FACTA UNIVERSITATIS
Series: **Working and Living Environmental Protection** Vol. 11, N^o 1, 2014, pp. 13 - 21

PARTICLES CHARGING IN TUBULAR ELECTROSTATIC PRECIPITATORS WITH POLYGONAL COLLECTION ELECTRODES

UDC 621.359:620.172.24:621.3.035.22

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Abstract. *The aim of this paper is to eliminate the ambiguities that appear in the literature relating to the saturation charge of particles in electrostatic precipitators. Investigation of the influence of collection electrode geometry on electric field strength in electrostatic precipitator, the saturate charge of particle that directly depends on, is the second main goal. The numerical results for electric field and potential distribution for various cross sections (polygonal) of tubular electrostatic precipitator geometry are shown. This analysis may lead to a quantification of the efficiency of the electrostatic precipitator depending on the geometry of collection electrodes, whose shape affects the distribution of the electric field, thus creating the possibility of design improvement.*

Key words: *electrostatic precipitator, collection electrode, polygonal cross section, saturation charge, point matching method.*

1. INTRODUCTION

The corona discharge in the electrical precipitation of smoke particles from gases was first described by the German mathematician M. Johann Cristoph Hohlfeld in 1824. The German physicist Robert Nahrwold, in 1878, noticed that the great collection of atmospheric dust appeared on electrified metal cylinder in the middle of which the sewing needle was placed. The first unsuccessful attempt to remove lead fume from smelting works was done by Sir Oliver Lodge in 1885. The first commercial electrostatic precipitator (ESP) was constructed in 1906 and patented in 1908 by an American Physical Chemist Dr Frederick Gardner Cottrell, [1].

Since then, efforts have been made to increase the efficiency of these precipitators, especially because it has already proven to be irreplaceable in collecting nanoparticles. Mathematical models of different complexity have been developed for the description of

Received June 30, 2014 / Accepted September 22, 2014
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the collecting particles mechanism and consequently for calculation of theoretical efficiency [2, 3]. On the other hand, much attention is given to experimental research which usually does not comply with theoretical results, [4, 5]. Finally, the rapid development of information technology has enabled a detailed simulation of work of ESP [6, 7], but theoretical approach is still a major challenge. In tubular ESP, the shape of collection electrode has strong influence on electric field strength and, therefore, on the overall efficiency. This influence is being considered in this paper.

1.1. Theoretical efficiency of ESP

A number of factors affect the ability of the ESP to collect particles. They can be divided into three main groups. The first group includes the mechanical and electrical characteristics of the particles. The second group covers physical and chemical properties of the carrier fluid. Finally, the third group implies structural design of ESP (tubular or plate), method of charging (single-stage or two-stage), the temperature of operation (cold-side or hot-side), the method of particle removal from collection areas (wet or dry). The most important features that are applied are electric potential, shape and size of discharge and collection electrodes.

Whilst the qualitative effects of these factors on the collection performance are well known, the quantitative effects are not so clear, and therefore, a well known Deutsch-Anderson collection formula [8], in spite of numerous flaws, is still the basic in design of the ESP.

$$\eta = 1 - e^{-\frac{wS}{Q}} \tag{1}$$

Where: η is particle collection efficiency,
S is total surface area of precipitator collection electrodes,
Q is gas volumetric flow rate through the ESP tubes, and
w is particle terminal velocity.

Volumetric flow rate and active surface of collection electrodes are easily changeable quantities. On the other hand, it is not easy to determine the velocity of the particles, and it is even more complex to affect its value.

1.2. Particle theoretical terminal velocity

Motion of particles in a fluid is determined by the balance between the internal and the external forces acting on a particle of the radius a , (see Fig. 1).

$$\sum F = 0 \text{, i.e. } F_E + F_G = F_I + F_B + F_D \tag{2}$$

Here: $F_E = qE$ is electrostatic force, q is particle charge and E is electric field strength.

$F_G = m_p g = \rho_p V_p g$ is gravitational force, where m_p, ρ_p, V_p are mass, density and volume of a particle respectively and g is average acceleration due to gravity.

$F_I = m_p \frac{\mathrm{d}w}{\mathrm{d}t}$ is inertial force on a particle moving with velocity w.

$F_B = m_f g = \rho_f V_p g$ is buoyant force where ρ_f is fluid density.

$F_D = 6\pi \mu a w / C_c$ is Stokes' drag force in a fluid with viscosity μ. Cunningham slip correction factor C_c is used for small particles drag force calculations.

In describing the settling of particles in air, the density of the air ρ_f is much less than the particle density ρ_P and there for $F_B << F_G$. Furthermore, gravitational force could be neglected due to small mass of a particle, e.g. in practice electrical force is essentially much stronger than the gravitational force ($F_E \sim 10^{39} F_G$). Finally, remaining forces give the equation of a particle motion and the corresponding solution.

$$\frac{\mathrm{d}w}{\mathrm{d}t} + w\frac{6\pi \mu a}{C_c m} = \frac{q}{m}E \qquad \Rightarrow \qquad w = \left(1 - \exp\left(-\frac{6\pi\mu a}{mC_c}t\right)\right)\frac{qEC_c}{6\pi\mu a} \tag{3}$$

After a short period of time exponential term can be neglected which means that

$$w = \frac{qEC_c}{6\pi\mu a} \tag{4}$$

The quantity w is theoretical migration velocity (component of particle velocity perpendicular to the collecting walls), also known as drift, terminal or settling velocity. Obviously, this is the solution of steady state problem. It should be emphasized that due to the electric force and the drag force of a medium which have the dominant values, inertial term could be omitted at the outset. This formula for theoretical terminal velocity do not take into account many factors such as inertia, fluid velocity or inhomogeneity of electric field strength distribution.

Diameter of the particles depends on the technological process. Slip factor is determined by the particle size and the mean free path length of the particles [8]. For this reason, these parameters can not be influenced upon. Definitely, in the design of ESP, it is possible to affect only the magnitude of charge and electric field strength.

2. Saturation Charge of a Particle

Dielectric sphere of permittivity ε and of radius a is placed in permittivity vacuum ε_0. External uniform electric field E_0 is oriented in z-direction. Due to the axial symmetry the potential φ does not depend on the azimuth coordinate and the distribution of electric scalar potential can be obtained by integration of two dimensional Laplace's equation in spherical coordinates. The governing equation and boundary conditions at origin, at the interface between the two dielectric media and at infinity lead to a well known solution for internal and external region, respectively, [9, 10].

$$\varphi(r,\theta) = \begin{cases} -\dfrac{3\varepsilon_0}{\varepsilon + 2\varepsilon_0} E_0 r\cos\theta, & r \le a \\ -E_0 r\cos\theta + \left(\dfrac{a}{r}\right)^3 \dfrac{\varepsilon - \varepsilon_0}{\varepsilon + 2\varepsilon_0} E_0 r\cos, & r \ge a \end{cases} \tag{5}$$

In the vicinity of the sphere the potential is equal to the sum of the potentials of the primary and induced electric fields.The influence of induced electrical charge can be replaced by an equivalent electric dipole with electric dipole moment $\vec{p}$.

$$\vec{p} = 4\pi\varepsilon_0 \frac{\varepsilon - \varepsilon_0}{\varepsilon + 2\varepsilon_0} a^3 \vec{E}_0 \tag{6}$$

The surface density of the induced charge and, consequently, the total or saturation charge of a particle are then

$$\eta = \frac{p}{V} = 3\varepsilon_0 \frac{\varepsilon - \varepsilon_0}{\varepsilon + 2\varepsilon_0} E_0 \quad \Rightarrow \quad q = \eta S = 12\pi\varepsilon_0 a^2 \frac{\varepsilon - \varepsilon_0}{\varepsilon + 2\varepsilon_0} E_0 \tag{7}$$

The formula (7) without ε_0 in numerator is commonly attributed to Pauthenier. Almost all the papers [11, 12, 13] relating to ESP, apparently suffering from the propagation of this result produce, as we might say, the effect of traveling mistake. In the limiting case when $\varepsilon \to \infty$, i.e. when the particle is perfect conductor, the above formula degenerates in form that can be found in literature.

As mentioned before, a diameter of particle is an unchangeable parameter as well as dielectric constant and the only point, in process of precipitation of particles at which it is possible to improve the efficiency of ESP is a strength of electric field. Substituting (7) in (4) gives,

$$w = \frac{2aC_c}{\mu} \frac{\varepsilon_0(\varepsilon - \varepsilon_0)}{\varepsilon + 2\varepsilon_0} E_0 E \tag{8}$$

and it is necessary to distinguish between two electric fields. The first E_0 makes the ionization of the particles in the vicinity of ionizing (inner, corona or discharge) electrode, while the second E attracts the particles to the collection (outer) electrode.

3. Electric Field in Tubular Electrode System of ESP

The electrode system of tubular electrostatic precipitator is composed of the inner tube at a high potential $\varphi = U$ and an outer coaxial cylinder, which is grounded $\varphi = 0$. The cross section of the inner electrode is a circle of radius R_1. The outer electrode has an equiangular polygon cross section which is inscribed in a circle of radius R_2. Cylindrical coordinate system is assumed. The central angle of N -gon is $\alpha = 2\pi / N$, where N is the number of vertices (Fig. 1).

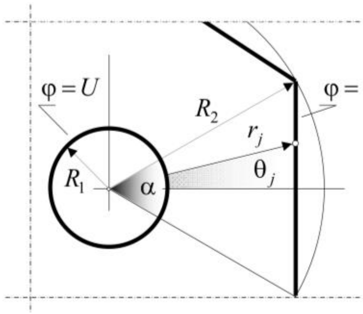

Fig. 1 Tubular ESP cross section geometry

The solution of Laplace's equation that satisfies boundary conditions on the inner electrode must be even and α-periodic function in all cases, including the circle $N \to \infty$, it can be obtained from the general solution, [10].

$$\varphi(r,\theta) = U + \varphi_0 \ln\frac{r}{R_1} + \sum_{i=1}^{\infty}\varphi_i\left[\left(\frac{r}{R_1}\right)^{iN} - \left(\frac{r}{R_1}\right)^{-iN}\right]\cos(iN\theta) \tag{9}$$

The constants of integration φ_0 and φ_i should be determined by satisfying boundary conditions $\varphi = 0$ on outer electrode $r = R_2$. Only in the trivial case, when the outer electrode is circle, this can be done exactly. In this case, potential depends only on the radial coordinate and the well known result will be obtained, [9].

$$\varphi(r) = U - \frac{U}{\ln\frac{R_2}{R_1}}\ln\frac{r}{R_1}, \qquad E_r = -\frac{\partial\varphi}{\partial r} = \frac{1}{r}\frac{U}{\ln\frac{R_2}{R_1}}, \qquad E_\theta = 0 \tag{10}$$

As there is no analytical solution of Laplace's equation for a circular-polygonal geometry, a numerical method should be used. Suitable choice is the method of moments [14]. In that sense, the approximate solution of the problem will be assumed in the form of the series (9) that should be truncated at some finite order M. Boundary condition should be satisfied in $M+1$ discrete points on outer electrode and this is equivalent to using a delta function as the weighting function in the method of moments. This choice is referred to as the point matching method. On that way, the ansatz (9) converts into a system of linear equations.

Matching points should be evenly spaced in the angle $0 \le \theta \le \alpha/2$.

$$\theta = \theta_j = \frac{j\pi}{NM}, \qquad r = r_j = R_2\frac{\cos(\pi/N)}{\cos\theta_j}, \qquad j = 0,1,\ldots,M \tag{11}$$

The system of linear equations has to be solved for the unknown coefficients φ_i. In the special case $M = 0$ the system of linear equations is reduced to the single linear equation. At the point given by $j = 0$ electric potential should be matched to zero and the solution gives zero order approximation for electric potential distribution and electric field strength. These results should be compared (10).

$$\varphi = U - U\frac{\ln\frac{r}{R_1}}{\ln\left(\frac{R_2}{R_1}\cos\frac{\pi}{N}\right)}, \ldots\ldots E_r = \frac{1}{r}\frac{U}{\ln\left(\frac{R_2}{R_1}\cos\frac{\pi}{N}\right)} \tag{12}$$

In the general case, the electric field has two components, which are determined as a potential degradient (9).

$$E_r = -\frac{\partial\varphi}{\partial r} = -\frac{\varphi_0}{r} - \sum_{i=1}^{M}\frac{iN}{R_1}\varphi_i\left[\left(\frac{r}{R_1}\right)^{iN-1} + \left(\frac{r}{R_1}\right)^{-iN-1}\right]\cos(iN\theta) \tag{13}$$

$$E_\theta = -\frac{\partial\varphi}{r\partial\theta} = \sum_{i=1}^{M}\frac{iN}{r}\varphi_i\left[\left(\frac{r}{R_1}\right)^{iN} - \left(\frac{r}{R_1}\right)^{-iN}\right]\sin(iN\theta) \tag{14}$$

On the surface of the inner electrode $r = R_1$ tangential component of electric field is equal to zero which is consistent with the boundary conditions. The charge per unit length on this conductor has to be calculated from the normal component of the electric field. Integration over Gaussian surface gives zero contribution for all terms but φ_0 .

$$q' = R_1\varepsilon_0 \int_0^{2\pi} E_r(r = R_1)R_1 \mathrm{d}\theta = -2\pi\varepsilon_0\varphi_0 = C'U \tag{15}$$

The capacitance per unit length should be used for testing the convergence of applied method. However, it can be shown that satisfactory accurate results are obtained with only a few matching points, [10, 15]

On the inner surface of the outer electrode $r = R_2$, boundary conditions for tangential component of an electric field and electric potential are satisfied only approximately. In fact, the boundary condition is satisfied exactly only at the matching points.

4. Numerical Results

This section deals with the presentation of numerical results for electric potential and electric field within tubular electrode system of ESP with collection electrodes of a polygonal type. These results are obtained using the proposed method. In addition, the presented illustrative results for distribution of electric potential are obtained using finite element method and shown in Fig. 3.

The numerical results for electric potential according to the zero order approximation formula (12) are given in Table 1, for various number of vertices N and ratio of radii R_2/R_1. As expected, the greatest potential gradient occurs in a triangular cross section, and is reasonable to assume that such an electrode system can be more efficient than one with a circular cross section.

The results for capacity per unit length, depending on the number of matching points are shown in Table 2. When N is a large number, polygon tends to circle and well known results for the capacitance per unit length of a conventional coaxial lines are obtained, even with a few matching points. Hence, it is also seen that the zero order approximation formula is quite accurate for design purposes. The strength of an electric field in the vicinity of the ionizing electrode is even greater if the number of vertices of the polygon is less. This fact again leads to the conclusion that greater efficiency can be expected with this kind of ESP

Table 1 Zero order approximation, $-\varphi_0 / U$

R_2 / R_1	$N = 3$	$N = 4$	$N = 6$	$N \to \infty$
2	∞	2.8854	1.8205	1.4427
5	1.0914	0.7919	0.6823	0.6213
10	0.6213	0.5112	0.4632	0.4343
20	0.4343	0.3775	0.3506	0.3338
50	0.3107	0.2805	0.2654	0.2556

Table 2 Test of convergence, $-C'/(2\pi\varepsilon_0)$, $R_2/R_1=5$

M	$N=3$	$N=4$	$N=6$	$N=10$
0	1.0914	0.7919	0.6823	0.6413
1	0.8955	0.6005	0.4663	0.3947
2	0.9329	0.6991	0.5882	0.5305
5	0.9590	0.7380	0.6446	0.6014
10	0.9610	0.7446	0.6581	0.6211

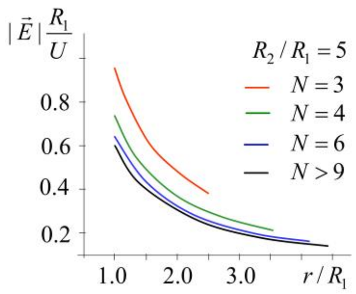

Fig. 2 Normalized electric field, $M=5$, $\theta=0$

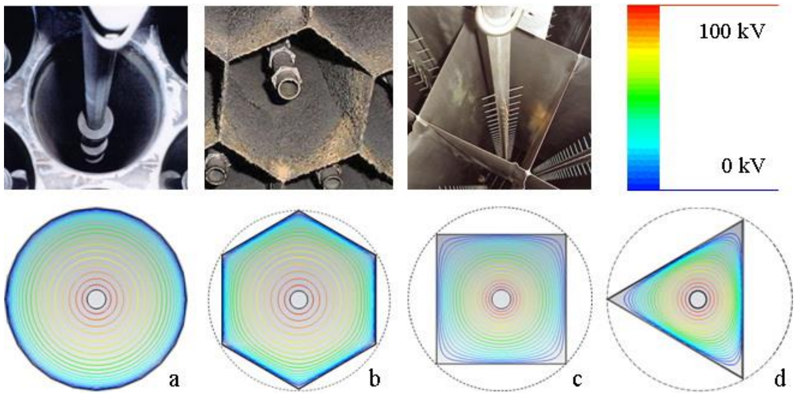

Fig. 3 Electric potential in tubular ESP electrode system
a) circular, b) hexagonal, c) square, d) triangular

5. Summary and General Conclusion

This paper highlights the need for analytical relationship between overall efficiency of a tubular ESP and the shape of the collection electrode. In the second part of the paper, the formula for saturation charge is developed. This should be useful in the correction of errors that appear in the reference lists. On the other hand, it is the quantity which can be directly affected by the constructive characteristics of the ESP, such as cross section pattern. The distribution of an electric potential and electric field in tubular ESP with collection electrodes of polygonal type are discussed in the third part of the paper. The approximate but satisfactory accurate formulas for these distributions have been derived. This could be further utilized to develop the relationship between electrode shape pattern and overall ESP efficiency. In the immediate vicinity of the electrodes, the angular component of the electric field can be ignored, and according to (8), expression (16) is approximately correct, as it was supposed to report to.

$$w = \frac{2aC_c}{\mu} \frac{\varepsilon_0(\varepsilon - \varepsilon_0)}{\varepsilon + 2\varepsilon_0} \frac{U^2}{R_1 R_2} \ln^{-2}\left(\frac{R_2}{R_1}\cos\frac{\pi}{N}\right) \tag{16}$$

Similar considerations can be carried out in other areas of applied electromagnetics, such as the analysis of atypical coaxial lines [16].

Acknowledgement. *The paper is a part of the research done within the project No. III 43012, The Ministry of Education, Science and Technological Development of the Republic of Serbia*.

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NAELEKTRISAVANJE ČESTICA U CEVASTIM ELEKTROSTATIČKIM FILETERIMA SA POLIGONALNIM TALOŽNIM ELEKTRODAMA

Cilj rada je da ukaze na nejasnoće koje se javljuju u literaturi vezano za izraze za brzinu taloženja čestica i količinu naelektrisanja u elektrostatičkim filterima. Ispitivanje uticaja geometrije taložne elektrode na jačinu električnog polja u elektrostatičkim filterima, od koje direktno zavisi i količina naelektrisanja, je drugi predmet istraživanja. U radu su prikazani numerički rezultati za raspodelu električnog polja i potencijala za različite poprečne preseke mnogougaone taložne elektrode. Ova analiza treba da dovede do konstitutivne veze između oblika taložne elektrode, koja najbitnije utiče na jačinu električnog polja, i efikasnosti elektrostatičkog filtera što pruža mogućnost poboljšanja performansi filtera u fazi projektovanja.

Ključne reči: *elektrostatički filter, taložna elektroda, zasićenje naelektrisanjem, poligonalni poprečni presek, metod podešavanja u tačkama.*