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STRESSING ISSUE OF A PIEZOCERAMIC CANTILEVER WITH ELECTRODE COATINGS AND TRANSVERSAL POLARIZATION

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Abstract. *This paper presents a general case of stressing a rectangular piezoceramic cantilever with transversal polarization which is loaded at the free end by a concentrated force. Two mutually opposite surfaces of the rectangular cantilever are with electrode coatings on which an excitation electric voltage is applied to. By applying the reverse method for solving the problems of electroelasticity theory, componential displacements, electric potential, specific strains, electric fields and piezoelectric displacements are determined for the rectangular piezoceramic cantilever made from PZT4 piezoceramic material.*

Key words: *stress, rectangular piezoceramic cantilever, transversal polarization, PZT4 piezoceramic material*

1. INTRODUCTION

In application of mathematical theory of electroelasticity where the subjects of studying are different piezoelectric bodies of concrete dimensions versatile tasks and problems can appear [1, 2]. In general, depending on what is known, all tasks of the linear theory of electroelasticity may be ranked into three groups: body loading (mechanical, electric, or combined), conditions on boundary surfaces of the observed body, or displacements of points on the surface of the electroelastic piezoelectric body [3].

Regarding the choice of unknown variables, there are three mathematical methods to solve the problems of the theory of electroelasticity: direct method, reverse method and semi-reverse method. Beside analytical and numerical methods, experimental examinations of stress and strain state of stressed electroelastic body take a significant place.

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Piezoelectric cantilever beams have received considerable attention for vibration-to-electric energy conversion [4, 5, 6 and 7]. The use of a piezoelectric unimorph cantilever allows both electrical actuation and electrical sensing. Cantilever piezoelectric power generators are being used because of their high strain and high power output even under lower acceleration amplitudes. This paper considers a general case of stressing rectangular prismatic piezoceramic cantilever with transversal polarization and electrode coatings on the two mutually opposite surfaces $z = \pm h/2$, loaded on the left free end by a concentrated force, vector of external loading $\vec{F}$ which is aimed in direction of the axis *Oz* (Fig. 1). It is assumed that electric potential difference $2U_0$ is applied on electrodes. Furthermore, it is also assumed that the effect of the electromechanical characteristics of the electrode coatings may be neglected. Coordinate system *Oxyz* is set at the free end of the cantilever. Axes *Oy* and *Oz* are main central axes of inertia of the cross section, while axis *Ox* is geometric axis. Axis *Oz* is directed downwards.

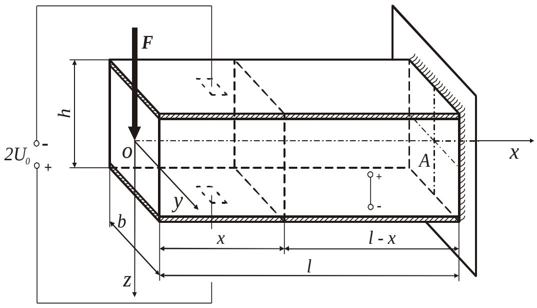

Fig. 1 Stressing of the piezoceramic cantilever with transversal polarization and electrodes

2. Fundamental Equations

According to the hypothesis of Журавский, for a cantilever loaded at the free end by a concentrated force $\vec{F}$, there are only normal stress σ_x in axial direction and tangential stress τ_{xz} in plane of the cross section, aimed in direction of the force $\vec{F}$ [8]. Volume forces are neglected, so the stresses are given by stress tensor matrix:

$$\mathbb{N} = \begin{bmatrix} \sigma_x & \tau_{xz} & 0 \\ \tau_{xz} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{1}$$

Since the dimensions of the cross section of the rectangular cantilever are small in regard to the length *l* ((*b*/*h*)<<1, (*b*/*l*)<<1), this stress state can be considered as planar, in plane *Oxz*. From strength of materials it is known that componential stresses for this case of stressing are [8]:

$$\sigma_x = -\frac{F}{I_y} xz\,, \qquad \tau_{xz} = -\frac{F}{8I_y}(h^2 - 4z^2) \tag{2}$$

while componential mechanical stresses on the sides y=±b/2 have values:

$$\sigma_y\Big|_{y=\pm\frac{b}{2}} = 0, \quad \tau_{yx}\Big|_{y=\pm\frac{b}{2}} = 0, \quad \tau_{yz}\Big|_{y=\pm\frac{b}{2}} = 0 \tag{3}$$

In agreement with the hypothesis of Журавский, componential mechanical stresses are equal to zero in the internal points of the piezoceramic rectangular cantilever. Also, according to this hypothesis, an assumption is introduced that componential displacements of the body points u and w are independent from coordinate y, i.e.:

$$u = u(x,\ z), \quad v = v(x,\ z) \tag{4}$$

For a piezoceramic cantilever with transversal or longitudinal polarization an additional assumption is introduced that the component of the piezoelectric displacement vector in direction of axis Oy is equal to zero:

$$D_y = 0 \tag{5}$$

therefore, the function of electrostatic potential ψ is independent from coordinate y:

$$\psi = \psi(x,\ z) \tag{6}$$

Boundary conditions on the sides z=±h/2 are expressed in the following way:

$$\sigma_z\Big|_{z=\pm h/2} = 0, \ \tau_{zx}\Big|_{z=\pm h/2} = 0, \ \psi\Big|_{z=\pm h/2} = \pm U_0 \tag{7}$$

On the surface x=0, stand following integral conditions:

$$\sigma_x\Big|_{x=0} = 0, \qquad -2b\int_{-h/2}^{h/2} \tau_{xz}\Big|_{x=0} dz = F \tag{8}$$

Conditions for the fixed end of the cantilever for the frontal surface x=l are:

$$u(l,0) = 0, \quad w(l,0) = 0, \quad \frac{\partial w}{\partial z}\Big|_{\substack{x=l\\z=0}} = 0 \tag{9}$$

The proposed task is solved by application of reverse method, such that components u and w of the displacement vector $\vec{s}$ and electric potential ψ are assumed in a polynomial form:

$$\begin{aligned} u &= a_0 + a_1x + a_2z + a_3z^3 + a_4x^2z \\ w &= b_0 + b_1z + b_2x + b_3x^3 + b_4z^2x \\ \psi &= c_1z + c_2x + c_3xz^2 \end{aligned} \tag{10}$$

Equations of electrostatics in absence of free electric charges, i.e. simplified Maxwell's partial differential equations are [3]:

$$div\vec{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = 0$$
$$\vec{E} = -grad\psi = -\left(\frac{\partial \psi}{\partial x}\vec{i} + \frac{\partial \psi}{\partial y}\vec{j} + \frac{\partial \psi}{\partial z}\vec{k} \right) \tag{11}$$

Cauchy's kinematic equations are:

$$\varepsilon_x = \frac{\partial u}{\partial x}, \quad \varepsilon_y = \frac{\partial v}{\partial y}, \quad \varepsilon_z = \frac{\partial w}{\partial z}$$
$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \quad \gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, \tag{12}$$
$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}$$

By substituting the assumed solutions (10) into expressions (11) and (12), respectively, following expressions are obtained:

$$E_x = -c_2 - c_3 z^2, \quad E_y = 0,$$
$$E_z = -c_1 - 2c_3 xz, \quad \varepsilon_x = a_1 + 2a_4 xz,$$
$$\varepsilon_y = 0, \quad \varepsilon_z = b_1 + 2b_4 xz, \tag{13}$$
$$\gamma_{xz} = a_2 + b_2 + (a_4 + 3b_3)x^2 + (b_4 + 3a_3)z^2,$$
$$\gamma_{xy} = 0, \quad \gamma_{yz} = 0$$

Expressions for specific strains (dilatations and slides) and components of the piezoelectric displacement vector are [1]:

$$\varepsilon_x = \varepsilon_{11}^E \sigma_x + \varepsilon_{12}^E \sigma_y + \varepsilon_{13}^E \sigma_z + b_{31} E_z,$$
$$\varepsilon_y = \varepsilon_{12}^E \sigma_x + \varepsilon_{11}^E \sigma_y + \varepsilon_{13}^E \sigma_z + b_{31} E_z,$$
$$\varepsilon_z = \varepsilon_{13}^E (\sigma_x + \sigma_y) + \varepsilon_{33}^E \sigma_z + b_{33} E_z,$$
$$\gamma_{xy} = \varepsilon_{66}^E \tau_{xy} = 2(\varepsilon_{11}^E - \varepsilon_{12}^E)\tau_{xy}, \tag{14}$$
$$\gamma_{xz} = \varepsilon_{44}^E \tau_{xz} + b_{15} E_x, \quad \gamma_{yz} = \varepsilon_{44}^E \tau_{yz} + b_{15} E_y,$$
$$D_x = d_{11}^\sigma E_x + b_{15} \tau_{xz},$$
$$D_y = d_{11}^\sigma E_y + b_{15} \tau_{yz},$$
$$D_z = d_{33}^\sigma E_z + b_{31}(\sigma_x + \sigma_y) + b_{33}\sigma_z$$

where: D_x, D_y, D_z are components of the piezoelectric displacement vector in C/m^2; ε_{11}^E, ε_{12}^E, ε_{13}^E, ε_{33}^E, ε_{44}^E are coefficients of elastic power at given electric field in m^2/N; b_{31}, b_{15}, b_{33} are coordinates of piezomodulus tensor in C/N; d_{11}^σ, d_{33}^σ are dielectric constant (dielectric permeability) at given mechanical stress in F/m.

Thirteen unknown coefficients: a_0, a_1, a_2, a_3, a_4, b_0, b_1, b_2, b_3, b_4, c_1, c_2, c_3, which enter into expressions (10) and (13), have to be determined in order to fulfill the system of equations of electroelasticity (14) and boundary conditions (7), (8) and (9):

$$a_1 + 2a_4 xz = -\varepsilon_{11}^E \frac{F}{I_y} xz - c_1 b_{31} - 2c_3 b_{31} xz, \ \Big|_{\varepsilon_x}$$

$$b_1 + 2b_4 xz = -\varepsilon_{13}^E \frac{F}{I_y} xz - c_1 b_{33} - 2c_3 b_{33} xz, \ \Big|_{\varepsilon_z}$$

$$a_2 + b_2 + (a_4 + 3b_3)x^2 + (b_4 + 3a_3)z^2 = -\varepsilon_{44}^E \frac{F}{8I_y} h^2 + \varepsilon_{44}^E \frac{F}{2I_y} z^2 - b_{15} c_2 - c_3 b_{15} z^2, \ \Big|_{\gamma_{xz}}$$

$$D_x = -\left(\frac{Fb_{15}}{8I_y} h^2 + c_2 d_{11}^\sigma \right) + \left(\frac{Fb_{15}}{2I_y} - c_3 d_{11}^\sigma \right) z^2, \quad (15)$$

$$D_z = -c_1 d_{33}^\sigma - \left(\frac{Fb_{31}}{I_y} + 2c_3 d_{33}^\sigma \right) xz,$$

$$U_0 = c_1 \frac{h}{2} + c_2 x + c_3 x \frac{h^2}{4}, \ \Big|_{z=\frac{h}{2}}$$

$$-U_0 = -c_1 \frac{h}{2} + c_2 x + c_3 x \frac{h^2}{4}, \ \Big|_{z=-\frac{h}{2}}$$

$$u\Big|_{\substack{x=l \\ z=0}} = a_0 + a_1 l = 0,$$

$$w\Big|_{\substack{z=0 \\ x=l}} = b_0 + b_2 l + b_3 l^3 = 0, \quad (16)$$

$$\frac{\partial w}{\partial x}\Big|_{\substack{z=0 \\ x=l}} = b_2 + 3b_3 l^2 = 0,$$

$$-2c_3 d_{33}^\sigma - b_{31} \frac{F}{I_y} = 0, \ \Big|_{div \vec{D}=0}$$

that is:

$$a_1 = -b_{31} c_1, \quad 2a_4 = -\varepsilon_{11}^E \frac{F}{I_y} - 2b_{31} c_3,$$

$$b_1 = -b_{33} c_1, \quad 2b_4 = -\varepsilon_{13}^E \frac{F}{I_y} - 2b_{33} c_3,$$

$$a_2 + b_2 = -\varepsilon_{44}^E \frac{F}{8I_y} h^2 - b_{15} c_2, \quad (17)$$

$$b_4 + 3a_3 = \varepsilon_{44}^E \frac{F}{2I_y} - b_{15}c_3,$$

$$a_4 = -3b_3, \quad c_2 = -c_3 \frac{h^2}{4}$$

From the system of equations (15), (16) and (17) unknown coefficients are determined as:

$$\begin{gathered}
a_0 = \frac{2U_0}{h} b_{31} l, \quad a_1 = -\frac{2U_0}{h} b_{31}, \\
a_2 = \frac{F\varepsilon_{11}^E}{2I_y}(1-k_{31}^2) \quad l^2 - \frac{F\varepsilon_{44}^E}{8I_y} h^2 (1-k_\tau^2), \\
a_3 = \frac{F\varepsilon_{13}^E}{6I_y}(1-k_S^2) \quad + \frac{F\varepsilon_{44}^E}{6I_y}(1-k_\tau^2), \\
a_4 = -\frac{F\varepsilon_{11}^E}{2I_y}(1-k_{31}^2), \quad b_0 = \frac{2}{3}\frac{F\varepsilon_{11}^E}{2I_y}(1-k_{31}^2) l^3, \\
b_1 = -\frac{2U_0}{h} b_{33}, \quad b_2 = -\frac{F\varepsilon_{11}^E}{2I_y}(1-k_{31}^2) l^2, \\
b_3 = \frac{1}{3}\frac{F\varepsilon_{11}^E}{2I_y}(1-k_{31}^2), \quad b_4 = -\frac{F\varepsilon_{13}^E}{2I_y}(1-k_S^2), \\
c_1 = \frac{2U_0}{h}, \quad c_2 = \frac{F}{2I_y}\frac{b_{31}}{d_{33}^\sigma}\frac{h^2}{4}, \\
c_3 = -\frac{F}{2I_y}\frac{b_{31}}{d_{33}^\sigma}, \quad k_{31}^2 = \frac{b_{31}^2}{\varepsilon_{11}^E d_{33}^\sigma}, \\
k_S^2 = \frac{b_{31} b_{33}}{\varepsilon_{13}^E d_{33}^\sigma}, \quad k_\tau^2 = -\frac{b_{15} b_{31}}{\varepsilon_{44}^E d_{33}^\sigma}
\end{gathered} \tag{18}$$

Coefficients k_{31}^2, k_s^2 and k_τ^2 are called coefficients of electromechanical static relations.

By introducing the obtained values for coefficients (18) into expressions (10) and (13), one obtains solutions for: componential displacements of the displacement vector $\vec{s}$, electric potential ψ, specific strains (dilatations ε_x and ε_z, and slide y_{xz}), electric fields E_x and E_z, and piezoelectric displacements D_x and D_z, for the rectangular prismatic cantilever with transversal polarization and electrode coatings on the sides z=±h/2, in the form of:

$$u = \frac{2U_0}{h} b_{31}(l-x) + \frac{F}{2I_y}\left\{\left[\varepsilon_{11}^E (1-k_{31}^2) l^2 - \varepsilon_{44}^E \frac{h^2}{4}(1-k_\tau^2)\right] z + \right.$$
$$\left. + \frac{1}{3}\left[\varepsilon_{13}^E (1-k_S^2) + \varepsilon_{44}^E (1-k_\tau^2)\right] z^3 - \varepsilon_{11}^E (1-k_{31}^2) x^2 z\right\}, \tag{19}$$

$$w = -\frac{2U_0}{h} b_{33} z + \frac{F}{2I_y} \left\{ \varepsilon_{11}^E (1-k_{31}^2) \left[\frac{2}{3} l^3 - l^2 x + \frac{1}{3} x^3 \right] - \varepsilon_{13}^E (1-k_S^2) z^2 x \right\},$$

$$\psi = \frac{2U_0}{h} + \frac{F}{2I_y} \frac{b_{31}}{d_{33}^\sigma} \left(\frac{h^2}{4} - z^2 \right) x,$$

$$\varepsilon_x = -\frac{2U_0}{h} b_{31} - \frac{F \varepsilon_{11}^E}{I_y} (1-k_{31}^2) xz,$$

$$\varepsilon_z = -\frac{2U_0}{h} b_{33} - \frac{F \varepsilon_{13}^E}{I_y} (1-k_S^2) xz,$$

$$\gamma_{xz} = -\frac{F \varepsilon_{44}^E}{8I_y} (1-k_\tau^2)(h^2 - 4z^2), \qquad (20)$$

$$E_x = -\frac{F}{8I_y} \frac{b_{31}}{d_{33}^\sigma} (h^2 - 4z^2),$$

$$E_z = -\frac{2U_0}{h} + \frac{F}{I_y} \frac{b_{31}}{d_{33}^\sigma} xz,$$

$$D_x = \frac{F}{2I_y} \left(b_{15} + \frac{b_{31}}{d_{33}^\sigma} d_{11}^\sigma \right) \left(z^2 - \frac{h^2}{4} \right),$$

$$D_z = -\frac{2U_0}{h} d_{33}^\sigma$$

3. Numerical Analysis And Discussion

Subject of observation in this paper is stressing of rectangular PZT4 piezoceramic cantilever [9], with the following dimensions: b=4.1mm, h=20.1mm and l=30.1mm, density ρ=7500kg/m^3, loaded by the concentrated force (Fig. 1). This material belongs to the hexagonal crystal system of crystal class 6mm (C_{6v}), and its material tensors are: matrix of elastic power constants tensor, matrix of piezomodulus tensor, and matrix of dielectric constants tensor, presented respectively as follows:

$$[b_{ij}] = \begin{bmatrix} 0 & 0 & 0 & 0 & b_{15} & 0 \\ 0 & 0 & 0 & b_{15} & 0 & 0 \\ b_{31} & b_{31} & b_{33} & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 496 & 0 \\ 0 & 0 & 0 & 496 & 0 & 0 \\ -123 & -123 & 289 & 0 & 0 & 0 \end{bmatrix} \cdot 10^{-12} \quad \left[\frac{C}{N} \right] \qquad (21)$$

$$[d_{ij}^{\sigma}]=\begin{bmatrix} d_{11}^{\sigma} & 0 & 0 \\ 0 & d_{11}^{\sigma} & 0 \\ 0 & 0 & d_{33}^{\sigma} \end{bmatrix}=\begin{bmatrix} 13{,}05 & 0 & 0 \\ 0 & 13{,}05 & 0 \\ 0 & 0 & 11{,}5 \end{bmatrix}\cdot 10^{-9}\left[\frac{F}{m}\right] \tag{22}$$

$$[\varepsilon_{ij}^{E}]=\begin{bmatrix} \varepsilon_{11}^{E} & \varepsilon_{12}^{E} & \varepsilon_{13}^{E} & 0 & 0 & 0 \\ \varepsilon_{12}^{E} & \varepsilon_{11}^{E} & \varepsilon_{13}^{E} & 0 & 0 & 0 \\ \varepsilon_{13}^{E} & \varepsilon_{13}^{E} & \varepsilon_{33}^{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \varepsilon_{44}^{E} & 0 & 0 \\ 0 & 0 & 0 & 0 & \varepsilon_{44}^{E} & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(\varepsilon_{11}^{E}-\varepsilon_{12}^{E}) \end{bmatrix}= \tag{23}$$

$$\begin{bmatrix} 12{,}3 & -4{,}05 & -5{,}31 & 0 & 0 & 0 \\ -4{,}05 & 12{,}3 & -5{,}31 & 0 & 0 & 0 \\ -5{,}31 & -5{,}31 & 15{,}5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 39 & 0 & 0 \\ 0 & 0 & 0 & 0 & 39 & 0 \\ 0 & 0 & 0 & 0 & 0 & 32{,}7 \end{bmatrix} 10^{-12}\left[\frac{m^2}{N}\right]$$

Based on the obtained solutions (19) and (20), numerical analysis was performed using Matlab software package, and biparametric surfaces of spatial state were obtained for componential displacement $u(x,z,F,U)$, componential displacement $w(x,z,F,U)$, electric potential $\psi(x,z,F,U)$, specific strain – dilatation $\varepsilon_x(x,z,F,U)$, specific strain – dilatation $\varepsilon_z(x,z,F,U)$, specific strain – slide $y_{xz}(z,F)$, electric field $E_x(z,F)$, electric field $E_z(x,z,F,U)$, and piezoelectric displacement $D_x(z,F)$. Due to the limited space, only few of the obtained results are presented in the following text.

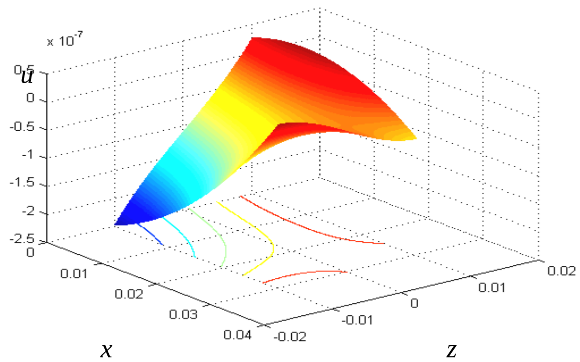

Fig. 2 Componential displacement $u=u(z, x)$

Fig. 2 shows biparametric surface of the componential displacement $u=u(z,x)$ in function of coordinate z and coordinate x, at dominant electric voltage U_0. Componential displacement has extreme values in points of the surface for $z=\pm h/2$ and in points of the frontal surface for $x=0$, while in the points of the fixed end cross section, for $x=l$, its value is equal to zero.

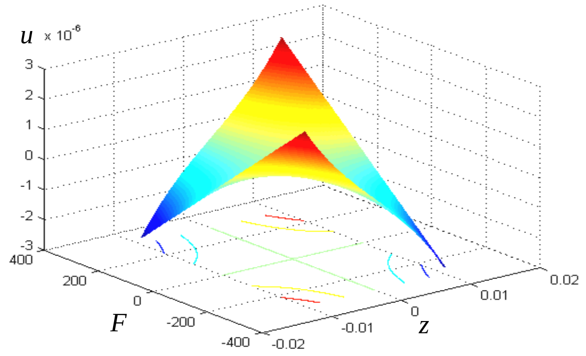

Fig. 3 Componential displacement $u=u(z, F)$

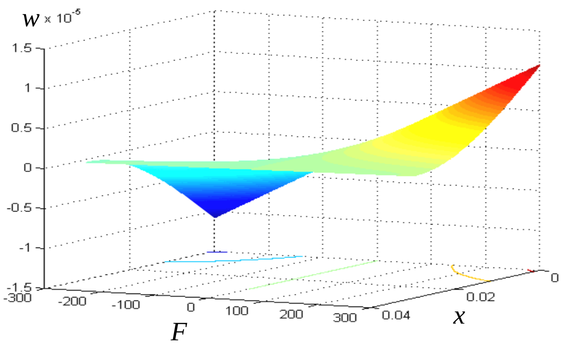

Fig. 4 Componential displacement $w=w(x, F)$

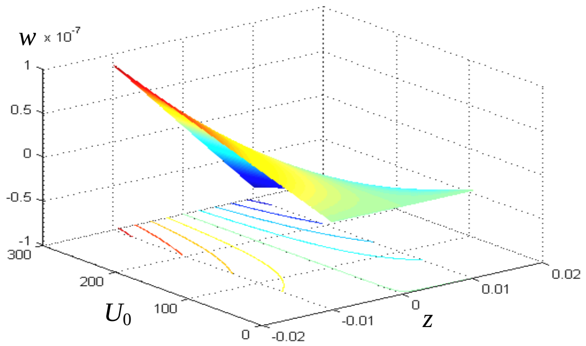

Fig. 5 Componential displacement $w=w(z, U_0)$

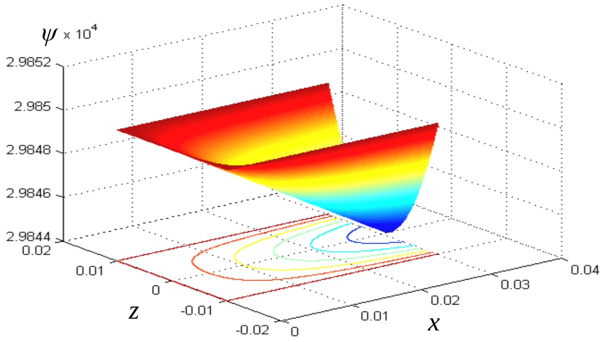

Fig. 6 Electric potential $\psi=\psi(x, z)$

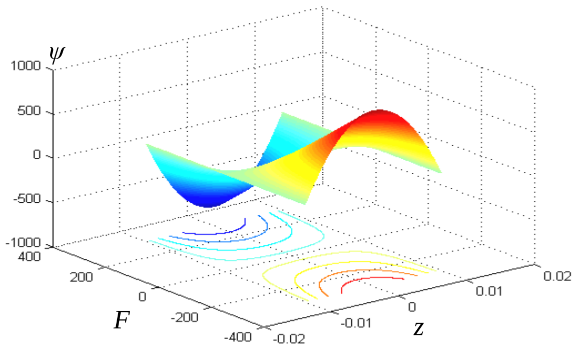

Fig. 7 Electric potential $\psi=\psi(z, F)$

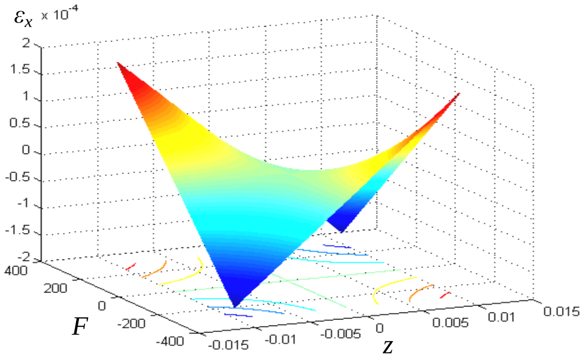

Fig. 8 Specific strain $\varepsilon_x=\varepsilon_x(z, F)$

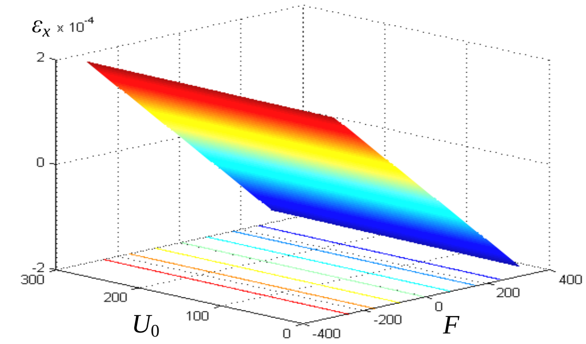

Fig. 9 Specific strain $\varepsilon_x=\varepsilon_x(F, U_0)$

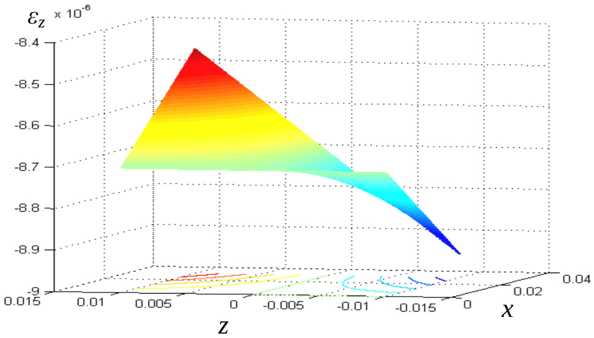

Fig. 10 Specific strain $\varepsilon_z=\varepsilon_z(x, z)$

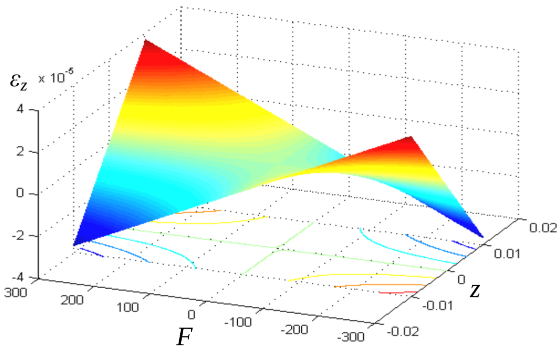

Fig. 11 Specific strain $\varepsilon_z=\varepsilon_z(z, F)$

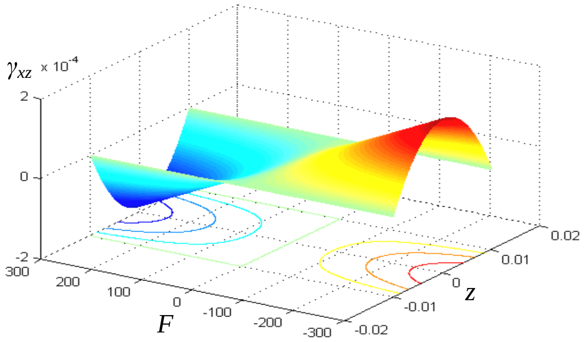

Fig. 12 Specific strain $\gamma_{xz}=\gamma_{xz}(z, F)$

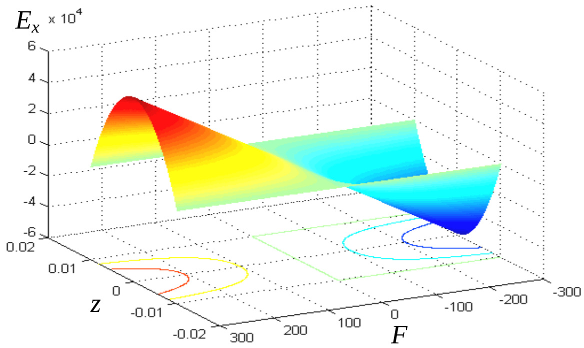

Fig. 13 Electric field $E_x=E_x(F, z)$

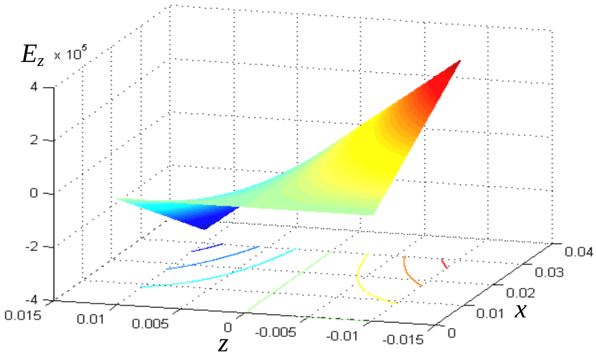

Fig. 14 Electric field $E_z=E_z(x, z)$

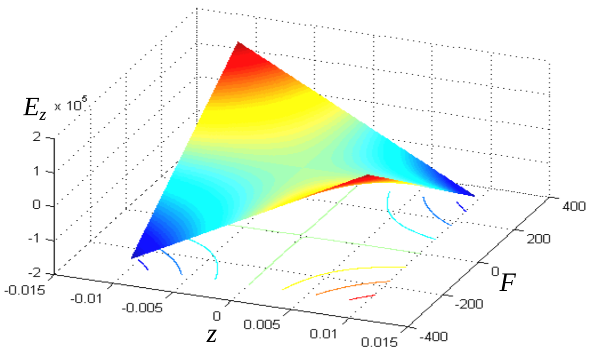

Fig. 15 Electric field $E_z=E_z(F, z)$

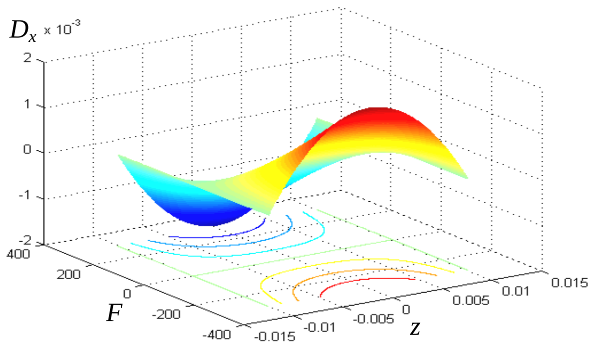

Fig. 16 Piezoelectric displacement $D_x=D_x(z, F)$

Biparametric surface of the componential displacement $u=u(z,F)$, in function of coordinate z and external concentrated force F, is presented on Fig. 3. Componential displacement has characteristic spatial surface in shape of a saddle. Extreme values of the componential displacement are achieved in surface points for $z=\pm h/2$, at maximum values of the concentrated force $\pm F$.

On Fig. 4 biparametric surface of the componential displacement $w=w(x,F)$ in function of coordinate x and external concentrated force F is presented. Componential displacement has extreme values for maximum intensity of the concentrated force $\pm F$ and in points of the frontal surface for $x=0$, while in the points of the fixed end cross section, for $x=l$, its value is equal to zero.

Biparametric surface of the componential displacement $w=w(z,U_0)$, in function of coordinate z and electric voltage U_0, is shown on Fig. 5.

Fig. 6 shows biparametric surface of the electric potential $\psi=\psi(x,z)$ in function of coordinate x and coordinate z. Electric potential has minimum value in point of frontal surface for $x=l$ when $z=0$.

In Fig. 7 biparametric surface of the electric potential $\psi=\psi(z,F)$, in function of coordinate z and external concentrated force F is shown. Extreme values of the electric potential are obtained in the sectional plane for $z=0$ and at maximum intensity of the force $\pm F$.

Fig. 8 illustrates biparametric surface of the specific saddle shaped strain, $\varepsilon_x=\varepsilon_x(z,F)$ in function of coordinate z and external concentrated force F.

Biparametric planar surface of the specific strain $\varepsilon_x=\varepsilon_x(F,U_0)$, in function of concentrated force F, and electric voltage U_0, is shown on Fig. 9.

On Fig. 10 biparametric surface of the specific strain $\varepsilon_z=\varepsilon_z(x,z)$ in function of coordinate x and coordinate z is presented.

Biparametric surface of the specific strain $\varepsilon_z=\varepsilon_z(z,F)$, in function of coordinate z and external concentrated force F, is illustrated in Fig. 11.

Fig. 12 illustrates biparametric surface of the specific strain $y_{xz}=y_{xz}(z,F)$ in function of coordinate z and external concentrated force F.

Fig. 13 shows biparametric surface of the electric field $E_x=E_x(F,z)$, in function of external concentrated force F and coordinate z.

On Fig. 14 is shown biparametric surface of the electric field $E_z=E_z(x,z)$ in function of coordinate x and coordinate z.

Biparametric saddle shaped surface of the electric field $E_z=E_z(F,z)$, in function of external concentrated force F and coordinate z, is presented on Fig. 15.

Biparametric surface of the piezoelectric displacement $D_x=D_x(z,F)$ in function of coordinate z and external concentrated force F is shown on Fig. 16. Extreme values of the piezoelectric displacement are obtained in the sectional points for $z=0$ and at maximum values of the concentrated force $\pm F$.

3. Conclusion

In solving problems of the theory of electroelasticity, for a general case of stressing three-dimensional electroelastic deformable bodies, one is faced with great mathematical problems. In this paper, the entire qualitative picture of stressed state of the loaded rectangular prismatic piezoceramic cantilever with transversal polarization and electrode coatings has been observed. For the particular piezoceramic cantilever, different state diagrams numerically processed with a PC were determined and presented. This kind of analysis enables to predict the characteristics of piezoceramic cantilevers with analyzed configuration before their construction. It is expected that the obtained solutions for this kind of task from the theory of oscillations can directly be applied in engineering practice.

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PROBLEM NAPREZANJA PIEZOKERAMIČKIH KONZOLA SA ELEKTRODNIM PREVLAKAMA I POPREČNOM POLARIZACIJOM

U ovom radu razmatra se opšti slučaj naprezanja pravougaone prizmatrične piezokeramičke konzole sa poprečnom polarizacijom, opterećene na slobodnom kraju koncentrisanom silom. Dve međusobno suprotne površi pravougaone konzole su sa elektrodnim prevlakama na koje se dovodi električni napon. Primenom obratne metode za rešavanje problema elektroelastične teorije određuju se električni potencijal, specifične deformacije, električna polja i piezoelektrični pomeraji za pravougaonu piezokeramičnu konzolu napravljenu od PZT4 piezokeramičkog materijala.

Ključne reči: *naprezanje, pravougaona piezokeramička konzola, poprečna polarizacija, PZT4 piezokeramički materijal*