}$$

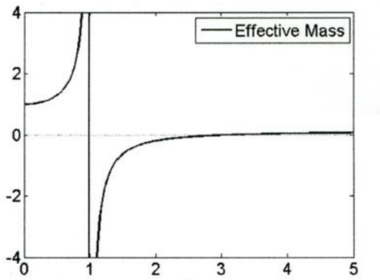

Fig. 2 Dimensionless effective mass m_{eff}/m_1 as a function of ω/ω_2 [3]

The effective mass is the ratio between the excitation force and acceleration of the mass m_1

$$m_{eff} = \frac{F}{\ddot{u}_1} = -\frac{F_0}{\omega^2 a_1} = -\frac{F}{\omega^2 u_1}. \tag{15}$$

If $\omega_2=\omega$ the effective mass tends to infinity. For that value the motion of mass m_1 is zero and the inertial force of the mass m_2 is equal to the excitation force: $F(t) = m_2\ddot{u}_2$.

So, the external force is eliminated with the inertia force $-m_2\ddot{u}_2$ through the spring k_2. This is the concept of vibration absorbers.

From the previous consideration and the Fig.2 following is concluded:

If $\omega<\omega_2$ and the effective mass m_{eff} is positive in the acoustic mode, the motions u_1 and u_2 are in phase. If $\omega>\omega_2$ and the effective mass m_{eff} is positive or negative in the optical mode, the displacements u_1 and u_2 are 180^0 out of phase. Then, the absorber works efficiently in the optical mode against the external acting on the mass m_1. The excitation is absorbed with the inertial force.

Remark: *The concept of vibration absorber from linear to nonlinear one is extended in the paper* [2].

3. METAMATERIAL STRUCTURES

Santillan and Bozhevolnyi [4,5] formed an acoustic metamaterial based on series of resonators described in the previous section. The resonators are arranged along a tube. This metamaterial realizes slow sound waves at a frequency between the resonant frequencies of detuned resonators. An improvement of the model is done by Li *et al*. [6]. They created an acoustic metamaterial on the basis of a simple and compact structure composed of solely one string of side pipes settled along a main wave guide other than asymmetric resonators. Using this metamaterial, the group velocities of acoustic waves can be manipulated to simultaneously realize diverse group velocities in one structure as negative group velocities: fast and slow waves. Near the resonant frequency in the metamaterial negative phase time is obtained and fast and slow acoustic waves are achieved.

In [7] a type of composite is presented which displays localized sonic resonances at 350-2000 Hz with a microstructure size in the millimeter to centimeter range. The schematic structure of the composite is plotted in Fig.3. “A” denotes a lead solid particle in centimeter diameter, “B” denotes a silicon rubber layer and “C” denotes the matrix material of epoxy.

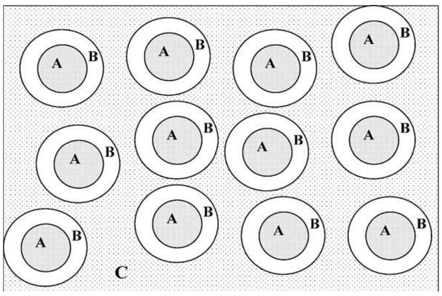

Fig. 3 Schematic structure of the composite [7]

Around the resonance frequencies the composite behaves as a metamaterial with effective negative elastic constraints and as a total wave reflector. When the microstructure is periodic this composite exhibits large elastic wave band gaps at the sonic frequency range.

Based on the theory given in [7], Sheng et al. [8] formed practically the composite with coated spheres which can exhibit resonance frequency in a fluid. The basic structure unit of locally resonant sonic material (LRSM) is a sphere core coated with a layer of silicon rubber (Fig. 4).

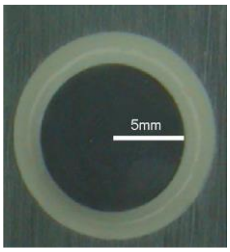

Fig. 4 Basic structure unit of LRSM [8]

As a resonator, the core particle acts as the mass of the oscillator and the coating is a spring. If the fluid medium exhibits a resonant behavior at the same frequency as the coated spheres the acoustic absorption occurs. The question remains: How this can be realized?

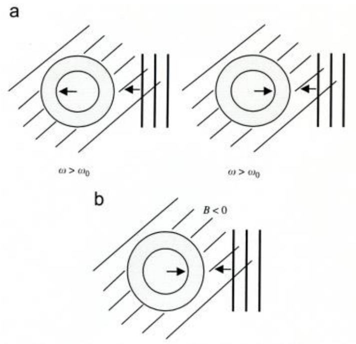

Fig. 5 a) Schemes of relations between coated particles and medium,
b) Scheme of acoustic metamaterial [8]

A dense collection of structure units LRSM with the matrix material, having a lighter density than the core particle, realizes negative dynamic density within the resonance frequency range. When the core particle oscillates in-phase with the wave in the matrix medium ($\omega<\omega_2$), the dynamic mass density must be positive, as shown in Fig. 5a left. When $\omega>\omega_2$ the core particle oscillates out of phase with the wave in the matrix medium as shown in Fig.5a right, the dynamic mass density can be negative, provided the density of the coated particles is sufficiently high. The core particles have a higher density than that of the matrix medium. Acoustic metamaterial is realized when the dynamic mass density is negative and addition the fluid medium is also negative with negative bulk modulus as a result (Fig. 5b).

In the paper [9] the realization of a structure which responds as if it had negative mass to oscillations at a fixed frequency above resonance is suggested (Fig.6). The spherical lead core oscillates out of phase with the rigid, but light, surrounding shell. For this time harmonic motion the inertial force on the outside shell is in the same direction as the acceleration of the outside shell (but in the opposite direction to the acceleration of the 'hiden' lead core). Thus if the body is shaken at that frequency it will feel like it has negative mass. If the core was ellipsoidal in shape, or the rubber was replaced by a suitably anisotropic material, the effective mass of the body would be anisotropic.

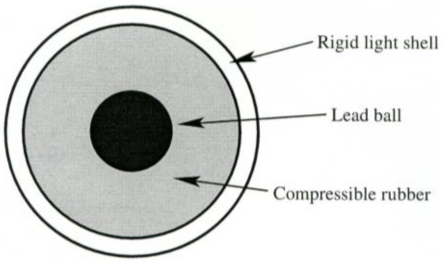

Fig. 6 An element with negative effective mass [9]

The suggested metamaterial made of elements given in [8] is experimentally tested in [10-12]. It was concluded that the density of metamaterial can be negative over a range of frequencies. Composites behave as anisotropic materials with complex density which depends on the frequency of oscillation. Milton and Willis [11] gave the theoretical consideration for dynamics of seemingly rigid bodies with composite structure. The system is modeled as one-dimensional (Fig. 7) where *n* cylindrical cavities of length *d* are carved out from a beam of rigid material. In the center of each cavity is a lead sphere of mass *m* and radius *r* which is attached to the ends of the cavity with two springs each having the same spring constant *K*. Excitation is harmonical with frequency ω. For the resonant case it is $\omega^2=2K/m$. It is worth to say that the model is formed using the model of semiconductor where it is already known that the electrons and holes can have anisotropic effective masses due to their interaction with periodic potential. According to (8) the effective mass of the model in Fig.6, i.e., effective density for the model is

$$m_{eff} = M_0 + nm\frac{2K}{2K - m\omega^2}, \tag{16}$$

where M_0 is mass of the rigid beam. The suggested model has the characteristic feature that its macroscopic properties depend on the resonant properties of substructures. Such resonant substructures give rise to new effects. In the models the motion of the rigid material violates Newton's law owing to the vibration of the internal masses: force equals mass times acceleration only applies if mass is replaced by effective mass which is non-local operator in time (a function of frequency under any purely harmonic excitation). In the model, the macroscopic velocity is not the averaged velocity in composites with voids, because it is unclear what to take for the velocity in the void phase. There was a need to generalize the continuum elastodynamic equations which govern the effective response of bodies with or without voids. The weighted average of the local velocity is introduced as the macroscopic velocity where the weighting function is zero in the void phase on in 'hidden regions' which are not accessible to measurement.

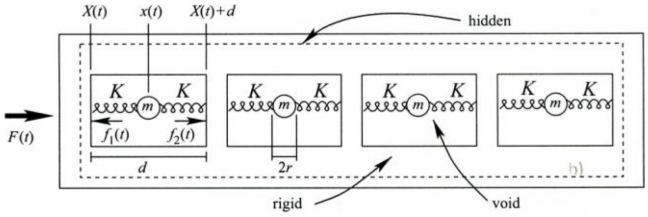

Fig.7 A one-dimensional material where the effective mass depends on the frequency ω and can be negative [13]

Instead of using springs one can fill each cavity with an elastically anisotropic (and possibly viscoelastic) material with the lead sphere being inserted in the center.

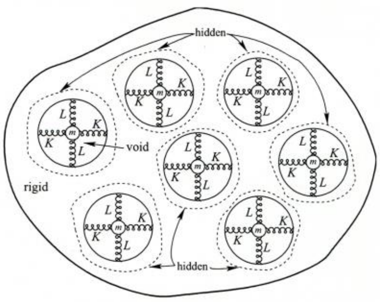

Fig. 8 The two-dimensional model [8]

In the paper [13] the model is extended to a two-dimensional one where the orthogonal springs in horizontal and vertical direction have the rigidity K and L (Fig.8). The dependence of the effective mass on frequency of the periodic force and the anisotropic property of the material is evident. The further extension is to a model with different unit cells (Fig. 9).

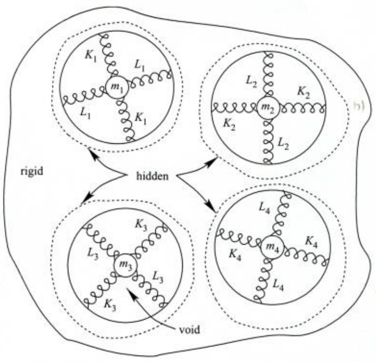

Fig. 9 The generalized two-dimensional model [6]

In the paper [9] a new unit cell model is suggested. The masses are approximated as point masses in the unit cell of the model of metamaterial (Fig.10). All springs which respond to elastic material have the same constant of rigidity.

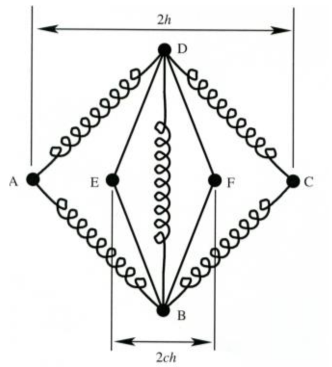

Fig. 10 A unit cell of the metamaterial [9]

The unit cell of a continuum model, which is conjectured that approximates the behavior of the discrete model, is plotted in Fig.11.

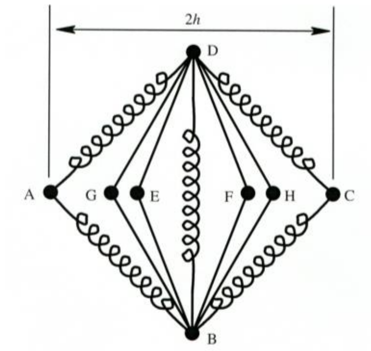

Fig. 11 A unit cell of a continuum model [9]

The black disks are heavy masses, and the remaining black areas are rigid but light material. The shaded areas are compressible material.

The material surrounding the two black disks on the left side of the unit cell is sufficiently stiff that their motion is in phase with that of the surrounding rigid material, giving a positive effective mass. The material surrounding the two black disks on the right side of the unit cell is sufficiently compliant that their motion is out of phase with that of the surrounding rigid material, giving a negative effective mass.

The triangular regions of material are highly compressible and are introduced so the junctions behave like hinges. The remaining area is void, or to make it a proper continuum, filled with a light highly compressible material. Unit cells are connected and form the model of the metamaterial (Fig. 12).

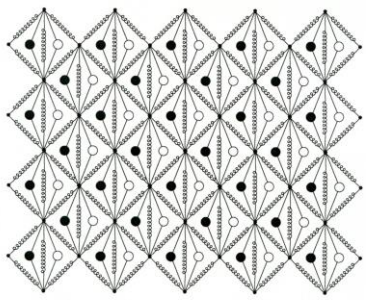

Fig. 12 A suggested model of metamaterial [7]

The model of metamaterial is expected to have strange elastic behavior. The large solid black circles have positive mass, while the neighboring large white circles have negative mass at the given frequency. They are connected to the spring network by rods of fixed length, which alternatively can be regarded as springs with infinite spring constants.

In Fig.13 the fabricated lattice metamaterial beam is shown.

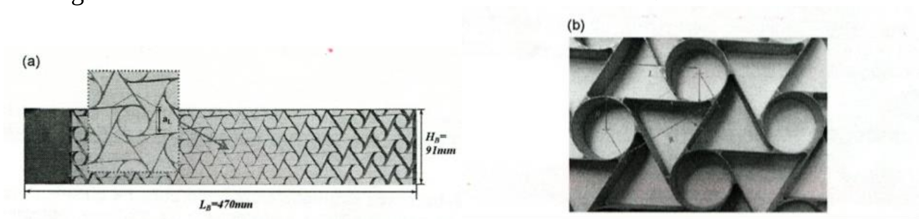

Fig. 13 a) The fabricated chiral lattice beam and its zoomed unit cell and b) the topology of the hexagonal chiral lattice [14]

The model is suitable to eliminate the propagation of the dispersive wave in the lattice for certain frequencies [14]. In Fig. 14 the measured (open circles) and calculated (solid line) transmission coefficients as a function of frequency is plotted (Fig.14). The experimentally obtained values are in good agreement with calculated values. The result proves the theoretical consideration.

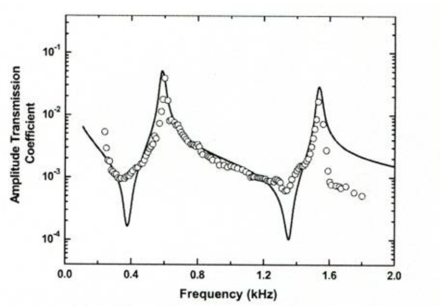

Fig. 14 Measured (open circles) and calculated (solid line) transmission coefficients as a function of frequency [7]

Finally, the structure in Fig. 13 is non-homogenous. If it is required that the metamaterial behave like a homogeneous material described by its averaged material properties, its subunits must be much smaller than the shortest wavelength of waves propagating in it. The averaging may result in the existence of a useful but mysterious phononic stop-band that allows no waves within that frequencies range to propagate forward, and most current designs of acoustic metamaterials are based on the stop-band effect [13]. To manufacture such metamaterials with tiny subunits in order to have stop-bands, expensive manufacturing techniques are required including micro and nano manufacturing technologies.

4. Conclusion

Theoretically a significant number of model of acoustic metamaterials are developed. Their property is quite strange as they have negative effective mass i.e., negative effective density. One and two dimensional models are developed. Based on this concept the three-dimensional models are also available. The experiments on some of the suggested artificial metamaterials are done. The results show their property to have a gap for certain frequency or frequency range which gives the possibility to be applied as acoustic absorbers. In future the presented metamaterial structures have to be realized and experimentally tested.

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AKUSTIČKI METAMATERIJALI: PREGLED VIBRACIONIH KARAKTERISTIKA

U ovom radu je dat pregled teorije akustičkih metamaterijala. Metamaterijali, koji su obično kompoziti, su veštački stvoreni materijali čije se osobine razlikuju od onih koji odgovaraju prirodnim materijalima. Navedeni su metamaterijali koji su pogodni za absorpciju akustičkih talasa. Akustički absorber je štap napravljen od čvrstog materijala za koji je vezan veći broj celina tzv. elementarnih absorbera sastavljenih od opruge i mase. Ugradnja ovih elementarnih absorbera obezbedjuje zaustavljanje akustičkih talasa u odrdjenoj oblasti frekvencija. U radu su razmotreni različiti tipovi veza elementarnih absorbera za osnovnu materiju forrmirajući metamaterijal. Na osnovu analogije izmedju elektromagnetnih i akustičkih talasa uveden je koncept negativne efektivne mase kao osnovnog elementa za modeliranje. U radu su prikazani akustički metamaterijali sa absorberima jedne, dve ili više frekvencija. U zavisnosti od veze absorbera u štapu struktura može da absorbuje talase u jednom pravcu (na primer, longitudinbalnom) ili u dva (poprečnom i longitudinalnom).U radu je dat pregled matematičkih modela koji opisuju vibraciona svojstva i daju smernice za dalja istraživanja.

Ključne reči: *elementarni absorber, struktura metamaterijala, rezonantne karakteristike, pojas absorbovanih frekvencija.*